Nature of the Spin Glass Phase in finite dimensional (Ising) Spin Glasses (part II)

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Outline of the Talk

- Wednesday
  - Fluctuation-Dissipation relations out of equilibrium.
    - Theoretical basis.
    - Static-Dynamics Relation: Computation of $P(q)$.
    - Measurement of $P(q)$.
  - Phases and Thermodynamic limit in Disordered systems: The Metastate.
    - Numerical construction of the Aizenman-Wehr Metastate
    - Observables and Numerical Simulations.
    - Results.
  - Conclusions and Open Problems.

Different Theories and Models

The Droplet Model

- Based on the Migdal-Kadanoff implementation (approximate) of the Renormalization Group (exact in $D = 1$).
- *Disguished Ferromagnet*: Only two pure states with order parameter $\pm q_{\mathrm{EA}}$ (related by spin flip).
- Compact Excitations of fractal dimension d_f. The energy of a excitation of linear size L grows as L^θ.
- Any amount of magnetic field destroys the spin glass phase (even for Heisenberg spin glasses).
- Trivial probability distributions of the overlaps.

Replica Symmetry Breaking

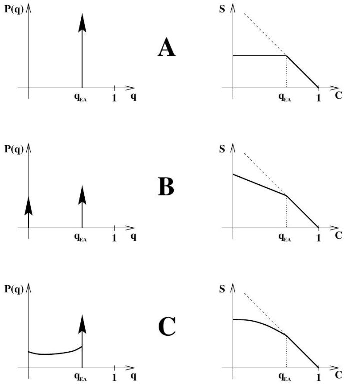
Summary

- Exact in $D = \infty$.
- Infinite number of phases (pure states) not related by any kind of symmetry.
- These (pure) states are organized in a ultrametric fashion.
- The spin glass phase is stable under (small) magnetic field.
- The excitations of the ground state are space filling.
- Overlap equivalence: All the definitions of the overlap are equivalent.
- Stochastic Stability. The Spin Glass Hamiltonian is "generic" under Random Perturbations.

Note: In a pure state, α, the clustering property holds:
$\langle S_i S_j \rangle_\alpha - \langle S_i \rangle_\alpha \langle S_j \rangle_\alpha \to 0$ as $|i - j| \to \infty$.

Different Theories (Comparison).

$$q_{\alpha,\beta} = \langle s_i \rangle_\alpha \langle s_i \rangle_\beta$$



Stochastic Stability

- A model is stochastically stable under a given class of random perturbations
$$\mathcal{H} \to \mathcal{H} + \epsilon \mathcal{H}_R$$
if its averaged free energy is differentiable with respect to ϵ and the thermodynamical limit commutes with $\partial/\partial\epsilon$.
- If we change the free energies of the states (F_α) by a random amount: $G_\alpha = F_\alpha + \epsilon r_\alpha$ (r_α are uncorrelated random numbers), then the probability distribution of the free energies is invariant:
$$\rho(F) = \rho(G)$$
The weight of the state α is $w_\alpha \propto \exp(-\beta F_\alpha)$.
- It is equivalent to the replica equivalence property of the Parisi's matrices Q_{ab} ($D = \infty$).
- Exact in $D = \infty$, and strongly tested in numerical simulations ($D = 3$ and 4.)

Order Parameter from Experiments?

- In experiments the magnetization (M) and susceptibility (χ) are measured.
- One can extract the spin glass susceptibility, $\chi_{\rm SG} = V\overline{\langle q^2 \rangle}$ via

$$\chi - \frac{M}{H} = \chi_2 H^2 + \chi_4 H^4 + O(H^6)$$

$$\chi_{\rm SG} \propto \chi_2$$

But:

- We need to compute the equilibrium susceptibility (low frequencies).
- In order to extract $P(q)$ we need to know the microscopic structure of the spins (the configurations). Solution: $\rightarrow$ FDR out of equilibrium!

