Misterios de la Física Cuántica: De la doble rendija a la integral de camino de Feynman

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Plan de la Charla

- Experimentos de interferencia: la doble rendija.
- Efectos cuánticos en sistemas “grandes”.
- Richard P. Feynman.
- Formulación de Feynman de la Mecánica Cuántica usando integrales de camino.
- Bibliografía.

Sobre la Mecánica Cuántica

- *I think I can safely say that nobody understands quantum mechanics.*

  (Richard Feynman)

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- *Very interesting theory- it makes no sense at all.*

  (Groucho Marx)

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  (Richard Feynman)

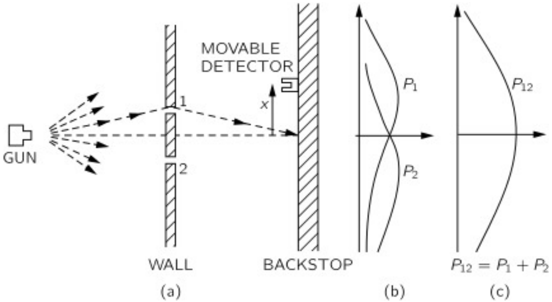
- *Very interesting theory- it makes no sense at all.*

  (Groucho Marx)

- *A child of five would understand this. Send someone to fetch a child of five.*

  (Groucho Marx)

Cañón de balas



- O bien la bala pasa por la rendija #1 (P_1).
- O en su defecto deberá pasar por la rendija #2 (P_2).

Por lo tanto (sucesos excluyentes):

$$P_{12} = P_1 + P_2$$

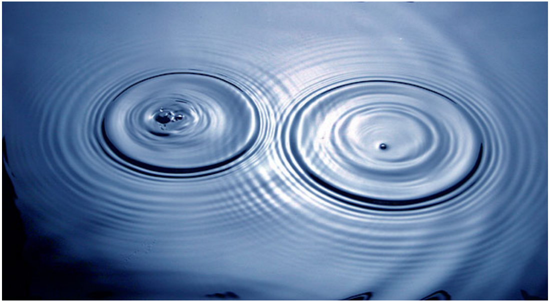
Ondas

La perturbación h, por ejemplo la altura de la ola, puede representarse mediante:

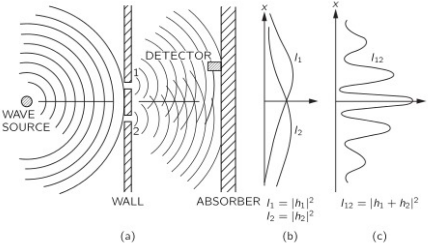
$$h(r,t) = A\cos(kr - \omega t) \propto \mathrm{Re}\left(Ae^{i(kr-\omega t)}\right)$$

$k = 2\pi/\lambda$ es el número de ondas y $\omega = 2\pi\nu$ es la frecuencia angular.
Intensidad $\propto h(x,t)^2 \to |h|^2$.
Recordar: $e^{i\theta} = \cos\theta + i\sin\theta$.
Si $z = x + iy$, entonces $|z|^2 = zz^* = x^2 + y^2$.

Interferencia



Interferencia



1. Desde la fuente, pasando por la rendija #1, y llegando al detector la onda recorre una distancia d: $h_1 \propto e^{ikd} e^{i\omega t}$.
2. Desde la fuente, pasando por la rendija #2, y llegando al detector la onda recorre una distancia $d+\delta$: $h_2 \propto e^{ik(d+\delta)} e^{i\omega t}$.

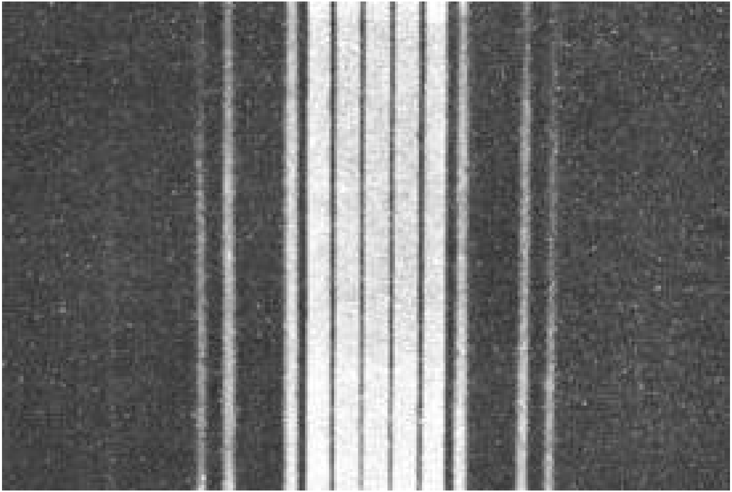
Por lo tanto:

$$\text{Intensidad} \propto |h_1 + h_2|^2 \propto |1 + e^{ik\Delta}| \propto 1 + \cos\left(\frac{2\pi\delta}{\lambda}\right)$$

