Ising spin-glass transition in magnetic field out of mean-field: Numerical simulations and experiments

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- What are spin glasses?
- Different Theories and Models (droplet, TNT and RSB).
- Spin Glasses with Long Range Interactions.
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  - Experiments.
- Conclusions.

What are Spin glasses

- Materials with disorder and fustration.
- Quenched disorder.
- Canonical Spin Glass: Metallic host (Cu) with magnetic impurities (Mn). RKKY interaction between magnetic moments: $J(r) \sim \frac{\cos(2k_F r)}{r^3}$.

Some Definitions

- The typical Spin Glass Hamiltonian:

$$\mathcal{H} = -\sum_{i,j} J_{ij}\sigma_i\sigma_j$$

- The order parameter is:

$$q_{\mathrm{EA}} = \overline{\langle\sigma_i\rangle^2}$$

Using two real replicas:

$$\mathcal{H} = -\sum_{i,j} J_{ij}\left(\sigma_i\sigma_j + \tau_i\tau_j\right)$$

Let $q_i = \sigma_i\tau_i$ be the normal overlap, then: $q_{\mathrm{EA}} = \overline{\langle\sigma_i\tau_i\rangle}$.
We also define the link overlap: $q_{i,\mu}^{\mathrm{link}} = q_i q_{i+\mu}$.

Different Theories and Models

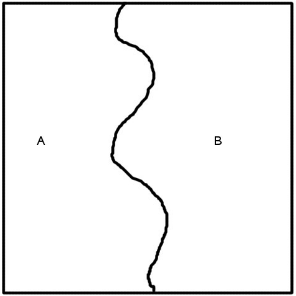
The Droplet Model

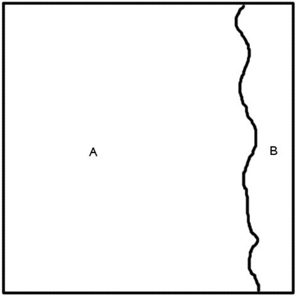
- Based on the Migdal-Kadanoff implementation (approximate) of the Renormalization Group (exact in $D = 1$).
- *Disguished Ferromagnet*: Only two pure states with order parameter $\pm q_{\rm EA}$ (related by spin flip).
- Compact Excitations of fractal dimension d_f. The energy of a excitation of linear size L grows as L^θ.
- Any amount of magnetic field destroys the spin glass phase (even for Heisenberg spin glasses).
- Trivial probability distributions of the overlaps (both the normal overlap and the link one).

Different Theories and Models

The Trivial Non Trivial (TNT) Model

- *Disguished Ferromagnet with Anti Periodic Boundary conditions*.
- Trivial probability distributions for the link overlap (the interfase has no effect) but Non Trivial probability distribution for the normal one (induced by the interface).





Different Theories and Models

Replica Symmetry Breaking (RSB) Theory

- Exact in $D = \infty$.
- Infinite number of phases (pure states) not related by any kind of symmetry.
- These (pure) states are organized in a ultrametric fashion.
- The spin glass phase is stable under (small) magnetic field.
- The excitations of the ground state are space filling.

Long Range Interactions

- Hamiltonian (Action) for the long range model ($J(r) \simeq r^{-\rho/2}$):

$$S_n = H_n \propto \int d^d k \left(k^{\rho-d} + \tau\right) \mathrm{Tr} Q^2 + \int d^D x \left[g_3 \mathrm{Tr}(Q^3) + \lambda \sum Q_{ab}^4\right]$$

- $\dim_p(g_3) = d - \frac{3}{4}\rho$. In MF: $\eta = d + 2 - \rho$ (holds in IRD!) and $1/\nu = \rho - d$.
- Hence, the Mean Field and Infrared region are ($d = 1$):

ρ	$D(\rho)$	transition type
≤ 1	∞	Bethe lattice like
$(1, 4/3]$	$[6, \infty)$	2^{nd} order, MF
$(4/3, 2]$	$[2.5, 6)$	2^{nd} order, non-MF
2	2.5	Kosterlitz-Thouless or $T = 0$-like
> 2	< 2.5	none

- It is possible to show (equivalence D-SR and $1d$-LR):

$$\frac{2 - \eta(D)}{D} = \rho - 1 \; ; \; \rho = 1.8 \rightarrow D = 3$$

Numerical Simulations

- The spins live on a finite connetivity network ($z = 6$) with periodic boundary conditions: $J_{ij} = 0, \pm 1$ with $P(J_{ij} \neq 0) \propto r_{ij}^{-\rho}$. With this choice one has $\overline{J_{ij}^2} \propto r_{ij}^{-\rho}$.
- We have implemented the Parallel Tempering Method.
- We have used multispin coding (64 bits) on a C++ code.
- We have simulated a Gaussian magnetic field and only two replicas.
- We have run on PC's Clusters.