FDR: Definitions.

- We start with the perturbed Hamiltonian $\mathcal{H}'$:

$$\mathcal{H}' = \mathcal{H} + \int h(t) A(t)\, dt \,,$$

- We can define the autocorrelation function, $C(t_1, t_2)$ and the response function $R(t_1, t_2)$,

$$C(t_1, t_2) \equiv \langle A(t_1) A(t_2) \rangle \ ,$$

$$R(t_1, t_2) \equiv \left. \frac{\delta \langle A(t_1) \rangle}{\delta h(t_2)} \right|_{h=0} \ .$$

In spin models: $A(t) = \sigma_i(t)$.

Equilibrium (Fluctuation-Dissipation Theorem)

$$R(t_1, t_2) = \frac{1}{T} \theta(t_1 - t_2) \frac{\partial C(t_1, t_2)}{\partial t_2}$$

FDR: Out of equilibrium

$t_1 > t_2$

$$R(t_1, t_2) = X(C(t_1, t_2)) \left(\frac{1}{T} \theta(t_1 - t_2) \frac{\partial C(t_1, t_2)}{\partial t_2} \right)$$

- At equilibrium $X = 1$.
- Mean Field. Cugliandolo and Kurchan. PRL 71, 173 (1993).
- Finite Dimensional spin glasses: Franz et al. PRL 81, 1758 (1998).

FDR: Connection Out of Equilibrium and Equilibrium

- If $C(t_1, t_2) = q$, then $X(C(t_1, t_2)) \to x(q)$
- Where $x(q)$ is the cumulative distribution of the overlap computed in the equilibrium regime:

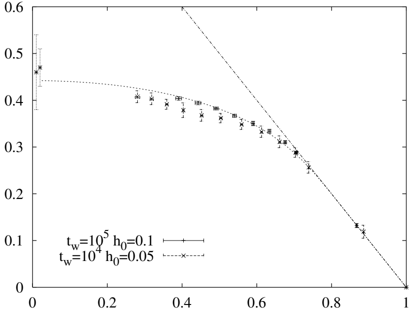
$$x(q) = \int_{-1}^{q} dq' \; P(q')$$

$$m(t, t_w) = h \int_{t_w}^{t} dt' R(t, t') \; , \; h(t) = h\theta(t - t_w)$$

$$m(t, t_w) \simeq h\beta \int_{t_w}^{t} dt' X[C(t, t')] \frac{\partial C(t, t')}{\partial t'} = h\beta \int_{C(t,t_w)}^{1} du X[u] \equiv h\beta S[C]$$

$$T\chi(t, t_w) = T \frac{m(t, t_w)}{h} = S[C(t, t_w)]$$

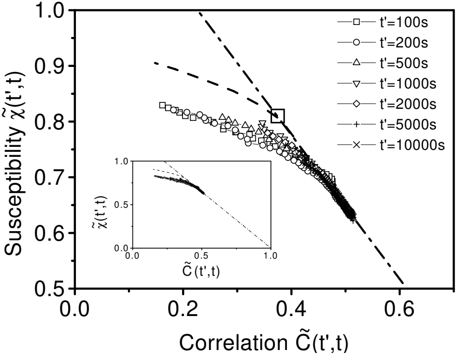
$P(q)$ from FDR



Marinari et al. JPA 31, 2611 (1998).

