Numerical characterization of the spin glass phase: a historical account

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Plan of the Talk

- Different Theories: Droplet/Scaling and RSB.
- Numerical studies of the phase transition in finite dimensional spin glass (EA) ($h = 0$).
- Numerical characterization of the spin glass phase in finite dimensions ($h = 0$): The replicon.
- Phase Transition on the Edwards-Anderson in a field ($D = 3$ and 4)?
- Conclusions/Open Problems.

Some related talks and posters

- Numerical simulations in the Sherrington-Kirkpatrick model (A. Maiorano's talk).
- Metastate (P. Young's talk).
- Behavior in presence of a Magnetic field (N. Read's talk).
- Numerical simulations in $2D$ spin glasses (M. Weigel's talk).
- Numerical Simulations in spin glass in a film geometry (I. Paga's poster).

Different Theories.

The Droplet/Scaling Theory.

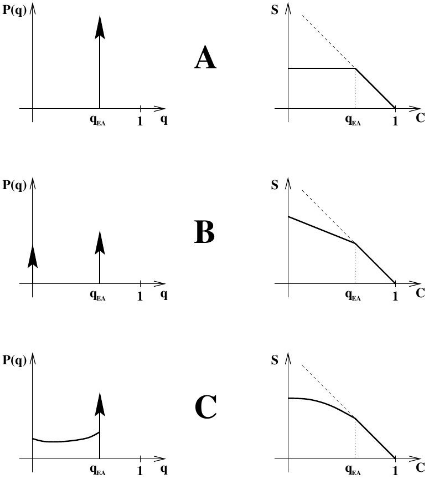
- Based on the Migdal-Kadanoff implementation (approximate) of the Renormalization Group (exact in $D = 1$).
- *Disguised Ferromagnet*: Only two pure states with order parameter $\pm q_{\text{EA}}$ (related by spin flip).
- Compact Excitations of fractal dimension d_f. The energy of a excitation of linear size L grows as L^θ. The free energy barriers (in the dynamics) grow as L^ψ. $\theta < (D-1)/2 < D-1 < d_f < D$ and $\psi \geq \theta$.
- Any amount of magnetic field destroys the spin glass phase (even for Heisenberg spin glasses).
- Trivial probability distributions of the overlaps (both normal overlap and link one).

Different Theories.

Replica Symmetry Breaking (RSB) Theory.

- Exact in $D = \infty$.
- Infinite number of pure states not related by any kind of symmetry.
- These pure states are organized in a ultrametric fashion.
- The spin glass phase is stable under (small) magnetic field. Phase transition in field: the de Almeida-Thouless line.
- The excitations of the ground state are space filling: e.g. the interface between two pure states is space filling.
- Overlap equivalence: All the definitions of the overlap are equivalent.

Different Theories (Comparison).



Different Theories

- Field Theory of the Replica Symmetry Breaking (RSB) Theory.
- Hamiltonian (Action)

$$S_n = H_n = \int d^D x \left[(\partial_\mu Q_{ab})^2 + \tau \mathrm{Tr} Q^2 + g_3 \mathrm{Tr}(Q^3) + \lambda \sum Q_{ab}^4 \right]$$

$a, b = 1, ..., n$. At the end, $n \to 0$! (The replica trick)

- Propagator ($T > T_c$):

$$G(p) = \frac{1}{p^2 + m^2}$$

- Propagator ($T = T_c$, λ is irrelevant: ϕ^3 theory, and the upper critical dimensions is $D = 6$):

$$G(p) = \frac{1}{p^{2-\eta}}$$

Different Theories.

- Propagators (Parisi Matrix, Dominicis, Kondor and Temesvári) ($T < T_c$ and λ is relevant):

$$G_q(r) \simeq q^2 + A(q) r^{-\theta(q)}$$

  where
  - $\theta(q_M) = D - 2$.
  - $\theta(q) = D - 3$ for $q_M > q > q_m$. This result should be modified below $D = 6$.
  - $\theta(q_m) = D - 4$.
- In the droplet/scaling Theory:

$$G(r) \simeq q_{EA}^2 + A r^{-\theta}$$

  where θ is the standard droplet exponent.

