The Last Survivor: a Spin Glass Phase in an External Magnetic Field.

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With Janus Collaboration (Rome-Ferrara-Zaragoza-Madrid-Extremadura)
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Plan of the Talk

- What are spin glasses?
- Different Theories: Droplet/Scaling and RSB.
- Results for the four dimensional Edwards-Anderson in a field.
- Results for the three dimensional Edwards-Anderson in a field.
- Conclusions.

What are Spin glasses

- Materials with disorder and frustration.
- Quenched disorder (similar to the Born-Oppenheimer in Molecular Physics).
- Canonical Spin Glass: Metallic host (Cu) with magnetic impurities (Mn).
- RKKY interaction between magnetic moments: $J(r) \sim \frac{\cos(2k_F r)}{r^3}$.
- Role of anisotropy: Ag:Mn at 2.5% (Heisenberg like), $CdCr_{1.7}IN_{0.3}S_4$ (also Heisenberg like) and $Fe_{0.5}Mn_{0.5}TiO_3$ (Ising like).

- Edwards-Anderson Hamiltonian:

$$\mathcal{H} = -\sum_{<ij>} J_{ij}\sigma_i\sigma_j + \sum h_i\sigma_i$$

J_{ij} are random quenched variables with zero mean and unit variance, $\sigma = \pm 1$ are Ising spins.

- The order parameter is:

$$q_{\rm EA} = \overline{\langle \sigma_i \rangle^2}$$

Using two real replicas:

$$\mathcal{H} = -\sum_{<ij>} J_{ij}\left(\sigma_i\sigma_j + \tau_i\tau_j\right) + \sum_i h_i(\sigma_i + \tau_i)$$

Let $q_i = \sigma_i\tau_i$ be the normal overlap, then: $q_{\rm EA} = \overline{\langle \sigma_i\tau_i \rangle}$.

Effective Hamiltonian:

$$\mathcal{H}_n = \int d^D x \left[(\partial_\mu Q_{ab})^2 + \tau {\rm Tr} Q^2 + g_3 {\rm Tr}(Q^3) + \lambda \sum Q_{ab}^4 + h^2 \sum Q_{ab}\right]$$

$a, b = 1, ..., n$. At the end, $n \to 0$!: $\log Z = \lim_{n\to 0} \frac{Z^n - 1}{n}$

Different Theories.

The Droplet/Scaling Theory.

- Based on the Migdal-Kadanoff implementation (approximate) of the Renormalization Group (exact in $D = 1$).
- *Disguised Ferromagnet*: Only two pure states with order parameter $\pm q_{\rm EA}$ (related by spin flip).
- Any amount of magnetic field destroys the spin glass phase (even for Heisenberg spin glasses).

Different Theories.

Replica Symmetry Breaking (RSB) Theory.

- Exact in $D = \infty$.
- Infinite number of phases (pure states) not related by any kind of symmetry.
- These (pure) states are organized in a ultrametric fashion.
- The spin glass phase is stable under (small) magnetic field. Phase transition in field: the de Almeida-Thouless line.

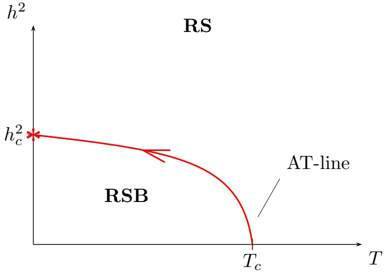
Different Theories: External Magnetic Field

RG from the paramagnetic phase:

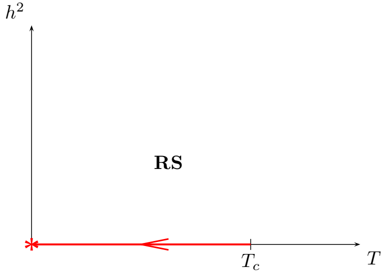
- The upper critical dimension in a field is still six (Bray and Moore).
- Due to a dangerous irrelevant variable, some observables change behavior at eight dimensions (Fisher and Sompolinsky).
- Projecting the theory (replicon mode) no fixed points were found (Bray and Roberts).
- However, starting with the most general Hamiltonian of the RS phase and relaxing the $n = 0$ condition a stable fixed point below six dimensions was found (Dominicis, Temesvári, Kondor and Pimentel)
- Temesvári is able to build the dAT slightly below $D = 6$ (but Bray and Moore, Temesvári and Parisi, Moore,...)