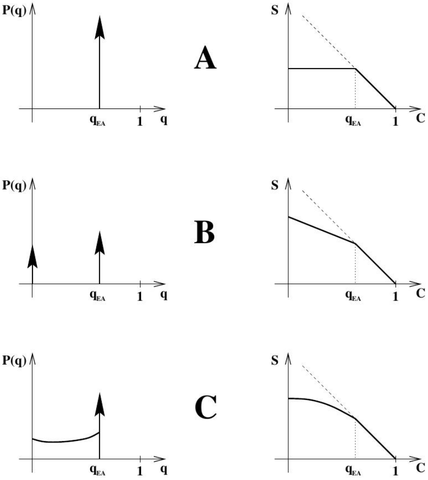
$$P(C) = -\frac{\partial^2 S(C)}{\partial C^2}$$

FDR: Experiments

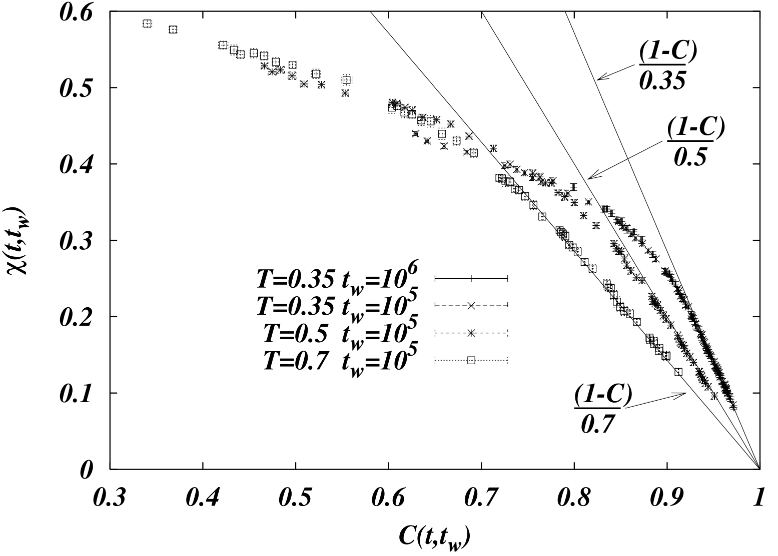


$CdCr_{1.7}In_{0.3}S_4$. $T_g = 16.2$K. $T = 0.8T_g$.
Hérisson and Ocio. PRL 88, 257202 (2002)

Different Theories (Comparison).

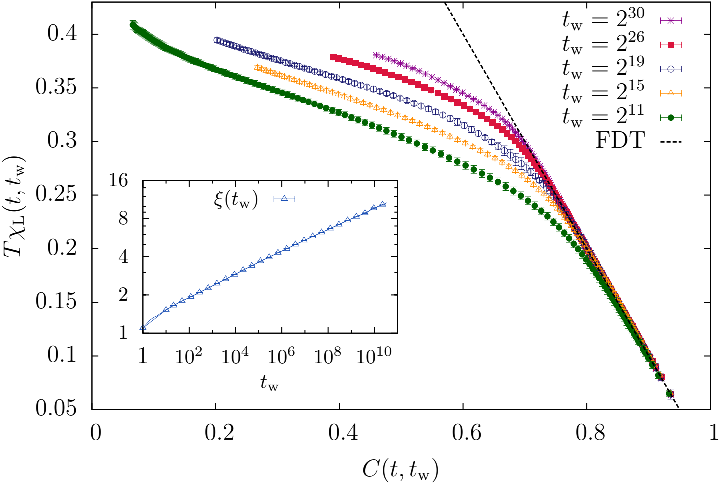


$P(q)$ from FDR



Marinari et al. JPA 33, 2373 (2000).

FDT: Numerical Results



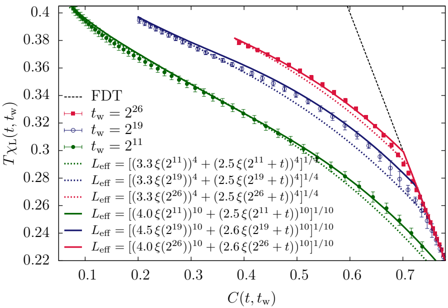
Janus Coll. Proc. Nat. Acad. Sci. USA. 114 (8) 1838-1843 (2017).