Some equations

- Edwards-Anderson Hamiltonian:

$$\mathcal{H} = -\sum_{<ij>} J_{ij}\sigma_i\sigma_j$$

J_{ij} are random quenched variables with zero mean and unit variance, $\sigma = \pm 1$ are Ising spins.

- The order parameter is:

$$q_{\mathrm{EA}} = \overline{\langle \sigma_i \rangle^2}$$

Using two real replicas:

$$\mathcal{H} = -\sum_{<ij>} J_{ij}\left(\sigma_i\sigma_j + \tau_i\tau_j\right)$$

Let $q_i = \sigma_i\tau_i$ be the normal overlap, then: $q_{\mathrm{EA}} = \overline{\langle \sigma_i\tau_i \rangle}$.
We also define the link overlap: $q^l_{i,\mu} = q_i q_{i+\mu}$.

Observables

- Correlation Function and susceptibility

$$G(\boldsymbol{r}) = \frac{1}{L^3} \sum_{\boldsymbol{x}} \overline{\langle S_{\boldsymbol{x}} S_{\boldsymbol{x}+\boldsymbol{r}} \rangle^2}\,, \quad \chi = \int G(\boldsymbol{r}) d^3 r\,.$$

- Binder Cumulant:

$$U_4 = \frac{\overline{\langle q^4 \rangle}}{(\overline{\langle q^2 \rangle})^2}\,.$$

  Universal at the critical point.

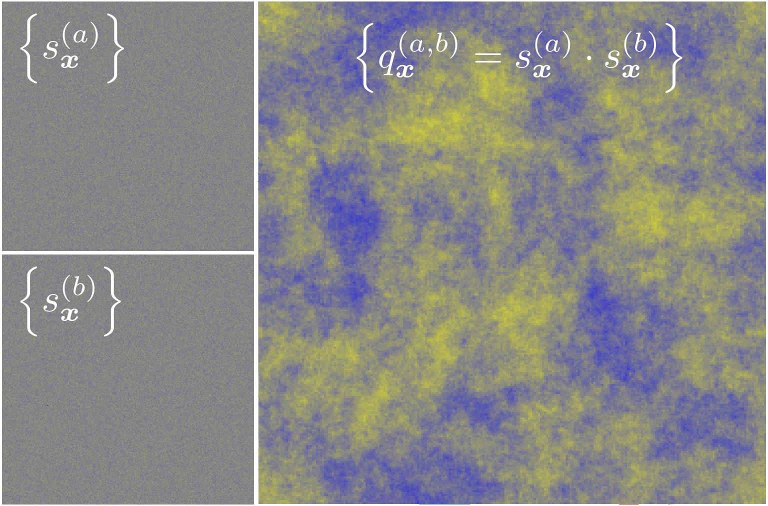
- Correlation Length (only in the PM phase):

$$\xi_2 = \frac{1}{2 \sin(\pi/L)} \left(\frac{\hat{G}(0)}{\hat{G}(\boldsymbol{k}_1)} - 1 \right)^{1/2}\,,$$

  where $\boldsymbol{k}_1 = (2\pi/L, 0, 0)$ (and two perm.)

- ξ_2/L should behave as the Binder cumulant. It is also universal at the critical point.

Overlap


$\left\{ s_{\boldsymbol{x}}^{(a)} \right\}$
$\left\{ s_{\boldsymbol{x}}^{(b)} \right\}$
$\left\{ q_{\boldsymbol{x}}^{(a,b)} = s_{\boldsymbol{x}}^{(a)} \cdot s_{\boldsymbol{x}}^{(b)} \right\}$

Ogielski-Morguenstern (1985)