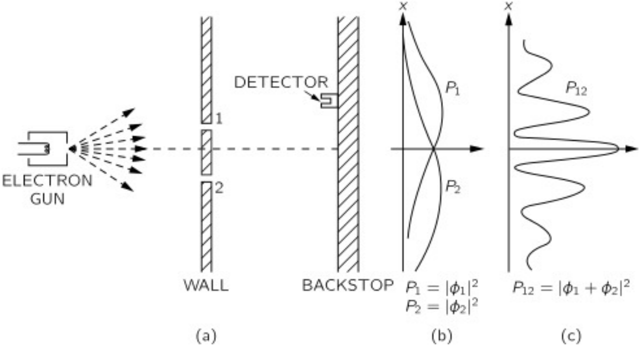
o equivalentemente

$$\text{Intensidad} \propto |h_1 + h_2|^2 = |h_1|^2 + |h_2|^2 + 2|h_1||h_2|\cos(2\pi\delta/\lambda)$$

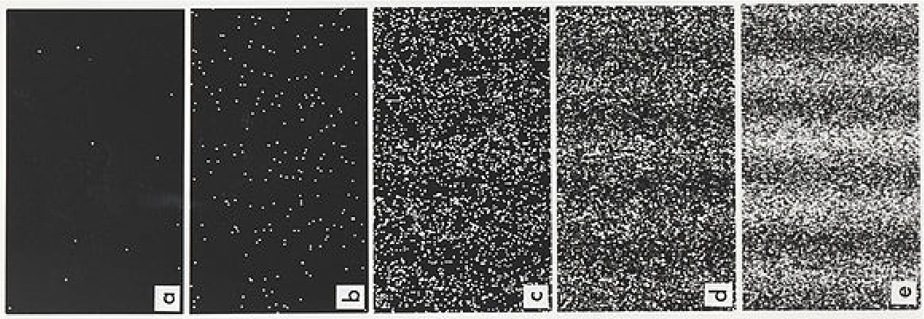
Interferencia



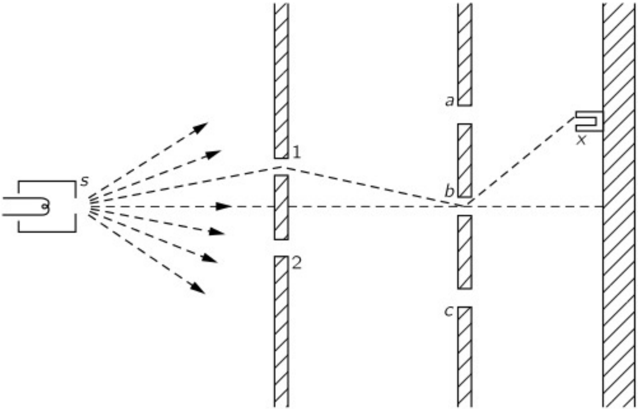
Cañón de electrones



Construcción de la Interferencia (electrones)

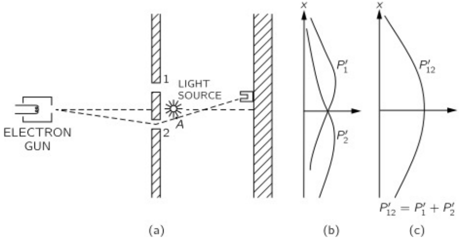


Cañón de electrones



Cañón de electrones

¿Podemos averiguar por dónde están pasando los electrones sin destruir la interferencia?

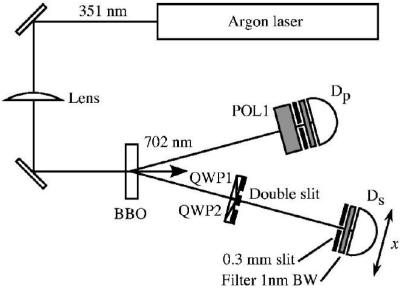


Para determinar la posición con una precisión menor que la distancia entre las dos rendijas Δ, necesitamos usar fotones con momento

$$p \simeq \frac{h}{\Delta}$$

Pero estos fotones destruyen la interferencia!!!

