New Results on Finite Dimensional Spin Glasses

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Phys. Rev. E 89. 032140 (2014)
and
J. Stat. Mech. P05014 (2014)

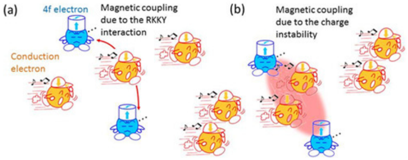
Plan of the Talk

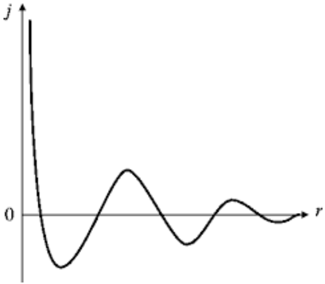
- What are spin glasses?
- Different Theories: Droplet/Scaling and RSB.
- Experiments.
- Results for the four dimensional Edwards-Anderson in a field.
- Some results for the three dimensional Edwards-Anderson in a field.
- Conclusions and open problems.

What are Spin glasses

- Materials with disorder and frustration.
- Quenched disorder (similar to the Born-Oppenheimer in Molecular Physics).
- Canonical Spin Glass: Metallic host (Cu) with magnetic impurities (Mn).
- RKKY interaction between magnetic moments: $J(r) \sim \frac{\cos(2k_F r)}{r^3}$.
- Role of anisotropy: Ag:Mn at 2.5% (Heisenberg like), $CdCr_{1.7}IN_{0.3}S_4$ (also Heisenberg like) and $Fe_{0.5}Mn_{0.5}TiO_3$ (Ising like).

RKKY interaction



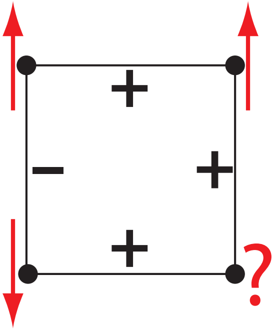


Simplification: We take the J_{ij} as random variables! For instance, Gaussian or from a bimodal probability distribution.

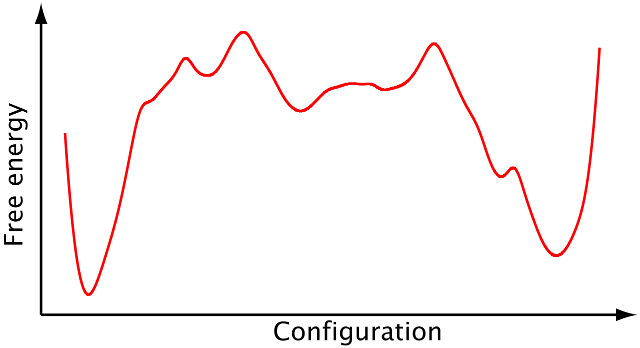
Frustration

Energy:

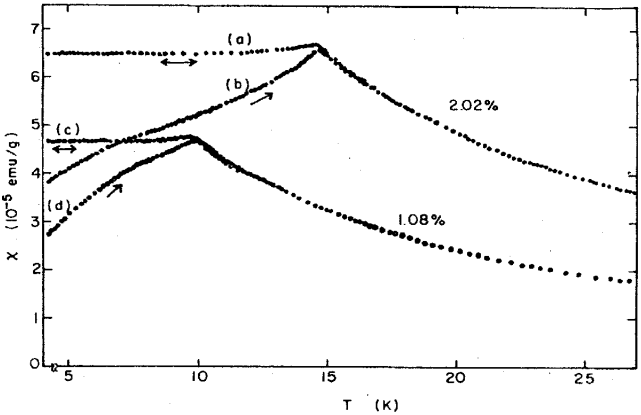
$$E = -\sum_{<ij>} J_{ij} S_i S_j$$



Free energy landscape



Some experiment (ZF and F-cooled susceptibility)