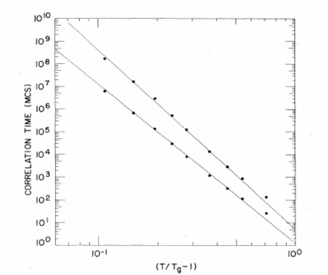
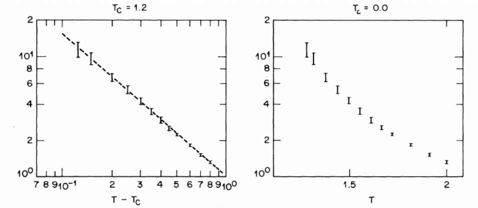
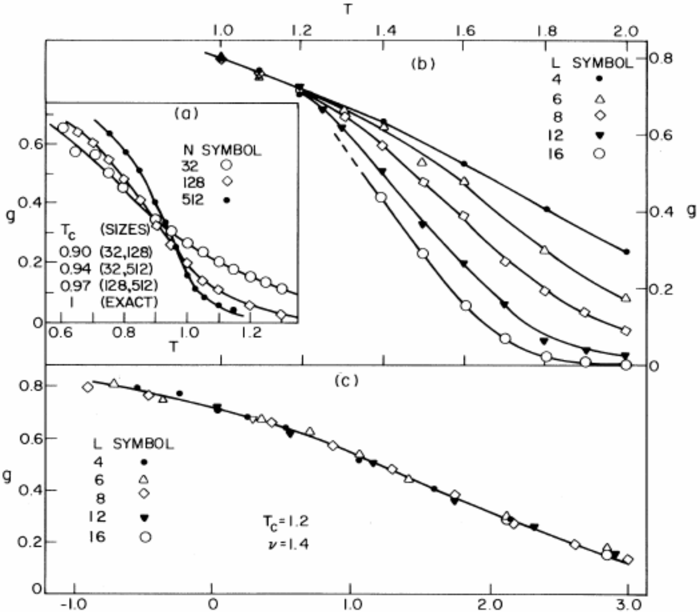


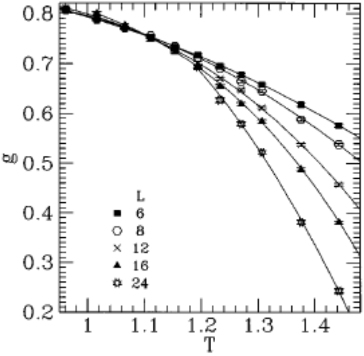
FIG. 16. Power-law fits $\tau = c(T-T_g)^{-z\nu}$ and $\tau_{av}=c_{av}(T-T_g)^{-z_{av}\nu}$ for the relaxation times. Lattice size 64^3, $T \geq 1.30$.

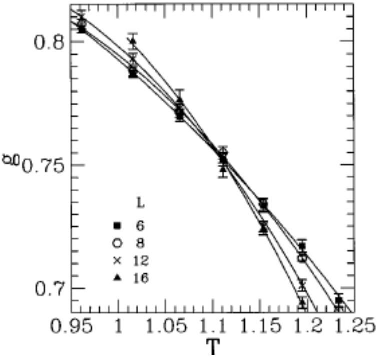


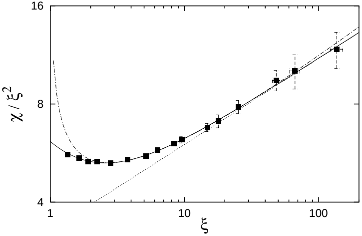
Bhatt-Young (1985)

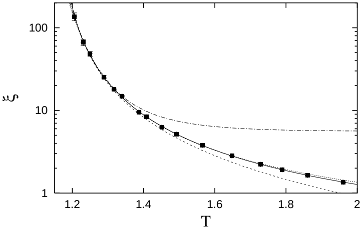


Kawashima-Young (1996)