FDT: Numerical Results: Synthetic $P(q)$

$$P_{\rm syn}(q,L) = (P(0,L) + P_1 q^2)\theta(q_{\rm EA}^{(L)}) + [1 - x(q_{\rm EA}^{(L)})]\delta(q - q_{\rm EA}^{(L)})$$

$$q_{\rm EA}^{(L)} = q_{\rm EA}^{(L=\infty)} + \frac{A}{L^{0.38}}$$

$$S_{\rm syn}(C,L) = \min[1 - C, S(0,L) - P_0 C^2 - P_1 C^4]$$



Conclusions

FDR in spin glasses

- Solid analytical base both in Mean Field and also at finite dimensions.
- It has been implemented in (difficult) experiments.
- Numerical simulations are reaching the experimental time region.
- The emerging picture points out a low temperature spin glass phase with Replica Symmetry Breaking properties.

Phases and Thermodynamic limit in Pure systems

- A state is a probability distribution (or an average, or a linear functional).
- In the non disordered Ising model, we can define two pure states

$$\langle(\cdots)\rangle_+ = \lim_{h\to 0+} \lim_{L\to\infty} \langle(\cdots)\rangle_{(L,h)} \ ,$$

$$\langle(\cdots)\rangle_- = \lim_{h\to 0-} \lim_{L\to\infty} \langle(\cdots)\rangle_{(L,h)}$$

- Mixtures can be analyzed via the decomposition:

$$\langle(\cdots)\rangle = \alpha\langle(\cdots)\rangle_+ + (1-\alpha)\langle(\cdots)\rangle_-$$

- In particular,

$$\lim_{L\to\infty} \langle(\cdots)\rangle_{(L,h=0)} = \frac{1}{2}\langle(\cdots)\rangle_+ + \frac{1}{2}\langle(\cdots)\rangle_-$$

Phases and Thermodynamic limit in Pure systems

- Dobrushin-Lanford-Ruelle states (locally equilibrium states).
- Finite volume pure states (conditional probabilities of DLR states).
- The states form a convex set. $\Gamma = \sum_i \alpha_i \Gamma_i$ with $\sum_i \alpha_i = 1$, $\alpha_i > 0$. (Mixtures)
- Pure states (phases): extremal points of the convex set.
- Inside a pure state, intensive magnitudes do not fluctuate, equivalently, the connected correlation functions verify the clustering property.

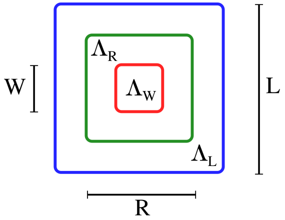
Phases and Thermodynamic limit in Disordered systems: The Metastate.

- Chaotic Size Dependence: The state $\Gamma_{L,J}$ does not approach a unique limit $\Gamma_J = \lim_{L\to\infty} \Gamma_{L,J}$ (when we increase the size we add additional random bonds to the Hamiltonian).
  1. Non-disordered Ising model with fixed boundary conditions (the values of the spins on the boundary change with L).
  2. The magnetization in the RFIM at low temperaures does not converge. (It is given by $\mathrm{sign}(\sum_i h_i)$ which is a random variable).
  3. Chaotic Pairs (CP) scenario. The model presents two states (spin flip related) for any large but finite size. This pair of states changes chaotically with L.

- Newman-Stein Metastate.
  Despite the lack of limit of $\Gamma_{L,J}$, one can compute the frequency of a given state appears as $L \to \infty$. The set of these frequencies is the Newman-Stein metastate.

Construction of the Aizenman-Wehr Metastate

- Internal disorder $\mathcal{I}$ in the region Λ_R.
- Outer disorder $\mathcal{O}$ in the region $\Lambda_L \backslash \Lambda_R$.
- Measurements in $\Lambda_W \subset \Lambda_R$.
- The wanted limit: $\Lambda_W << \Lambda_R << \Lambda_L$.



