Numerical test of the Cardy-Jacobsen conjecture in the site-diluted Potts model in three dimensions

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Outline of the Talk

- Effect of the disorder on a first order phase transition ($D = 2$).
  - The Aizenman-Wehr theorem ($D = 2$).
  - The Cardy-Jacobsen conjecture ($D > 2$).
- The three-dimensional diluted Potts model with $Q = 4$ and 8 states:
  - Numerical Simulations.
  - Observables.
  - Scaling near a tricritical point.
  - Our results: Testing the Cardy-Jacobsen conjecture.
- Conclusions.

Effect of Quenched disorder on first order phase transitions.

Some examples:

1. Tilt ordering.
2. Ferroelectrics.
3. Random block copolymers.
4. Topological phases in correlated electron systems.
5. Surface waves.
6. Multiplicative noise in RLC circuits.
7. Liquid crystals.

Aizenman and Wehr Theorem

In TWO dimensions the slightest concentration of impurities switches the transition from first order to second order.

The Cardy-Jacobsen conjecture ($D > 2$) (I).

[PRL 79, 4063 (1997), NPB 515, 701 (1998) and arXiv:9806355]

The models

- RFIM: $\mathcal{H} = -J \sum_{<i,j>} s_i s_j - \sum_i h_i s_i - H \sum_i s_i$.
  h_R^2 is the variance of the random fields $\{h_i\}$.
  (see Natterman and Belanger reviews in *Spin Glasses and Random Fileds* (A.P. Young editor)).
- DAFF: $\mathcal{H} = +J \sum_{<i,j>} \epsilon_i \epsilon_j s_i s_j - H \sum_i s_i$.
  (see Fernandez et al. PRB 84, 100408(R) (2011)).
- Q-states diluted Potts model: $\mathcal{H} = -\sum_{<i,j>} \epsilon_i \epsilon_j \delta_{s_i,s_j}$.
  $\epsilon_i = 1$ with probability p. In addition $w \propto p$.

The Cardy-Jacobsen conjecture ($D > 2$) (II).

CJ mapping in the limit of a strong first order phase transition (FOT)

Strong FOT		Random Field
Σ/kT_c	$\Longleftrightarrow$	J/kT
$(L/kT_c)w$	$\Longleftrightarrow$	h_{RF}/kT
$(T - T_c)L$	$\Longleftrightarrow$	HM

For the $Q \gg 1$ diluted Potts Model

$$w \log Q \Longleftrightarrow h_{RF}$$

$$\log Q \Longleftrightarrow J$$

$$t \log Q \Longleftrightarrow H$$

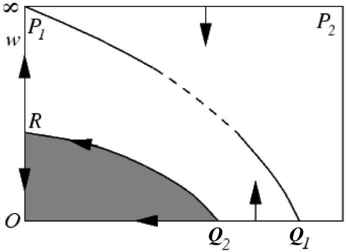
The Cardy-Jacobsen conjecture ($D > 2$) (III).

The critical exponents are:

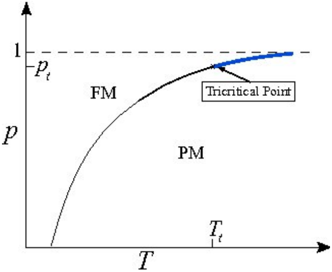
- $y_p = y_{h_R/J}^{\mathrm{RFIM}} = \frac{1}{\nu^{\mathrm{RFIM}}}$.
- $y_T = y_H^{\mathrm{RFIM}} - \theta = \frac{1}{2}\left(D - \theta + 2 - \eta^{\mathrm{RFIM}}\right)$.
- The latent heat vanishes with β^{RFIM}.
- The surface tension exponent μ is given by $\mu = D - \theta - 1$ (modified Widom law).

The Cardy-Jacobsen conjecture ($D > 2$) (IV).

RG behavior (Cardy & Jacobsen)



Phase diagram



Some previous work:

- 2nd Order part. Ballesteros et al. PRB 61, 3215 (2000).
- 1st Order part. Chatelain et al. Nucl. Phys. B719, 275 (2005).
- Tricritical point. Mercaldo et al. PRE 73, 026126 (2006); Fernandez et al. PRL 100, 57201 (2008).