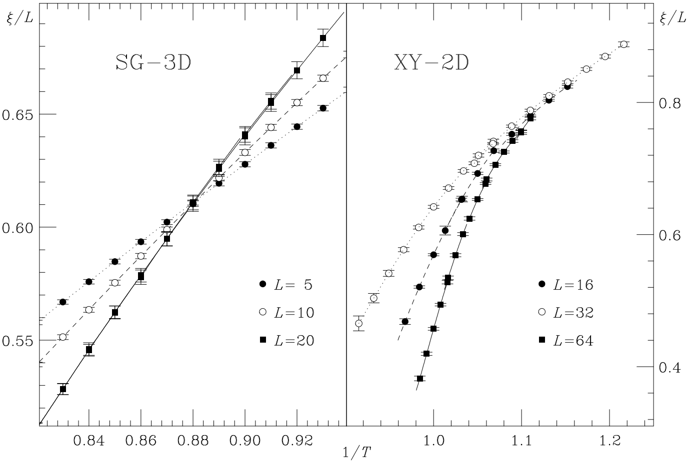




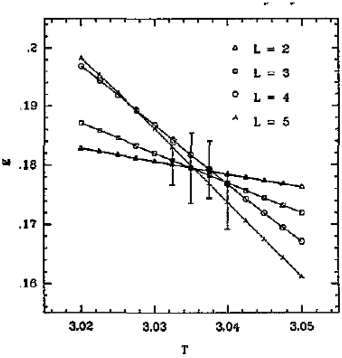


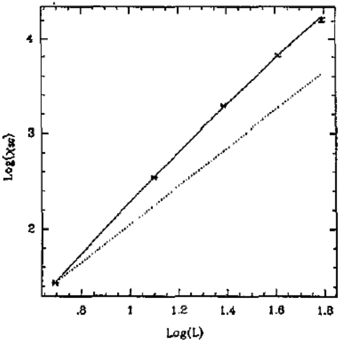
- Simulations only in the PM phase.
- Extrapolations, using the method of Caracciolo et al, to infinte volume.

SUE Coll. (2000)



Upper Critical Dimension (Wang-Young, 1992)





- $\chi(L) = L^2 (\log L)^c$, $c = 0.64$.
- Theory-RG (RL, 2000): $c = 2/3$.

Lower Critical Dimension, $D_l = 2.5$

- Boettcher (2005). Using the behavior of the stiffness exponent, θ, with the dimension.

$$\theta(D_L) = 0\,.$$

- Franz, Parisi and Virasoro (1994) studyng interfaces:

$$\delta F \propto |q_1 - q_2|^{5/2} L^{D-5/2}$$

- For more recent work, see Astuti, Franz and Parisi (2018).
- See also, Maiorano and Parisi (2018).

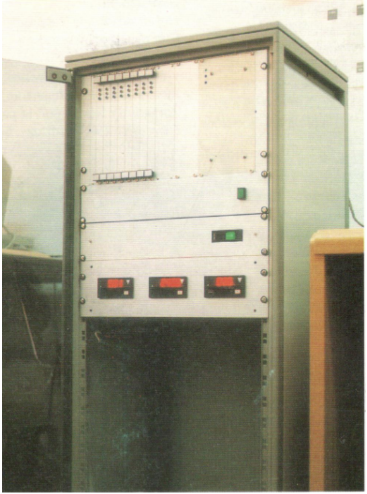
Summary Phase Transiton

- (Standard) Phase transition between a paramagnetic phase and a spin glass one.
- $\nu = 2.562(42)$, $\eta = -0.3900(36)$ (Janus col, 2010).
- Universality (Katzgraber, Koerner and Young, 2006).
- $D_U = 6$ and $D_l = 2.5$.

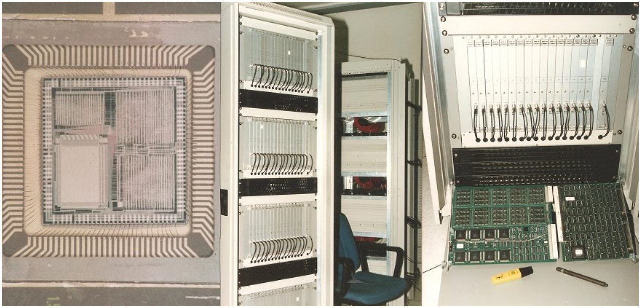
Dedicated Computers.

- Ogielski's dedicated computer (1985).
- Reconfigurable Transputer Network (RTN): Triviality issue in U(1)-Higgs model (lattice field theory) and spin glasses (1989).
- APE100. Quantum Chromodynamics (mainly) and spin glasses (1994).
- Spin Update Engine (SUE) (2000).
- Super Spin Update Engine (SSUE) (2003).
- Janus I (2008).
- Janus II (2013).