Construction of the Aizenman-Wehr Metastate

- Let us compute

$$\kappa_{\mathcal{I},R}(\Gamma) = \lim_{L\to\infty} \mathbb{E}_{\mathcal{O}}\left[\delta^{(F)}\left(\Gamma - \Gamma_{\mathcal{J},L}\right)\right]$$

- If the limit

$$\kappa(\Gamma) = \lim_{R\to\infty} \kappa_{\mathcal{I},R}(\Gamma)$$

exists, it does not longer depend on the internal disorder $\mathcal{I}$ and provides the AW metastate.

- The metastate-averaged state (MAS), $\rho(\underline{s})$, is defined via $\langle\cdots\rangle_\rho \equiv [\langle\cdots\rangle_\Gamma]_\kappa$
- Restricted to Λ_W, a state $\Gamma(\underline{s})$ is a set of probs. $\{p_\alpha\}_{\alpha=1,\ldots,2^{W^d}}$. This is a point of the hyperplane $\sum_\alpha p_\alpha = 1$.
- The metastate is a probability distribution on this hyperplane.
- The MAS $\rho(\underline{s})$ is the average of this distribution, and it is itself a point on the hyperplane (hence, the MAS is a state itself).

Some Observables

- The MAS spin glass correlation function:

$$C_\rho(x) = \overline{[\langle s_0 s_x \rangle_\Gamma]^2_\kappa} = \frac{1}{\mathcal{N}_\mathcal{I}} \sum_{\mathtt{i}} \left(\frac{1}{\mathcal{N}_\mathcal{O}} \sum_{\mathtt{o}} \langle s_0^{\mathtt{i;o}} s_x^{\mathtt{i;o}} \rangle \right)^2 =$$
$$= \frac{1}{\mathcal{N}_\mathcal{I}} \sum_{\mathtt{i}} \frac{1}{\mathcal{N}_\mathcal{O}^2} \sum_{\mathtt{o,o'}} \langle s_0^{\mathtt{i;o}} s_x^{\mathtt{i;o}} s_0^{\mathtt{i;o'}} s_x^{\mathtt{i;o'}} \rangle \sim |x|^{-(d-\zeta)} \ ,$$

- Remember $\langle \cdots \rangle_\rho \equiv [\langle \cdots \rangle_\Gamma]_\kappa$.
- ζ is the Read's exponent.
- $\mathtt{i} = 0, \ldots, \mathcal{N}_\mathcal{I}$. $\mathcal{N}_\mathcal{I} = 10$ instances of internal disorder ($\mathcal{I}$).
- $\mathtt{o} = 0, \ldots, \mathcal{N}_\mathcal{O}$. $\mathcal{N}_\mathcal{O} = 1280$ instances of outer disorder ($\mathcal{O}$).

Physics behind the ζ-exponent

- $\log \mathcal{N}_{\text{states}}(W) \sim W^{d-\zeta}$. $\zeta \geq 1$.
- If $\zeta < d$ we have a dispersed metastate.
- Reid's conjecture $\zeta = \zeta_{q=0}$.
- The constrained (on q) equilibrium overlap-overlap correlation function is defined as:

$$G(\boldsymbol{r}, q) \equiv \overline{\langle q(\boldsymbol{r}) q(0) \rangle}_q - q^2 \sim \frac{1}{r^{d-\zeta_q}}$$

- Above the upper critical dimension (de Dominicis et al.):
  - $\zeta_{q=0} = 4$.
  - $\zeta_q = 3$, $0 < q < q_{\text{EA}}$.
  - $\zeta_{q_{\text{EA}}} = 2$.
- Dynamical interpretation: $G_d(\boldsymbol{r}, q, t) \equiv \overline{\langle q(\boldsymbol{r}, t) q(0, t) \rangle}$ plays the role of $C_\rho(\boldsymbol{r})$, with $R \sim \xi(t)$. [Manssen, Hartmann and Young].