Quenched and Annealed Disorder

- We compute the partition function for a given realization of the couplings:

$$\mathcal{Z}_J = \sum_{[S_i]} \exp(-\beta \mathcal{H}_J) = \sum_{[S_i]} \exp(\beta \sum_{ij} J_{ij} S_i S_j)$$

- Next, we compute its free energy:

$$F_J = -\frac{1}{\beta} \log \mathcal{Z}_J$$

- Finally, we average the different free energies

$$F = \overline{F_J} = \int d[J] p[J] F_J$$

- Annealed average:

$$F = -\frac{1}{\beta} \log \int d[J] p[J] \sum_{[S_i]} \exp(-\beta \mathcal{H}_{\mathcal{J}})$$

Mini-review on Phase Transitions (Ferromagnets)

- Order parameter: Magnetization

$$M(T) =< S(r) >= \begin{cases} (T_c - T)^\beta & T \lesssim T_c \\ 0 & T > T_c \end{cases}$$

- Correlation function (leading behavior):

$$G(r,T) =< S(r)S(0) >\propto \begin{cases} \frac{1}{r^{(D-1)/2}} \exp(-r/\xi(T)) & T > T_c \\ \frac{1}{r^{D-2+\eta}} & T = T_c \\ M(T)^2 + O(\exp(-r/\xi(T))) & T < T_c \end{cases}$$

- Correlation length

$$\xi(T) = |T - T_c|^{-\nu}$$

- Magnetic Susceptibility

$$\chi(T) = |T - T_c|^{-\gamma}$$

- Specific Heat

$$C(T) = |T - T_c|^{-\alpha}$$

- Only two critical exponents are independent, e.g, ν and η.

- The magnetization is zero ($\forall T$).

$$M = \overline{\langle S(r) \rangle} = 0$$

- All other possible definitions of the magnetization are also zero:

$$M_{\boldsymbol{k}} = \frac{1}{V} \sum_{\boldsymbol{r}} \overline{\langle S(r) \rangle} \exp(i\boldsymbol{k}\boldsymbol{r})$$

- The order parameter is

$$q_{\mathrm{EA}} = \overline{\langle S(r) \rangle^2}$$

Some equations

- Edwards-Anderson Hamiltonian:

$$\mathcal{H} = -\sum_{<ij>} J_{ij}\sigma_i\sigma_j + h\sum_i \sigma_i$$

J_{ij} are random quenched variables with zero mean and unit variance, $\sigma = \pm 1$ are Ising spins.

- The order parameter is:

$$q_{\mathrm{EA}} = \overline{\langle \sigma_i \rangle^2}$$

Using two real replicas:

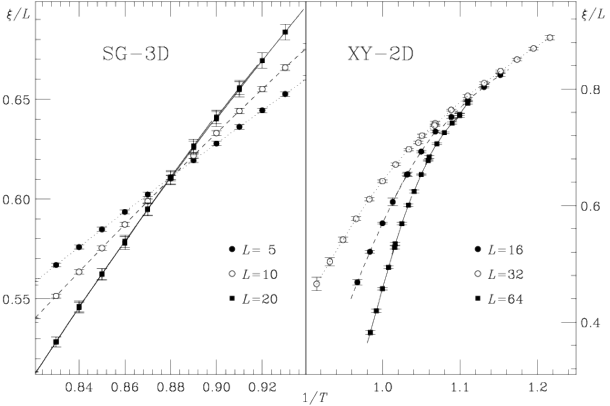
$$\mathcal{H} = -\sum_{<ij>} J_{ij}\left(\sigma_i\sigma_j + \tau_i\tau_j\right) + h\sum_i \sigma_i + h\sum_i \tau_i$$

Let $q_i = \sigma_i\tau_i$ be the normal overlap, then: $q_{\mathrm{EA}} = \overline{\langle \sigma_i\tau_i \rangle}$.