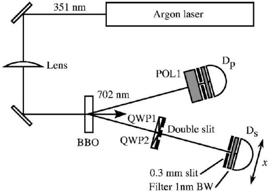
Enredando la doble rendija...con gafas...



BBO: Cristal de Borato de Bario-beta. Se generan dos fotones entrelazados (up y down).
QWP: Lámina de un cuarto de longitud de onda.
QWP1: Convierte un fotón con polarización y en uno con polarización circular derecha (CD) y si llega uno con polarización x lo convierte en uno con polarización circular izquierda (CI).
QWP2: $x \to$ CD, $y \to$ CI.

Enredando la doble rendija...con gafas...

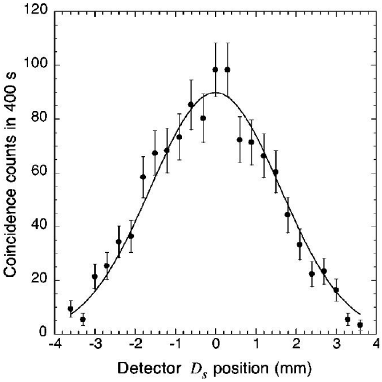
¿Podemos averiguar por dónde están pasando los electrones sin destruir la interferencia?



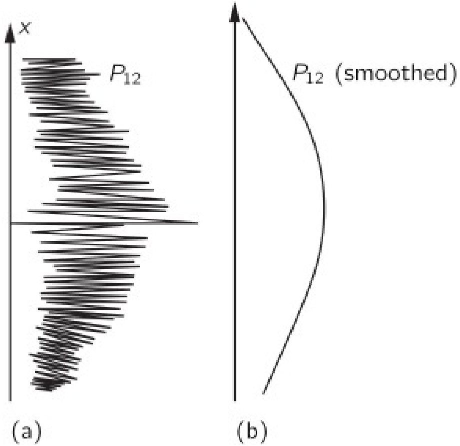
Polarizaciones:

up	down (antes de QWPs)	down (QWP1-s1)	down (QWP2-s2)
x	y	CD	CI
y	x	CI	CD

Enredando la doble rendija...con gafas



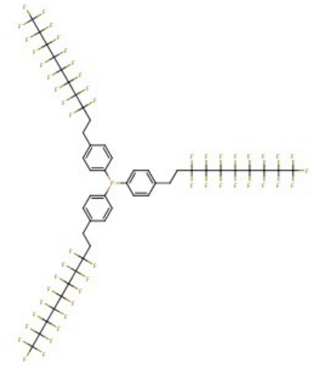
Y las balas?



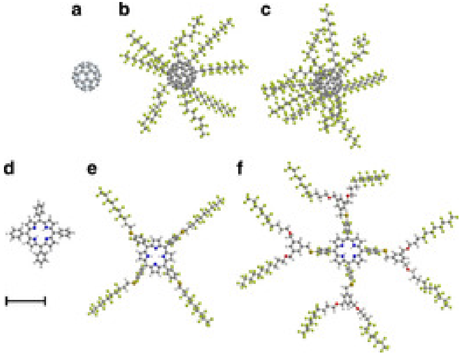
$$\lambda = \frac{h}{mv} \simeq 10^{-33}\text{m}$$

Interferencia moléculas orgánicas

Molécula: $C_{48}H_{24}F_{51}P$
Tris[4-(1H,1H,2H,2H-perfluorodecyl)phenyl]phosphine
Masa: 1600.60 AMU