Some Observables

- The (generalized) overlap on the box Λ_W:

$$q_{\mathtt{i};\mathtt{o},\mathtt{o}'} \equiv \frac{1}{W^3} \sum_{x\in\Lambda_W} \sigma_x^{\mathtt{i};\mathtt{o}} \tau_x^{\mathtt{i};\mathtt{o}'} \ .$$

- Probability density functions of $q_{\mathtt{i};\mathtt{o},\mathtt{o}'}$:

$$P(q) = \frac{\sum_{\mathtt{i}} P_{\mathtt{i}}(q)}{\mathcal{N}_{\mathcal{I}}} \quad , \quad P_{\mathtt{i}}(q) = \frac{1}{\mathcal{N}_{\mathcal{O}}} \sum_{\mathtt{o}} \langle \delta(q - q_{\mathtt{i};\mathtt{o},\mathtt{o}}) \rangle,$$

$$P_\rho(q) = \frac{\sum_{\mathtt{i}} P_{\rho,\mathtt{i}}(q)}{\mathcal{N}_{\mathcal{I}}} \quad , \quad P_{\rho,\mathtt{i}}(q) = \frac{1}{\mathcal{N}_{\mathcal{O}}^2} \sum_{\mathtt{o},\mathtt{o}'} \langle \delta(q - q_{\mathtt{i};\mathtt{o},\mathtt{o}'}) \rangle.$$

- $P(q)$ is the standard probability distribution of the overlap.

Some Observables

- Although $P_\rho(q) \to \delta(q)$ as $L \to \infty$, the scaling of its variance provides us with useful information:

$$\chi_\rho = \sum_{x \in \Lambda_W} C_\rho(x) = W^d \int q^2 P_\rho(q)\, dq \sim W^\zeta \,.$$

- $P_\rho\big(q/(W^{-(\zeta-d)/2})\big)$ is Gaussian.

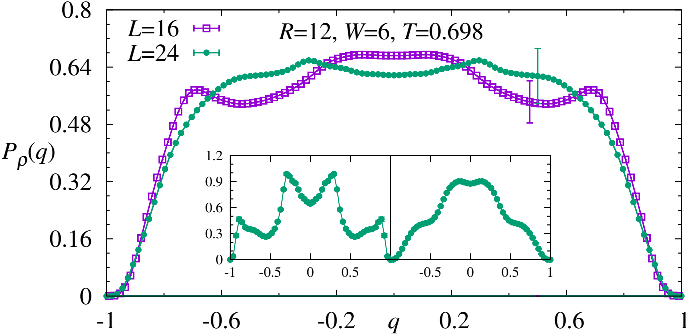
Numerical Simulations and Observables

Numerical Simulations

- We have simutated the three-dimensional Edwards-Anderson model with periodic boundary conditions and bimodal disorder.
- We have implemented the Parallel Tempering Method (with Metropolis single spin-flip).
- We have used multispin coding (128 bits).
- Equilibration was assessed on a sample-by-sample basis.
- For large systems, the worse samples were simulated using multisite multispin coding.
- We have run on conventional supercomputers.
- We have simulated $L = 8, 12, 16$ and 24.
- The lowest temperature $T_{\min} = 0.698 = 0.64 T_c$

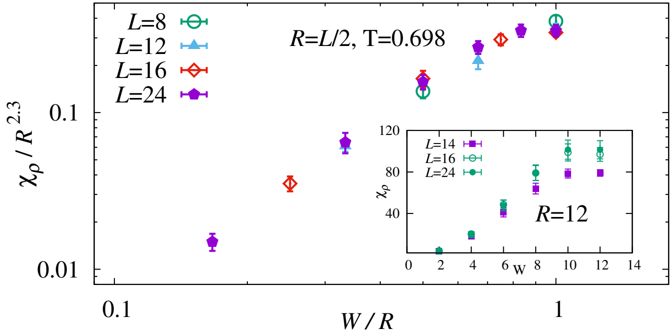
A. Billoire , L. A. Fernández, V. Martín-Mayor, E. Marinari, A. Maiorano, J. Moreno-Gordo, G. Parisi, F. Ricci-Tersenghi and J. J. Ruiz-Lorenzo Phys. Rev. Lett. **119**, 0327203(2017)