Some Observables

- The spin glass correlation function:

$$C(x) = \sum_{i=1}^{L} \overline{\left(\langle \sigma_i \sigma_{i+x} \rangle - \langle \sigma_i \rangle \langle \sigma_{i+x} \rangle\right)^2}$$

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- FSSA in the MF regime ($1 < \rho \leq 4/3$):

$$\frac{\chi_{\mathrm{sg}}}{L^{1/3}} = \tilde{\chi}\left(L^{\frac{1}{3}}(T - T_c)\right), \quad \frac{\xi}{L^{\nu/3}} = \tilde{\xi}\left(L^{\frac{1}{3}}(T - T_c)\right)$$

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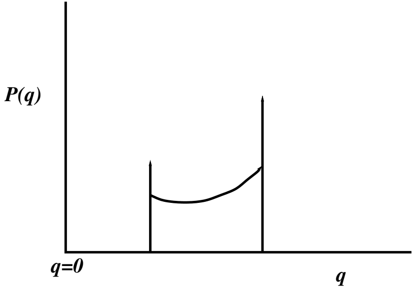
- FSSA in the IRD regime ($\rho > 4/3$):

$$\frac{\chi_{\mathrm{sg}}}{L^{2-\eta}} = \tilde{\chi}\left(L^{\frac{1}{\nu}}(T - T_c)\right), \quad \frac{\xi}{L} = \tilde{\xi}\left(L^{\frac{1}{\nu}}(T - T_c)\right).$$

The negative overlap problem

- $P(q)$ in a magnetic field: SK results and numerical ones.



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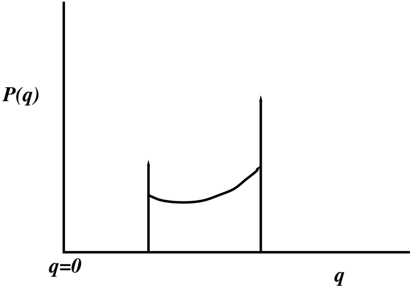

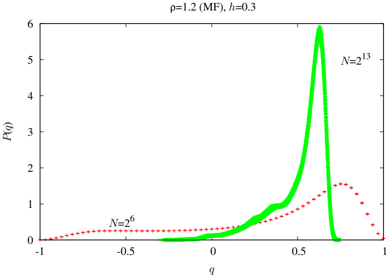

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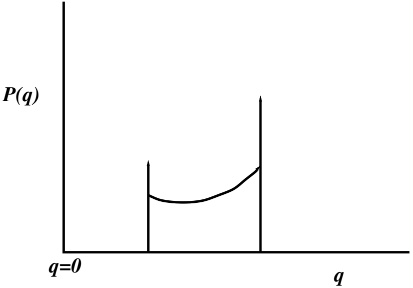
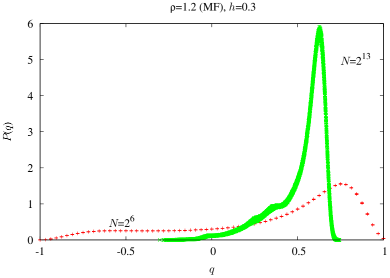
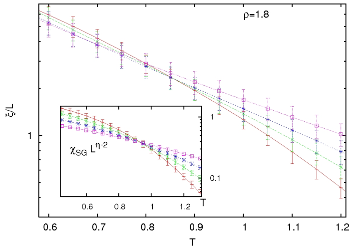




- The negative overlap region induces large corrections in $\tilde{C}(0)$!!

Numerical Analysis of the Correlation function ($h = 0$)

- $h = 0$ and $\rho = 1.8$.



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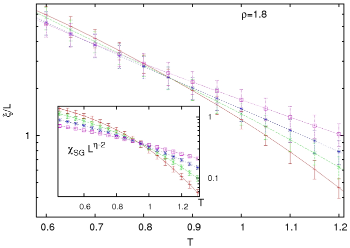

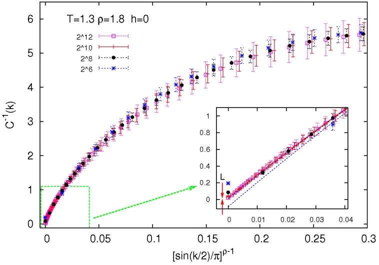

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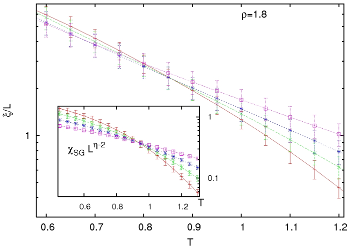
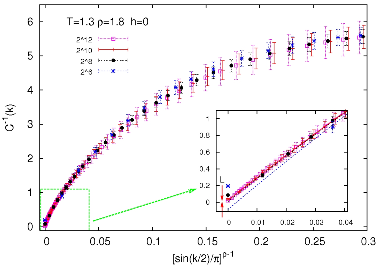
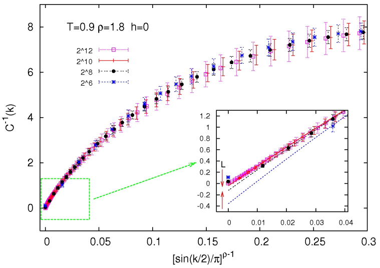