Simulations Details

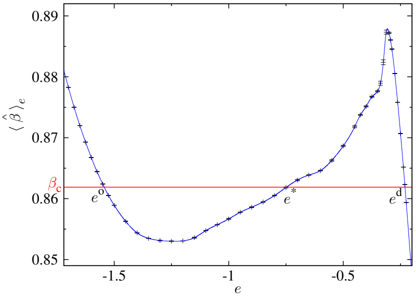
Simulations

- We have used the Extended Microcanonical Approach (Martín-Mayor, PRL 98, 137207 (2007)).
- Spin Update: Swendsen-Wang and Metropolis.
- $12 \leq L \leq 64$ and $0.6 \leq p \leq 1$.
- 500 samples on each lattice size.
- $Q = 8$. However we have reanalyzed our old $Q = 4$ data.

Ibercivis Citizen Computer

- Boinc based volunteer platform.
- Stable and open infrastructure.
- Around 10^5 cores and 2×10^5 users.
- 3×10^6 CPU hours [2.4 GHz single CPU].
- More information `www.ibercivis.com`

$Q = 8$, $L = 24$ and $p = 0.95$.

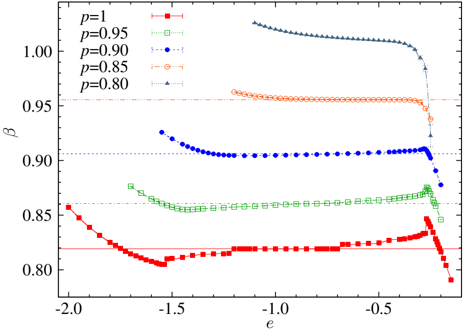


- We compute β_c^L using the Maxwell rule.
- In addition:

$$\Delta e = e_d - e_o\,; \;\; \Sigma(L) = \frac{N}{2L^{D-1}} \int_{e_L^*(\beta_c^L)}^{e_L^d(\beta_c^L)} de \left(\langle\hat{\beta}\rangle_e - \beta_c^L \right)$$

Maxwell Construction (II)

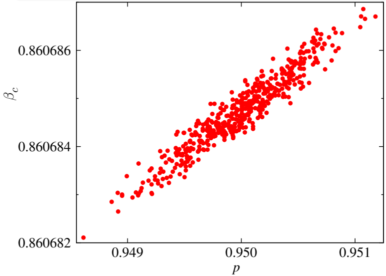
Behavior of the Maxwell construction ($Q = 8$ and $L = 48$) varying the dilution:

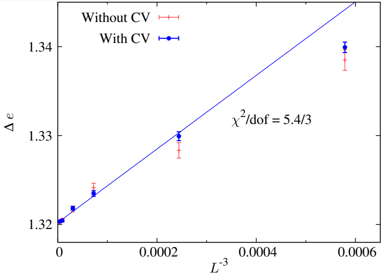


Control Variates

- We want to improve A using its correlations with B (Fernandez et al., PRE 79, 051109 (2009)).
- If $\langle B \rangle = 0$, we can define $\hat{A} \equiv A + \alpha B$, so $\langle \hat{A} \rangle = \langle A \rangle$. But depending on α: $\mathrm{var}(\hat{A}) < \mathrm{var}(A)$. The optimal value being:

$$\alpha_{\mathrm{opt}} = \frac{\mathrm{cov}(A, B)}{\sqrt{\mathrm{var}(A)\mathrm{var}(B)}}$$





$$\beta'_c = \beta_c + \alpha(p_i - p)\,,\ p_i = N_i/N\,,\ p = 0.95$$

Scaling Near a tricritical point

- Close to the tricritical point at $(p_\mathrm{t}, T_\mathrm{t} = T_\mathrm{c}(p_\mathrm{t}))$

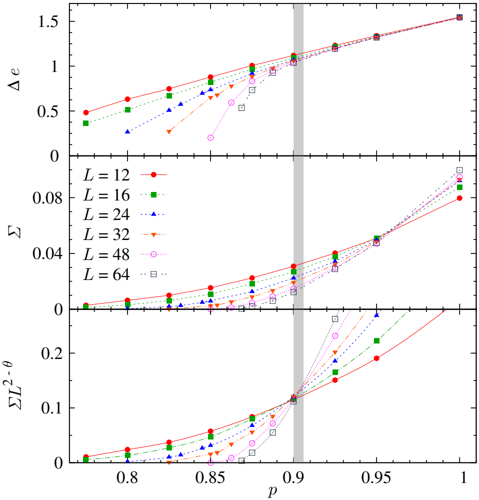
$$O(L, p_\mathrm{t} + \delta p, T_\mathrm{t} + \delta T) = L^x G(L^{y_T} u_T, L^{y_p} u_p)\,,$$