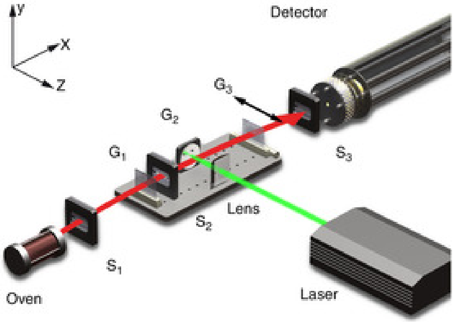


Interferencia moléculas orgánicas



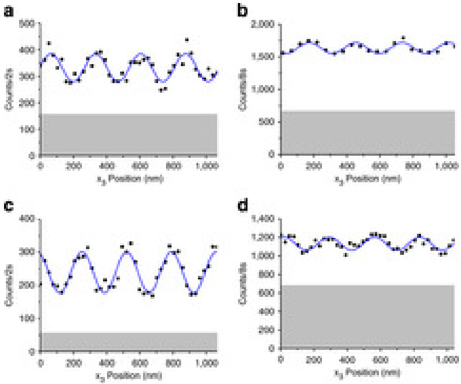
(a) The fullerene C60 (m=720 AMU, 60 atoms) serves as a size reference and for calibration purposes; (b) The perfluoroalkylated nanosphere PFNS8 (C60[C12F25]8, m=5,672 AMU, 356 atoms) is a carbon cage with eight perfluoroalkyl chains. (c) PFNS10 (C60[C12F25]10, m=6,910 AMU, 430 atoms) has ten side chains and is the most massive particle in the set. (d) A single tetraphenylporphyrin TPP (C44H30N4, m=614 AMU, 78 atoms) (e) TPPF84 (C84H26F84N4S4, m=2,814 AMU, 202 atoms) and (f) TPPF152

Interferencia moléculas orgánicas



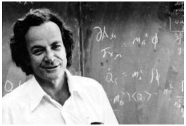
Periodicidad las rendijas: 266 nm.
Anchura de la rendija: 90 nm.
Longitud de onda de de Broglie 1 pm.
Tamaño máximo de la molécula: 6 nm.

Interferencia moléculas orgánicas

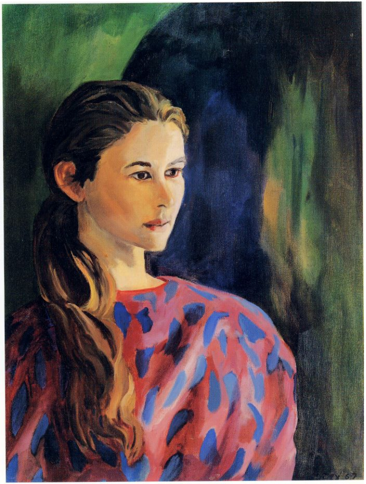


Richard Phillips Feynman

- Richard Phillips Feynman (1911-1988).
- Graduado en el MIT (1939).
- Doctorado en Princeton (1941).
- Proyecto Manhattan.
- Premio Nobel de Física (1965).
- Profesor en U. Cornell y en CalTech.
- Miembro de la comisión de investigación del Challenger.



Feynman Pintor



Richard Phillips Feynman: Logros

- Integral de caminos.
- QED.
- Diagramas de Feynman.
- Helio Superfluido.
- Computación Cuántica.
- Partones.
- Interacciones débiles.

Nacimiento de la Mecánica Cuántica

- W. Heisenberg. Mecánica Matricial (Julio, 1925).
- Principio de incertidumbre (Marzo, 1927).

- E. Schrödinger. Mecánica Ondulatoria (Enero, 1926).
- E. Schrodinger. Equivalencia entre las dos Mecánicas (Mayo, 1926).

Heisenberg

Über quantentheoretische Umdeutung kinematischer und mechanischer Beziehungen.

Von **W. Heisenberg** in Göttingen.

(Eingegangen am 29. Juli 1925.)

Fragen wir weiter nach dem Repräsentant der Größe $x(t)^3$, so finden wir ohne Schwierigkeit:

Klassisch:

$$\mathfrak{C}(n, \gamma) = \sum_{-\infty}^{+\infty} \sum_{-\infty}^{+\infty}{}_{\alpha, \beta}\ \mathfrak{A}_\alpha(n)\, \mathfrak{A}_\beta(n)\, \mathfrak{A}_{\gamma-\alpha-\beta}(n). \qquad (9)$$

Quantentheoretisch:

$$\mathfrak{C}(n, n-\gamma) = \sum_{-\infty}^{+\infty} \sum_{-\infty}^{+\infty}{}_{\alpha, \beta}\ \mathfrak{A}(n, n-\alpha)\, \mathfrak{A}(n-\alpha, n-\alpha-\beta)\, \mathfrak{A}(n-\alpha-\beta, n-\gamma) \quad (10)$$

bzw. die entsprechenden Integrale.

3. *Quantisierung als Eigenwertproblem;*
von E. Schrödinger.

(Erste Mitteilung.)

Die Konstante K muß aus dimensionellen Gründen eingeführt werden, sie hat die Dimension einer *Wirkung*. Damit erhält man

$$H\left(q, \frac{K}{\psi}\frac{\partial\psi}{\partial q}\right) = E \,. \tag{1'}$$

Es muß also erstens

$$\Delta\psi + \frac{2m}{K^2}\left(E + \frac{e^2}{r}\right)\psi = 0 \tag{5}$$

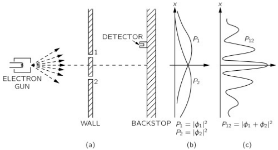
Zürich, Physikalisches Institut der Universität.