ρ=1.8
ξ/L
T
χ_{SG} $L^{\eta-2}$


T=1.3 ρ=1.8 h=0
2^12
2^10
2^8
2^6
$C^{-1}(k)$
$[\sin(k/2)/\pi]^{\rho-1}$
L


T=0.9 ρ=1.8 h=0
2^12
2^10
2^8
2^6
$C^{-1}(k)$
$[\sin(k/2)/\pi]^{\rho-1}$
L

Numerical Analysis of the Correlation function

- We will avoid the $k = 0$ value by fitting ($k > 0$):

$$\left(\frac{1}{\tilde{C}_4(k)}\right)^{\mathrm{fit}} = A(L,T) + B(L,T)[\sin(k/2)/\pi]^{\rho-1}$$

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- We fix the L-dependent critical temperature by means:

$$A(L, T_c(L)) = 0$$

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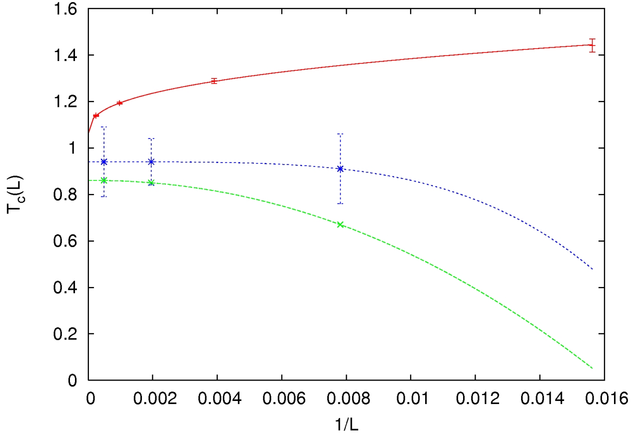
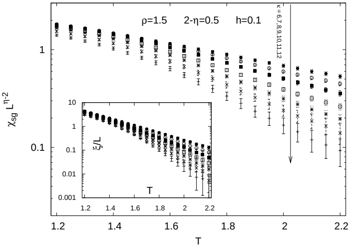


Numerical Analysis of the Correlation function ($h \neq 0$)

- $h = 0.1$ and $\rho = 1.5$.



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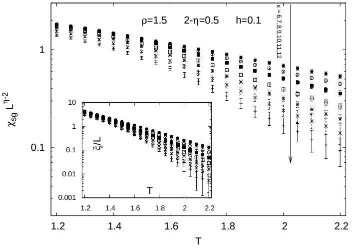

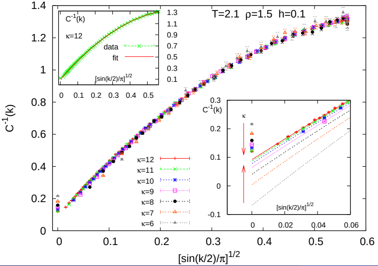

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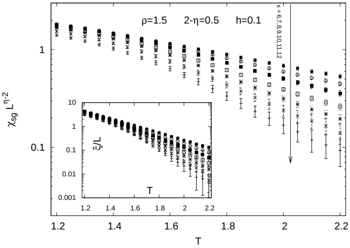
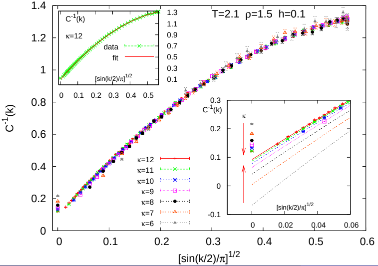
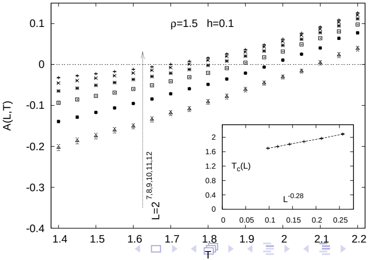