Results: the MAS overlap probability distribution



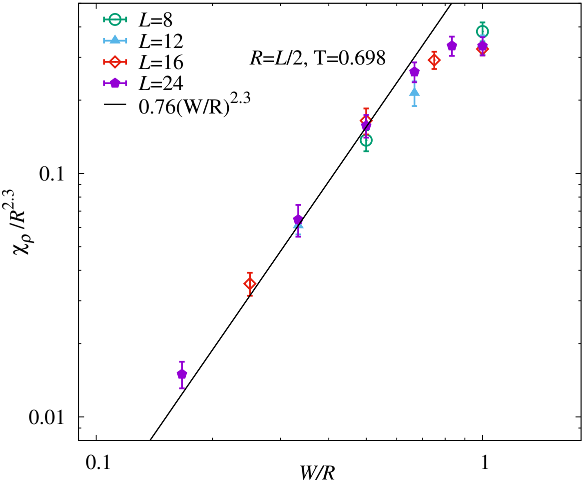
Notice that for $R/L = 3/4$ there are no finite size effects. We will take in the following the safe ratio $R/L = 1/2$.

Results: Scaling



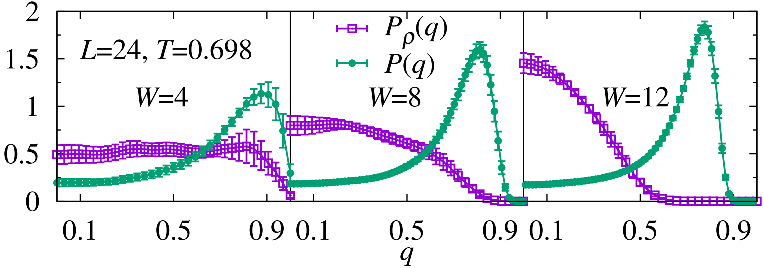
The scaling regime extends to $W/R = 0.75$.

Results: Scaling



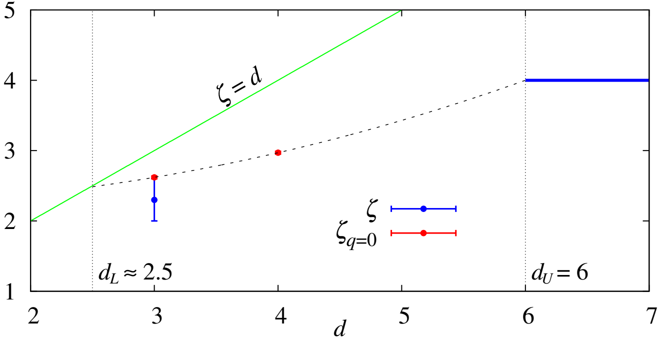
$\zeta = 2.3(3)$, to be compare with $\zeta_{q=0} = 2.62(2)$

Results: Comparison $P(q)$ and $P_\rho(q)$



$P(q)$ and $P_\rho(q)$ are different: Dispersed Metastate.

Results: ζ-exponent



Conclusions

- We have constructed numerically the Aizenman-Wehr metastate.
- We have found strong evidences for a dispersed metastate.
- Only RSB and CP have a dispersed metastate.
- Strong numerical support on the Reid's conjecture $\zeta = \zeta_{q=0}$.

Open Problems

- Understand the strong coupling behavior of the RG of the model in a field.
- Numerical characterization of the Phase Transition in an external magnetic field $d = 3$.
- Numerical characterization of the ultrametricity in $d = 3$ and $h = 0$.
- Numerical and analytical calculation of the correlation function exponents below T_c and $h = 0$.