Phase transition: Finite size scaling

$$G_1(\boldsymbol{r}) = \frac{1}{V}\sum_{\boldsymbol{x}} \overline{\langle S_{\boldsymbol{x}} S_{\boldsymbol{x}+\boldsymbol{r}}\rangle^2}\ ,\ \xi_2 = \frac{1}{2\sin(\pi/L)}\left(\frac{\hat{G}(0)}{\hat{G}(\boldsymbol{k}_1)} - 1\right)^{1/2}$$

where $\boldsymbol{k}_1 = (2\pi/L, 0, 0)$ (and two perm.)



Different Theories.

The Droplet/Scaling Theory.

- Based on the Migdal-Kadanoff implementation (approximate) of the Renormalization Group (exact in $D = 1$).
- *Disguised Ferromagnet*: Only two pure states with order parameter $\pm q_{\mathrm{EA}}$ (related by spin flip).
- Compact Excitations of fractal dimension d_f. The energy of a excitation of linear size L grows as L^θ. The free energy barriers (in the dynamics) grow as L^ψ. $\theta < (D-1)/2 < D-1 < d_f < D$ and $\psi \geq \theta$.
- Any amount of magnetic field destroys the spin glass phase (even for Heisenberg spin glasses).
- Trivial probability distributions of the overlaps (both normal overlap and link one).

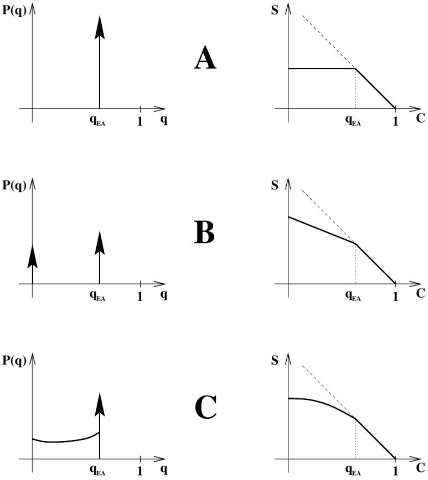
Different Theories.

Replica Symmetry Breaking (RSB) Theory.

- Exact in $D = \infty$.
- Infinite number of phases (pure states) not related by any kind of symmetry.
- These (pure) states are organized in a ultrametric fashion.
- The spin glass phase is stable under (small) magnetic field. Phase transition in field: the de Almeida-Thouless line.
- The excitations of the ground state are space filling: e.g. the interface between two pure states is space filling.
- Overlap equivalence: All the definitions of the overlap are equivalent.

Note: In a pure state, α, the clustering property holds: $\langle S_i S_j \rangle_\alpha - \langle S_i \rangle_\alpha \langle S_j \rangle_\alpha \to 0$ as $|i - j| \to \infty$.

Different Theories (Comparison).



Different Theories

- Field Theory of the Replica Symmetry Breaking (RSB) Theory.
- Hamiltonian (Action)

$$S_n = H_n = \int d^D x \left[(\partial_\mu Q_{ab})^2 + \tau \mathrm{Tr} Q^2 + g_3 \mathrm{Tr}(Q^3) + \lambda \sum Q_{ab}^4 \right]$$

$a, b = 1, ..., n$. At the end, $n \to 0$! (The replica trick)

- Propagator ($T > T_c$):

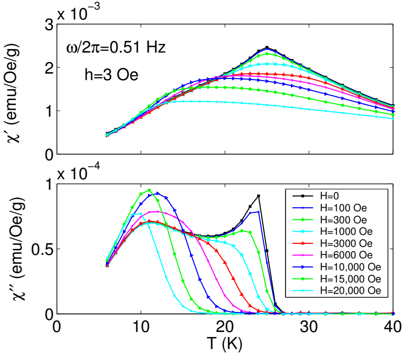
$$G(p) = \frac{1}{p^2 + m^2}$$

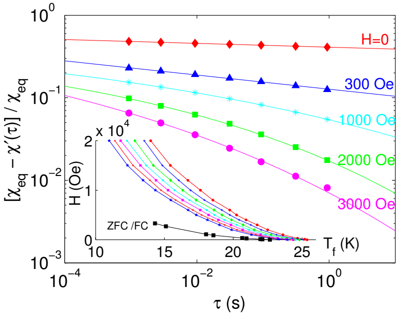
- Propagator ($T = T_c$, λ is irrelevant: ϕ^3 theory, and the upper critical dimensions is $D = 6$):

$$G(p) = \frac{1}{p^{2-\eta}}$$

Experiments

- Experimental data from $Fe_{0.5}Mn_{0.5}TiO_3$ (Jönsson et al.).