(Eingegangen 27. Januar 1926.)

Dirac y Feynman (Varsovia 1962)

F: I am Feynman.
D: I am Dirac. (Silence.)
F: It must be wonderful to be the discoverer of that equation.
D: That was long time ago. (Pause.) What are you working on?
F: Mesons.
D: Are you trying to discover an equation for them?
F: It is very hard.
D: One must try.

Cañón de electrones



Notación para la amplitud de probabilidad:

$$\langle x|s\rangle \equiv \langle \text{Partícula que llega a } x|\text{Partícula que sale de } s\rangle$$

Podemos reescribir P_{12} en la nueva notación:

$$P_{12} = |\langle x|s\rangle_{\text{ambos}}|^2$$

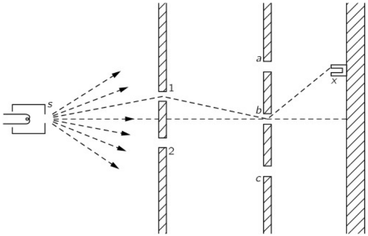
con (suma de las amplitudes)

$$\langle x|s\rangle_{\text{ambos agujeros abiertos}} = \langle x|s\rangle_{\text{a través de 1}} + \langle x|s\rangle_{\text{a través de 2}}$$

Además (producto de las amplitudes)

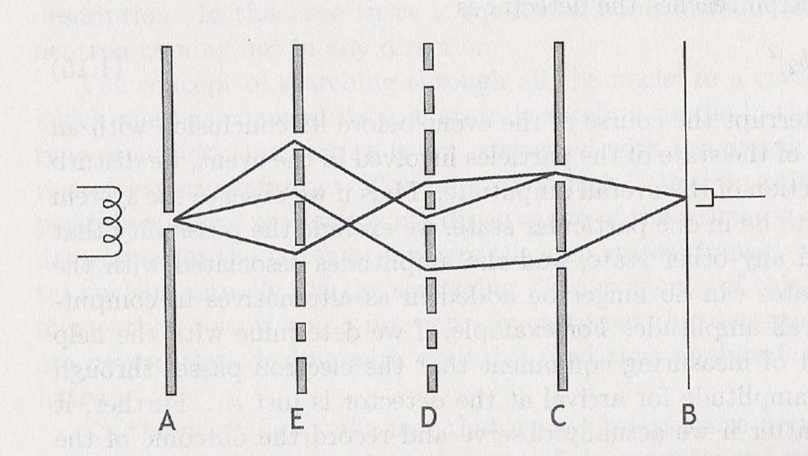
$$\langle x|s\rangle_{\text{a través de 1}} = \langle x|1\rangle\langle 1|s\rangle$$

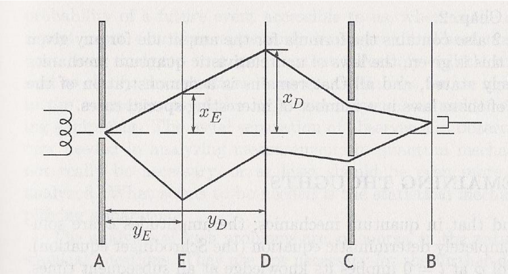
$$\langle x|s\rangle_{\text{ambos}} = \langle x|1\rangle\langle 1|s\rangle + \langle x|2\rangle\langle 2|s\rangle$$



La amplitud de probabilidad se escribe como:

$$\langle x|s\rangle = \sum_{\substack{i=1,2 \\ \alpha=a,b,c}} \langle x|\alpha\rangle\langle\alpha|i\rangle\langle i|s\rangle$$


A
E
D
C
B

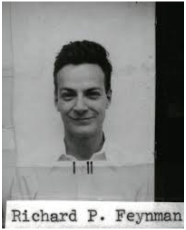


$$\langle x|s\rangle = \sum_{i_c=1,2} \int dx_E \int dx_D \langle x|i\rangle\langle i|x_D\rangle\langle x_D|x_E\rangle\langle x_E|s\rangle$$

Pero, ¿cómo se calculan las amplitudes de probabilidad?

The Development of the Space-Time View of Quantum Electrodynamics.
R. P. Feynman. Nobel Lecture, December 11, 1965.