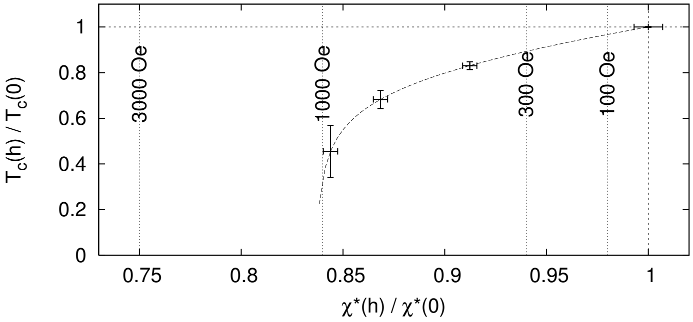


Characterization of the de Almeida-Thouless line

	ρ	"D"	h	T_c from $\tilde{C}(0)$	T_c from $A(L,T)$
MF	1.2	10	0.0	2.24(1)	2.34(3)
	1.2	10	0.1	2.02(2)	1.9(2)
	1.2	10	0.2	1.67(3)	1.4(2)
	1.2	10	0.3	1.46(3)	1.5(4)
	1.25	8	0.0	2.191(5)	2.23(2)
IRD	1.4	5	0.0	1.954(3)	1.970(2)
	1.4	5	0.1	~ 1.5	1.67(7)
	1.4	5	0.2	~ 1.1	1.2(2)
	1.5	4	0.0	1.758(4)	1.770(5)
	1.5	4	0.1	—	1.46(3)
	1.5	4	0.15	—	1.20(7)

Experiments

- Relative decrease of $T_c(h)/T_c(0)$ with increase field for $\rho = 1.5$ and $h = 0, 0.1, 0.15$ and 0.2 versus the relative decrease of χ^* (ZFC susceptibility). Experimental data from $Fe_{0.5}Mn_{0.5}TiO_3$ (Jönsson et al.).



- Hence, the critical field should be $H_c < 1000$ Oe.

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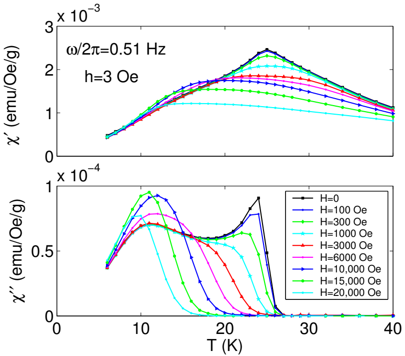

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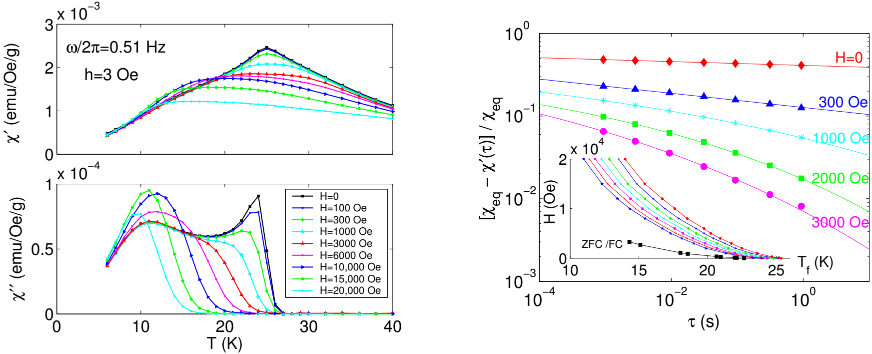
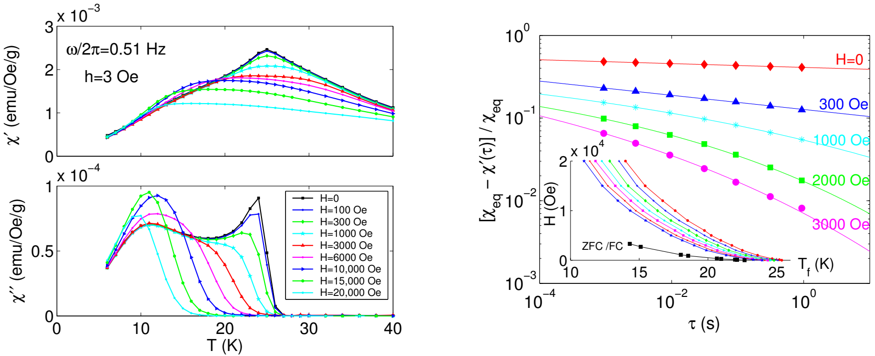


More on Experiments

- Experimental data from $Fe_{0.5}Mn_{0.5}TiO_3$ (Jönsson et al.).



- $q(t) \simeq 1/t^x$ clear signature of a Spin Glass Phase (Ogileski).

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- Recent experiments find spin glass order in a magnetic field for small external fields ($H \simeq 500$ Oe) in RKKY Spin Glasses.

Some (additonal) References:

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