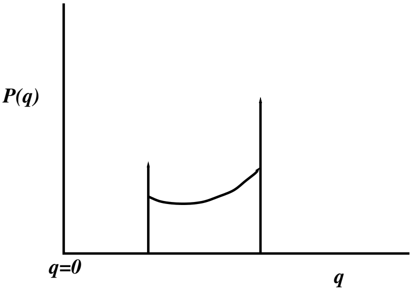


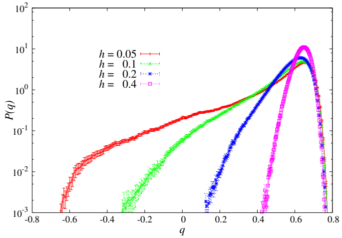


- $q(t) \simeq 1/t^x$ clear signature of a Spin Glass Phase (Ogileski).

The negative overlap problem

- $P(q)$ in a magnetic field: SK results and numerical ones.





- The negative overlap region induces large corrections in $\tilde{G}(0)$!!

The correlation length

- Correlation Functions ($D = 4$): The replicon Propagator:

$$
\begin{aligned}
G_1(\boldsymbol{r}) &= \frac{1}{L^4} \sum_{\boldsymbol{x}} \overline{\left(\langle S_{\boldsymbol{x}} S_{\boldsymbol{x}+\boldsymbol{r}} \rangle - \langle S_{\boldsymbol{x}} \rangle \langle S_{\boldsymbol{x}+\boldsymbol{r}} \rangle \right)^2} \,, \\
G_2(\boldsymbol{r}) &= \frac{1}{L^4} \sum_{\boldsymbol{x}} \overline{\left(\langle S_{\boldsymbol{x}} S_{\boldsymbol{x}+\boldsymbol{r}} \rangle^2 - \langle S_{\boldsymbol{x}} \rangle^2 \langle S_{\boldsymbol{x}+\boldsymbol{r}} \rangle^2 \right)} \,.
\end{aligned}
$$

- Correlation Length:

$$
\xi_2 = \frac{1}{2 \sin(\pi/L)} \left(\frac{\hat{G}(0)}{\hat{G}(\boldsymbol{k}_1)} - 1 \right)^{1/2} ,
$$

where $\boldsymbol{k}_1 = (2\pi/L, 0, 0, 0)$ (and three perm.)

A new observable R_{12}

- R_{12}:

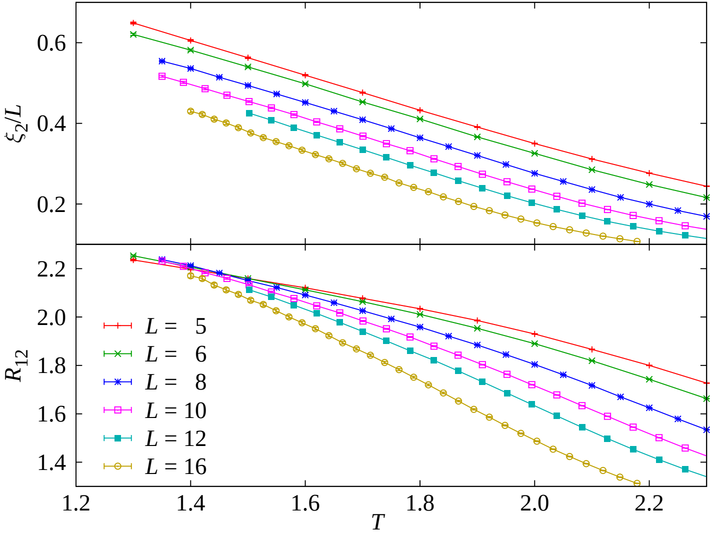
$$R_{12} = \frac{\hat{G}(\boldsymbol{k}_1)}{\hat{G}(\boldsymbol{k}_2)},$$