I went to a beer party in the Nassau Tavern in Princeton. There was a gentleman, newly arrived from Europe (Herbert Jehle) who came and sat next to me. Europeans are much more serious than we are in America because they think that a good place to discuss intellectual matters is a beer party. So, he sat by me and asked, "what are you doing" and so on, and I said, "I'm drinking beer." Then I realized that he wanted to know what work I was doing and I told him I was struggling with this problem, and I simply turned to him and said, "listen, do you know any way of doing quantum mechanics, starting with action - where the action integral comes into the quantum mechanics?" "No", he said, "but Dirac has a paper in which the Lagrangian, at least, comes into quantum mechanics. I will show it to you tomorrow."



Nassau Tavern (Princeton, 1946)

THE LAGRANGIAN IN QUANTUM MECHANICS.

By P. A. M. Dirac.

(Received November 19, 1932).

Quantum mechanics was built up on a foundation of analogy with the Hamiltonian theory of classical mechanics. This is because the classical notion of canonical coordinates and momenta was found to be one with a very simple quantum analogue, as a result of which the whole of the classical Hamiltonian theory, which is just a structure built up on this notion, could be taken over in all its details into quantum mechanics.

Now there is an alternative formulation for classical dynamics, provided by the Lagrangian. This requires one to work in terms of coordinates and velocities instead of coordinates and momenta. The two formulations are, of course, closely related, but there are reasons for believing that the Lagrangian one is the more fundamental.

where L is the Lagrangian. If we take T to differ only infinitely little from t, we get the result

$$(q_{t+dt} \mid q_t) \text{ corresponds to } \exp\,[iL\,dt/h]. \qquad (9)$$

Physikalische Zeitschrift der Sowjetunion, Band 3, Heft 1 (1933).

Dirac y Feynman (Princeton 1948)

I said, "Do you know, in your book you make the relationship that the action in classical mechanics is analogous to ei over h times the action, times the Legrangian, is analogous to the operator carrier for an infinitesimal times from one position to another, from one wave function to the next, really from one position to another." I said, "Did you know that they're not only analogous but they're equal? Or rather proportional?" He said, "No, are they? Are they proportional?" I said, "Yes." So at least I found out that the discovery which I had made which led me on, that I told you about, that they were really proportional, was really a new thing. He himself hadn't noticed this, but I didn't know. For all I knew, he always thought they were proportional. See, I still had the belief that he thought they were proportional and simply was explaining that. But anyway he said that he didn't know they were. He said, "Oh, are they?" That's what he said. I said, "Yes they are." "That's very interesting." That was the end of the conversation...

Oral History Transcript — Dr. Richard Feynman

Amplitudes de Probabilidad

Lagrangiano: $L = T - V$.

- $\delta t \ll 1$.
- Dirac (1932):

$$\langle x', t + \delta t | x, t \rangle \Longleftrightarrow \exp\left(i \delta t L / \hbar\right)$$

- Feynman (1941):

$$\langle x', t + \delta t | x, t \rangle = \frac{1}{A} \exp\left(i \delta t L / \hbar\right)$$

  con $A = \sqrt{2\pi \delta t \hbar / M}$.
- Con L dado por:

$$L = \frac{M}{2} \left(\frac{x' - x}{\delta t} \right)^2 - V \left(\frac{x + x'}{2} \right)$$

- $h = 6.6 \times 10^{-34}$ J s. $\hbar = h/(2\pi)$.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Amplitudes de Probabilidad

- El Lagrangiano

$$L(x(t), \dot{x}(t)) = T - V$$

Ejemplo, el oscilador armónico:

$$L = \frac{m}{2}\left(\frac{dx}{dt}\right)^2 - \frac{k}{2}x^2$$

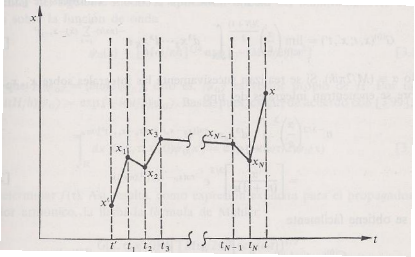
- La acción (es un funcional)

$$S[x(t)] = \int_{t_1}^{t_2} dt L$$

La amplitud de probabilidad se escribe

$$\langle u, t_2 | s, t_1 \rangle \propto \sum_{\substack{\text{Todos los} \\ \text{caminos que} \\ \text{conectan} \\ u \text{ y } s}} \exp\left(\frac{i}{\hbar} S(\text{camino})\right)$$