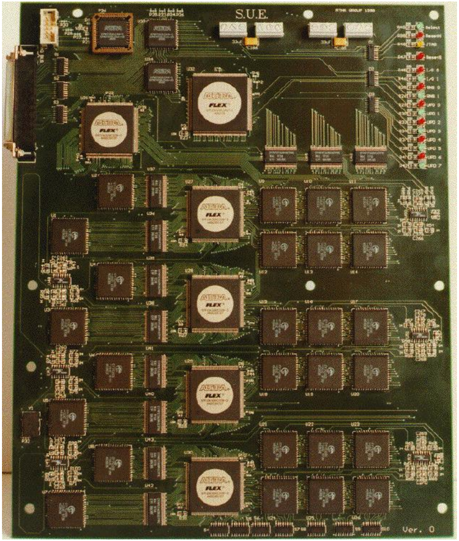
RTN (Zaragoza, Madrid, Roma)



APE100 (INFN, Italy)



SUE (Zaragoza, Madrid)



Janus I (Ferrara-Roma-Madrid-Zaragoza-Badajoz)

Janus II (Ferrara-Roma-Madrid-Zaragoza-Badajoz)

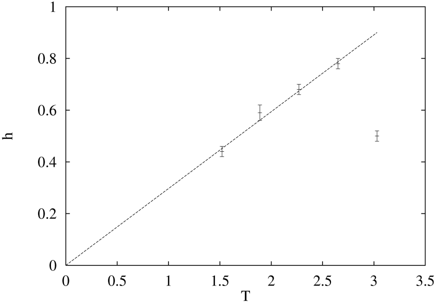
Evolution of (some) Algorithms (only $D \geq 3$ and $T \neq 0$).

- Normal Monte Carlo.
- Simulated Annealing.
- Microcanonical methods (Berg).
- Simulated Tempering (Marinari and Parisi).
- Monte Carlo Exchange (Parallel Tempering) (Hukushima and Nemoto).
- Population Annealing (Hukushima and Iba; Machta).
- Multispin coding in higher dimensions (different implementations).

Upper Critical Dimension.

The replicon in $D = 6$.

- Assuming $z(T) = 4\frac{T_c}{T}$
- The out of equilibrium behavior of $\chi(t)$, should follow that of the critical propagator $p^{-2}(r^{-4})$ at $T = T_c$ and the replicon one below T_c: $p^{-4}(r^{-2})$.
- The behavior of $\chi(t) \simeq t^{h(T)}$ is:
  - $h(T_c) = 1/2$.
  - $h(T) = 4/z(T)$ si $T < T_c$.



- Note: $\chi(t) = \int^{\xi(t)} d^D x \; G(x)$.
  (G. Parisi, P. Ranieri, F. Ricci-Tersenghi, JJRL (1997).)

Out of equilibrium: Replicon.

- The overlap-overlap correlation function:

$$C_4(x, t_\mathrm{w}) = \frac{1}{L^3} \overline{\sum_i q_i(t_\mathrm{w}) q_{i+x}(t_\mathrm{w})}.$$

$T < T_\mathrm{c}$: $C_4(r, t_\mathrm{w}) \simeq r^{-\theta(0)} e^{-[r/\xi(t_\mathrm{w})]^b}$ (Parisi, Marinari, Ritort and JJRL (1996).)

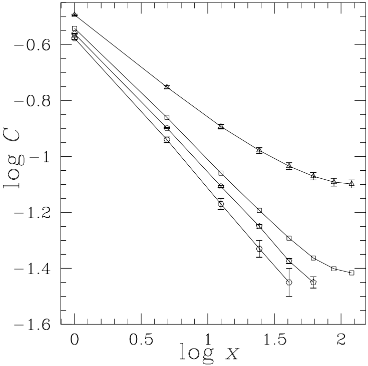
- At equilibrium and in the critical point: $a = 1 + \eta = 0.6100(36)$.
- The correlation length can be computed using (Janus Coll.):

$$I_k(t_\mathrm{w}) = \int_0^\infty dr\ r^k C_4(r, t_\mathrm{w})$$

assuming that $C_4 \simeq r^{-\theta(0)} f(r/\xi)$, then

$$\xi_{k,k+1}(t_\mathrm{w}) = I_{k+1}(t_\mathrm{w})/I_k(t_\mathrm{w}) \propto \xi(t_\mathrm{w}), \quad (\xi \ll L)$$

Replicon: Equilibrium and out of equilibrium



(Marinari, Parisi and RL (1998))