  where $\boldsymbol{k}_1 = (2\pi/L, 0, 0, 0)$, $\boldsymbol{k}_2 = (2\pi/L, 2\pi/L, 0, 0)$ (and permutations)
- We have checked the behavior of this observable in the EA model in $D = 3$ and $D = 4$ ($h = 0$).
- And in the two dimensional (ordered) Ising model. We have been able to compute its value at criticality using Conformal Field Theory:

$$R_{12} = 1.694\ 024...$$

- In a paramagnetic phase, for large L: $R_{12} \to 1$.

$D = 4$ $(h = 0.15)$



0.6
0.4
0.2
ξ_2/L
2.2
2.0
1.8
1.6
1.4
R_{12}
L = 5
L = 6
L = 8
L = 10
L = 12
L = 16
1.2
1.4
1.6
1.8
2.0
2.2
T

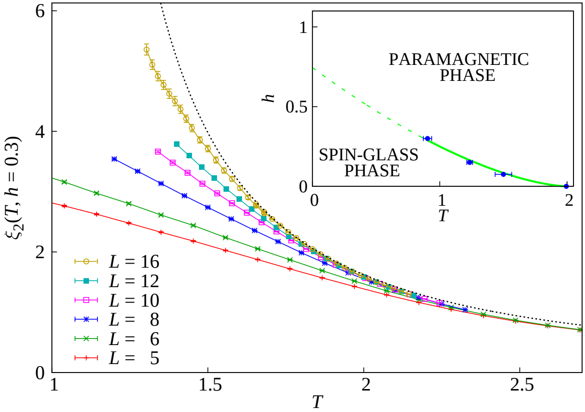
$D = 4$ ($h \neq 0$): Critical exponents

Parameter	$h = 0.3$	$h = 0.15$	$h = 0.075$
$T_c(h)$	0.906(40)[3]	1.229(30)[2]	1.50(7)
ν	1.46(7)[6]		—
η	−0.30(4)[1]		—
ω	1.43(37)		—

For reference ($h = 0$):

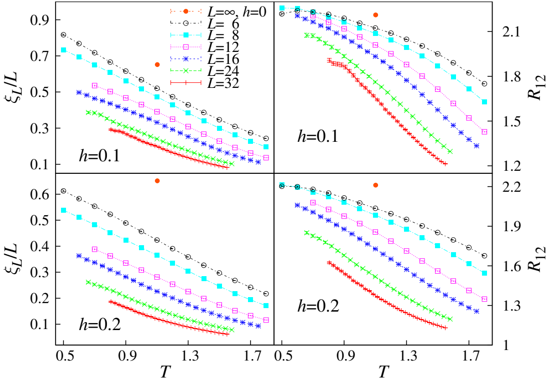
$T_c^{(0)} = 2.03(3)$, $\nu^{(0)} = 1.025(15)$, $\eta^{(0)} = -0.275(25)$

$D = 4$ $(h \neq 0)$: Summary



Fisher-Sompolinsky relation: $h^2(T) \simeq A|T - T_c^{(0)}|^{\beta^{(0)}+\gamma^{(0)}}$

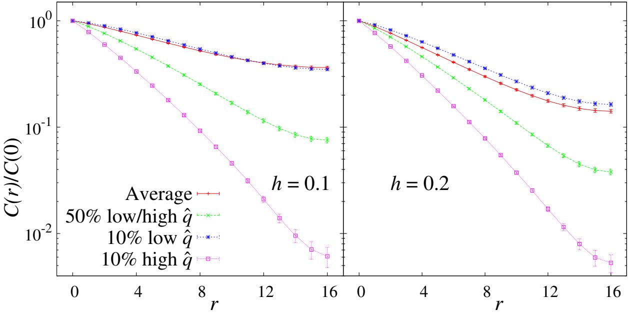
Spin Glass behavior in $D = 3$ $(h \neq 0)$?