Amplitudes de Probabilidad

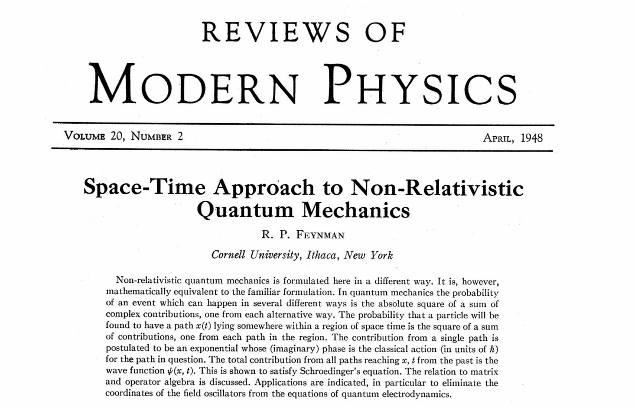


$$\langle x,t|x',t'\rangle = \lim_{\epsilon\to 0}\int_{-\infty}^{\infty} dx_1 \cdots \int_{-\infty}^{\infty} dx_N \ \langle x',t'|x_1,t_1\rangle\langle x_1,t_1|x_2,t_2\rangle \cdots \langle x_N,t_N|x,t\rangle$$

$$= \lim_{\epsilon\to 0}\frac{1}{A}\int_{-\infty}^{\infty}\frac{dx_1}{A}\cdots\int_{-\infty}^{\infty}\frac{dx_N}{A}\exp\left(\frac{i}{\hbar}S[x]\right)$$

$$\equiv \int_{x(t)=x}^{x(t')=x'} \mathcal{D}x \ \exp\left(\frac{i}{\hbar}\int_{t_1}^{t_2} dt L\right)$$

$t_{i+1}-t_i=\epsilon$ y $A=\sqrt{2\pi i\hbar\epsilon/M}$.



REVIEWS OF
MODERN PHYSICS

VOLUME 20, NUMBER 2 APRIL, 1948

Space-Time Approach to Non-Relativistic Quantum Mechanics

R. P. FEYNMAN

Cornell University, Ithaca, New York

Non-relativistic quantum mechanics is formulated here in a different way. It is, however, mathematically equivalent to the familiar formulation. In quantum mechanics the probability of an event which can happen in several different ways is the absolute square of a sum of complex contributions, one from each alternative way. The probability that a particle will be found to have a path $x(t)$ lying somewhere within a region of space time is the square of a sum of contributions, one from each path in the region. The contribution from a single path is postulated to be an exponential whose (imaginary) phase is the classical action (in units of $\hbar$) for the path in question. The total contribution from all paths reaching x, t from the past is the wave function $\psi(x, t)$. This is shown to satisfy Schroedinger's equation. The relation to matrix and operator algebra is discussed. Applications are indicated, in particular to eliminate the coordinates of the field oscillators from the equations of quantum electrodynamics.

Relación con la funciones de onda

La solución de la ecuación de Scrhrödinger nos dice:

$$|\psi(t_2)\rangle = U(t_2, t_1)|\psi(t_2)\rangle$$

[Si H no depende del tiempo $U(t_2, t_1) = \exp\left(-iH(t_2 - t_1)/\hbar\right)$]
que en la representación de posiciones se puede escribir como

$$\langle x_2|\psi(t_2)\rangle = \psi(x_2, t_2) = \langle x_2|U(t_2, t_1)|\psi(t_1)\rangle$$

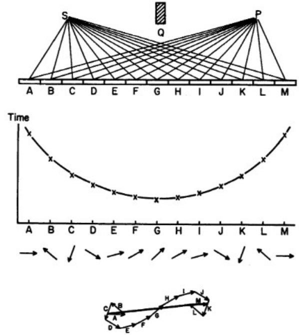
Introduciendo la identidad $1 = \int dx_1\ |x_1\rangle\langle x_1|$:

$$\psi(x_2, t_2) = \int dx_1\ G(x_2, t_2; x_1, t_1)\psi(x_1, t_2)$$

con

$$G(x_2, t_2; x_1, t_1) = \langle x_2|U(t_2, t_1)|x_1\rangle = \langle x_2, t_2|x_1, t_1\rangle$$

Principio de Fermat



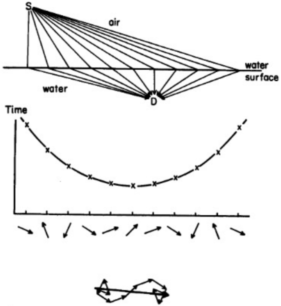
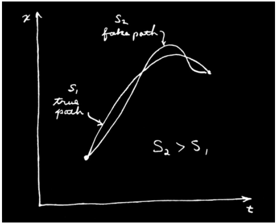