Summary Replicon

- $T < T_c$: Behavior of $\chi(t)$ in six dimensions is compatible with a propagator $1/p^4$.
- Clear signatures at equilibrium and out of equilibrium in three dimensions for the replicon mode. (Janus)
- $\theta(0) = 0.38(6)$ (out eq); $\theta(0) = 0.377(14)$ (eq).(Janus 2010, 2009))
- By studying the Metastate in $3D$ we have computed the Read's exponent: $\zeta = 2.3(3) < 3$. (Billoire et al (2017))
- Read conjectured that $\zeta = D - \theta(0)$. So, $\zeta_{q=0} = 2.62(5) < 3$.
- $\theta(q_{\rm EA}) \simeq 0.511(16)(60)$ (eq). (Janus 2010)
- We have not been able to compute $\theta(q)$ for $0 < q < q_{\rm EA}$. (Janus 2009, 2010)

$D = 4$ $(h \neq 0)$: Observables

- Correlation Functions

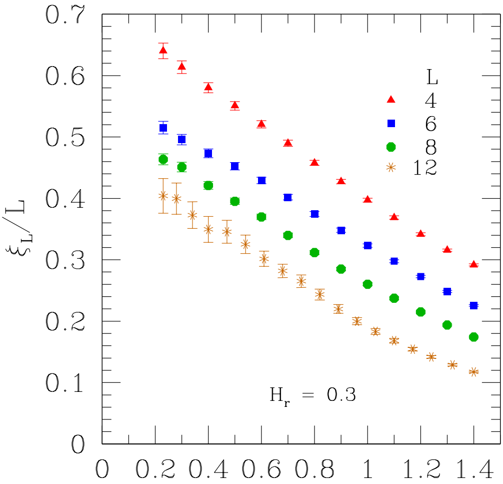
$$
\begin{aligned}
G_1(\boldsymbol{r}) &= \frac{1}{L^3} \sum_{\boldsymbol{x}} \overline{\left(\langle S_{\boldsymbol{x}} S_{\boldsymbol{x}+\boldsymbol{r}} \rangle - \langle S_{\boldsymbol{x}} \rangle \langle S_{\boldsymbol{x}+\boldsymbol{r}} \rangle\right)^2}\,, \\
G_2(\boldsymbol{r}) &= \frac{1}{L^3} \sum_{\boldsymbol{x}} \overline{\left(\langle S_{\boldsymbol{x}} S_{\boldsymbol{x}+\boldsymbol{r}} \rangle^2 - \langle S_{\boldsymbol{x}} \rangle^2 \langle S_{\boldsymbol{x}+\boldsymbol{r}} \rangle^2\right)}\,.
\end{aligned}
$$

- Correlation Length:

$$
\xi_2 = \frac{1}{2 \sin(\pi/L)} \left(\frac{\hat{G}(0)}{\hat{G}(\boldsymbol{k}_1)} - 1 \right)^{1/2} ,
$$

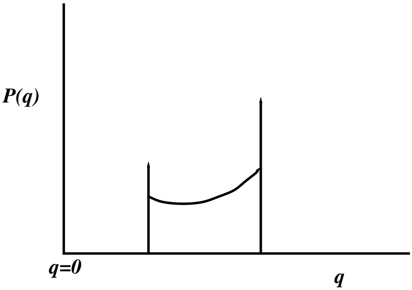
where $\boldsymbol{k}_1 = (2\pi/L, 0, 0)$ (and two perm.)

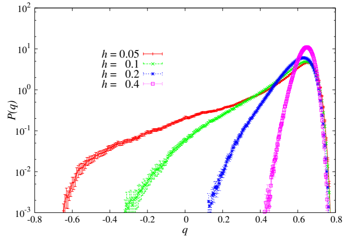
Spin Glass behavior in $D = 3$ $(h \neq 0)$? (Katzgraber and Young)



The negative overlap problem

- $P(q)$ in a magnetic field: RSB prediction and numerical results.





- The negative overlap region induces large corrections in $\tilde{G}(0)$!!

“Improved” Observable. Behavior in $D = 4$.