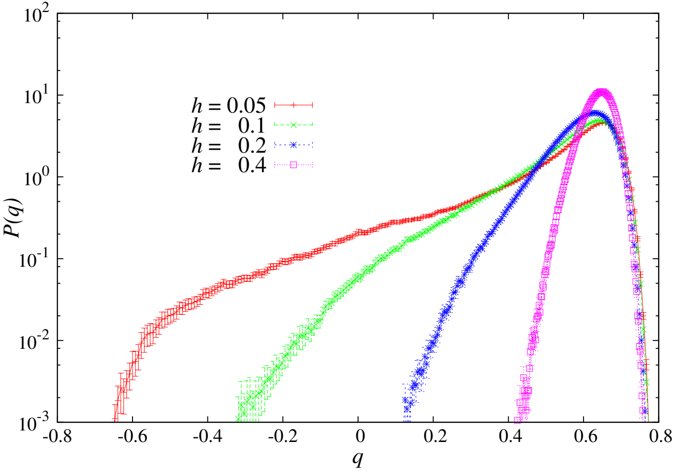
No signal of a phase transition in the ξ_L/L and R_{12}-channels!

The *fauna* of measurements $D = 3$ ($h \neq 0$)?

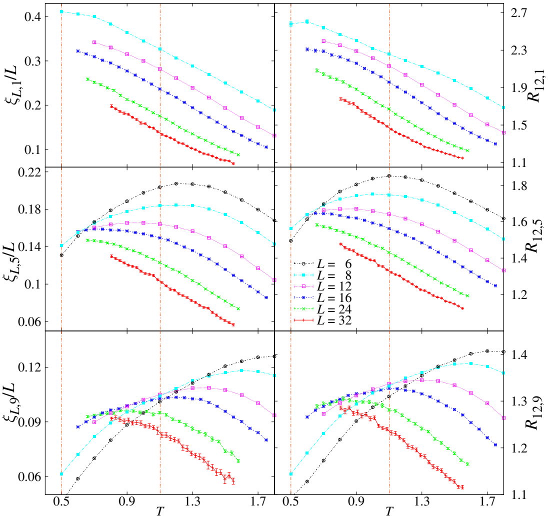
Study of the point-to-plane correlation function $C(r)$:



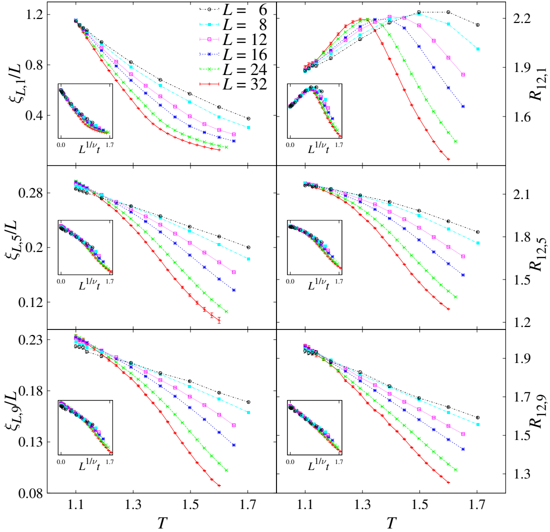
Quantile analysis in $D = 3$ ($h = 0.2$)



Quantile analysis in $D = 3$ $(h = 0.2)$



Test: Quantile analysis in $h = 0$



Conclusions

1. We have shown strong numerical evidences which support a dAT line below the upper critical dimension:
   - In $D = 4$ for the EA model.
2. However the situation in $D = 3$ dimensions is not yet clear:
   - Quantile analysis (equilibrium) shows traces of a phase transition.
   - But, will this picture (quantiles) survive for larger lattice sizes?
   - **Maybe Janus-II will be able to provide the solution!**