FIGURE 24. *Each path the light could go (in this simplified situation) is shown at the top, with a point on the graph below it showing the time it takes a photon to go from the source to that point on the mirror, and then to the photomultiplier. Below the graph is the direction of each arrow, and at the bottom is the result of adding all the arrows. It is evident that the major contribution to the final arrow's length is made by arrows E through I, whose directions are nearly the same because the timing of their paths is nearly the same. This also happens to be where the total time is least. It is therefore approximately right to say that light goes where the time is least.*

FIGURE 29. *Quantum theory says that light can go from a source in air to a detector in water in many ways. If the problem is simplified as in the case of the mirror, a graph showing the timing of each path can be drawn, with the direction of each arrow below it. Once again, the major contribution toward the length of the final arrow comes from those paths whose arrows point in nearly the same direction because their timing is nearly the same; once again, this is where the time is least.*

Acción Extrema



Ejemplo. Oscilador armónico ($k = m = 1$, $\omega = 1$):

$$\frac{d^2x}{dt^2} + x = 0,\ \ x(0) = 0,\ \ x(\pi/2) = 1$$

$$x(t) = \sin t$$

Lagrangiano:

$$L = T - V = \frac{1}{2}\left(\frac{dx}{dt}\right)^2 - \frac{1}{2}x^2$$

Acción:

$$S[x(t)] = \int_0^{\pi/2} L dt$$

Acción Extrema

La acción calculada sobre la solución de las ecuaciones de Newton es:

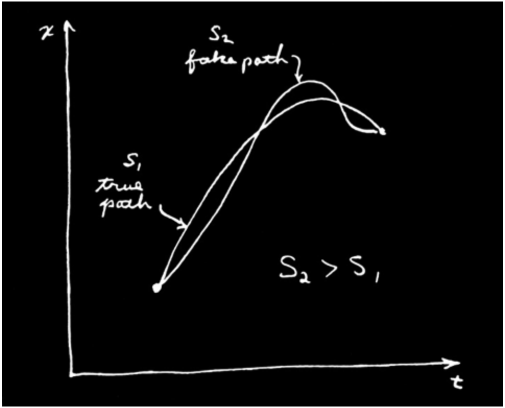
$$S_1 = S[\sin t] = 0$$

Calculemos ahora la acción sobre la función $\sin^2 t$ que satisface $x(0) = 0,\ x(\pi/2) = 1$, pero NO las ecuaciones de Newton:

$$S_2 = S[\sin^2 t] = \frac{\pi}{32}$$

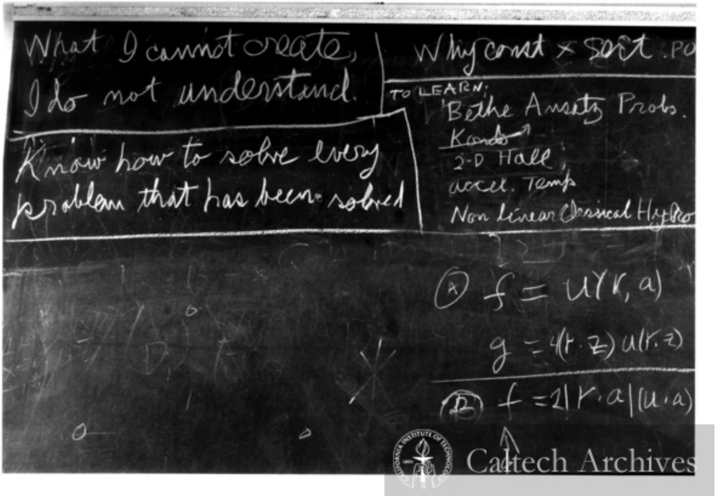
Nótese que $S_1 < S_2$. En general se puede demostrar que la $x(t)$, solución de las ecuaciones de Newton, es un extremo (en general mínimo, pero también podrí ser también máximo o punto de inflexión).

Límite Clásico



$$\langle u, t_2 | s, t_1 \rangle \propto \sum_{\substack{\text{Todos los} \\ \text{caminos que} \\ \text{conectan} \\ u \text{ y } s}} \exp\left(\frac{i}{\hbar} S(\text{camino})\right)$$

La última pizarra de Feynman




THE THING THAT'S EXCITING ABOUT THIS IS THAT NATURE IS STRANGE!

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Diagramas de Feynman