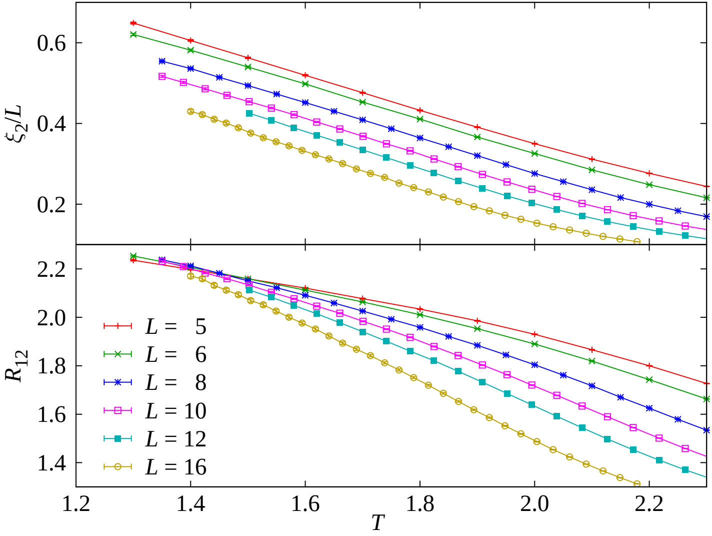
- R_{12}:

$$R_{12} = \frac{\hat{G}(\boldsymbol{k}_1)}{\hat{G}(\boldsymbol{k}_2)},$$

  where $\boldsymbol{k}_1 = (2\pi/L, 0, 0, 0)$, $\boldsymbol{k}_2 = (2\pi/L, 2\pi/L, 0, 0)$ (and permutations)
- We have checked the behavior of this observable in the EA model in $4d$ $(h = 0)$.
- And in the two dimensional (ordered) Ising model. We have been able to compute its value at criticality using the Onsager solution on a torus:

$$R_{12} = 1.694\ 024...$$

$D = 4$ $(h = 0.15)$



0.6
0.4
0.2
ξ2/L
2.2
2.0
1.8
1.6
1.4
R12
L = 5
L = 6
L = 8
L = 10
L = 12
L = 16
1.2
1.4
1.6
1.8
2.0
2.2
T

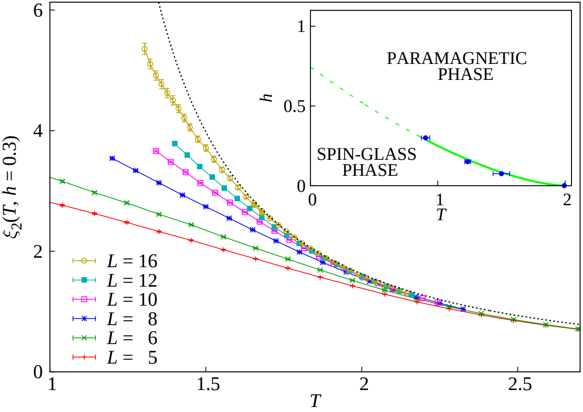
$D = 4\ (h \neq 0)$

Parameter	$h = 0.3$	$h = 0.15$	$h = 0.075$
$T_c(h)$	0.906(40)[3]	1.229(30)[2]	1.50(7)
ν	1.46(7)[6]		—
η	−0.30(4)[1]		—
ω	1.43(37)		—

For reference ($h = 0$):
$T_c^{(0)} = 2.03(3)$, $\nu^{(0)} = 1.025(15)$, $\eta^{(0)} = -0.275(25)$

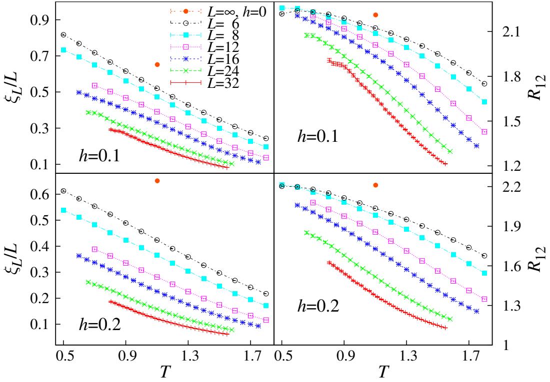
$D = 4\ (h \neq 0)$

Summary:



Fisher-Sompolinsky relation: $h^2(T) \simeq A|T - T_c^{(0)}|^{\beta^{(0)}+\gamma^{(0)}}$

Spin Glass behavior in $D = 3$ ($h \neq 0$)? (Janus)



No signal of a phase transition in the ξ_L/L and R_{12}-channels!

Conclusions

- We have shown strong numerical evidences which support a dAT line below the upper critical dimension:
- In three dimensions there are no signals of a phase transition.
- Lower critical dimension for the Ising spin glass in a field $D_l \geq 3$ or $D_l < 3$?