- $u_T = f_T(\delta T, \delta p)$ and $u_p = f_p(\delta T, \delta p)$.
- The Maxwell construction enforces $u_T = 0$ (with accuracy of order $O(L^{-D})$), so $u_p \propto \delta p$
-

$$O(L, p, \mathrm{Maxwell}) = L^x \left(\tilde{G}\big(L^{y_p}(p - p_\mathrm{t})\big) + \mathcal{O}(L^{y_T - D}) \right)\,.$$

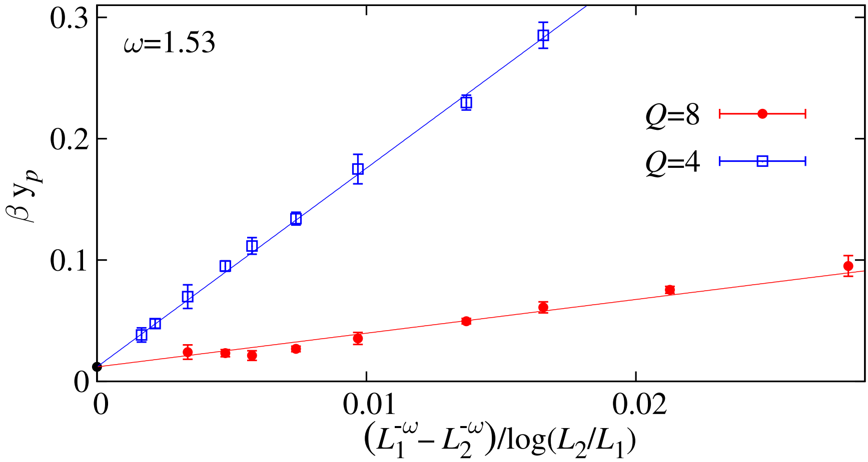
- So, the Maxwell construction allows us to employ standard FSS, with an effective scaling-corrections exponent $\omega = D - y_T$.
- Notice that $u_Q = 1/\log Q$ is irrelevant with exponent $-\theta$. However, numerically $\theta \simeq \omega$.

Results (I)



The RFIM value of $\theta = 1.469(20)$ has been used.

Results (II)



Fixing $\beta y_p = 0.0119(4)$: $\omega = 1.53(5)(3)$ $(\chi^2/\mathrm{d.o.f.} = 14/15)$.

Results (III): Testing C-J Conjecture

- From the Latent Heat:
  - $\beta y_p = 0.0022(48)(3)$ and $\omega = 1.36(8)(1)$.
  - Fixing $\beta y_p = 0.0119(4)$:
    - $\omega = 1.53(5)(3)$.
    - $y_T = D - \omega$ so $y_T = 1.47(8)$. [Mercaldo et al: $y_T = 1.49(9)$.]
- From the Surface Tension:
  - $y_p = 0.775(46)(1)$ fixing $\omega = 1.36(8)(1)$.
  - $y_p = 0.779(41)(4)$ fixing $\omega = 1.53(5)(3)$.
  - $\alpha = (2y_p - D + \theta)/y_p = 0.030(10)$.
  - By fitting $\Sigma(L, p_t^{L,2L}) = A_Q L^{\theta-2}(1 + B_Q L^{-\omega})$ using $\omega = 1.5(1)$ we obtain $\theta = 1.52(11)(2)$.
- RFIM values:
  - $\theta = 1.469(20)$ and $\beta/\nu = 0.0119(4)$.
  - $0.73 \leq 1/\nu \leq 1.12$.
  - $y_H - \theta = 1.52(2)$; $\omega = 1.48(2)$.
  - Experimentally $\alpha \simeq 0$ (maybe a log div.).

Conclusions

- We have presented a finite size scaling study of the $3D$ $Q = 4$ and 8 diluted Potts model.
- We have used the citizen computer IBERCIVIS (`www.ibercivis.com`) for the equivalent of 3×10^6 CPU hours.
- We have run the extended microcanonical method.
- By considering leading scaling corrections we have shown that the Universality class for the tricritical point is that of the RFIM such as was predicted by Cardy and Jacobsen.