Different Theories: External Magnetic Field

Renormalization group predictions (from Temesvári and Parisi):



(a)



(b)

$D = 4$ ($h \neq 0$): Observables

- We have simulated using the JANUS computer.
- $L = 5, 6, 8, 10, 12$ and 16.
- Three (uniform) magnetic Fields: $h = 0.075, 0.150$ and 0.3.
- Parallel Tempering in Temperature (e.g. 32 temperatures in $L = 16$)
- Single sample thermalization protocol.

Dedicated Computers: Janus.

$D = 4$ $(h \neq 0)$: Observables

- Correlation Functions

$$G_1(\boldsymbol{r}) = \frac{1}{L^4} \sum_{\boldsymbol{x}} \overline{\left(\langle S_{\boldsymbol{x}} S_{\boldsymbol{x}+\boldsymbol{r}} \rangle - \langle S_{\boldsymbol{x}} \rangle \langle S_{\boldsymbol{x}+\boldsymbol{r}} \rangle\right)^2},$$

$$G_2(\boldsymbol{r}) = \frac{1}{L^4} \sum_{\boldsymbol{x}} \overline{\left(\langle S_{\boldsymbol{x}} S_{\boldsymbol{x}+\boldsymbol{r}} \rangle^2 - \langle S_{\boldsymbol{x}} \rangle^2 \langle S_{\boldsymbol{x}+\boldsymbol{r}} \rangle^2\right)}.$$

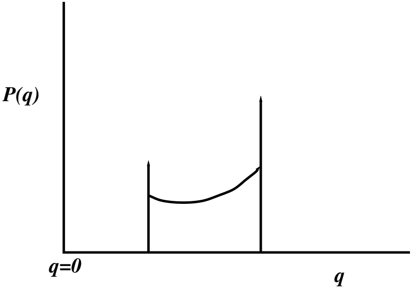
- Correlation Length:

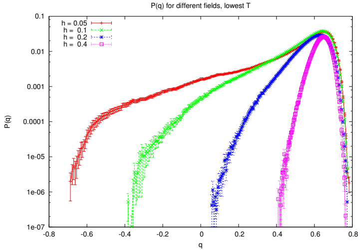
$$\xi_2 = \frac{1}{2\sin(\pi/L)} \left(\frac{\hat{G}(0)}{\hat{G}(\boldsymbol{k}_1)} - 1 \right)^{1/2},$$

where $\boldsymbol{k}_1 = (2\pi/L, 0, 0, 0)$ (and two perm.)

The negative overlap problem

- $P(q)$ in a magnetic field: SK results and numerical ones.





- The negative overlap region induces large corrections in $\tilde{G}(0)$!!
- We should avoid the mode $\boldsymbol{k} = 0$ in the analysis.

$D = 4$ $(h \neq 0)$: Observables

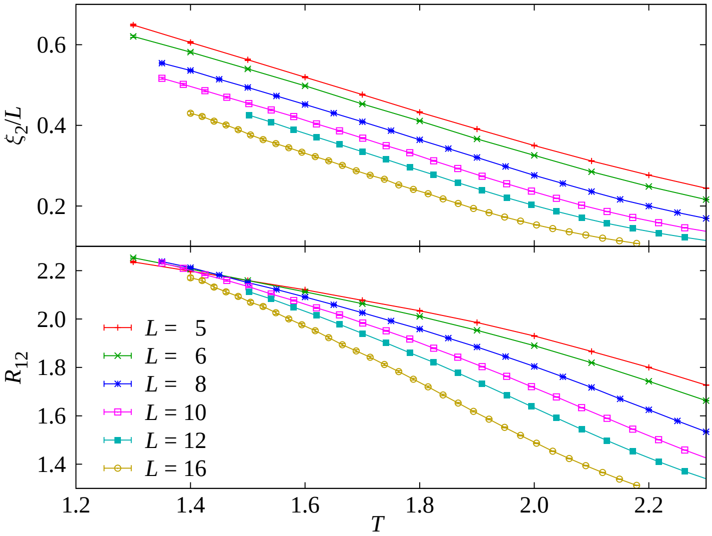
- R_{12}:

$$R_{12} = \frac{\hat{G}(\boldsymbol{k}_1)}{\hat{G}(\boldsymbol{k}_2)},$$

  where $\boldsymbol{k}_1 = (2\pi/L, 0, 0, 0)$, $\boldsymbol{k}_2 = (2\pi/L, 2\pi/L, 0, 0)$ (and permutations)
- We have checked the behavior of this observable in the EA model in $4d$ $(h = 0)$.
- And in the two dimensional (ordered) Ising model. We have been able to compute its value at criticality using Conformal Field Theory:

$$R_{12} = 1.694\ 024...$$

$D = 4 \ (h = 0.15)$


0.6
0.4
0.2
ξ_2/L
2.2
2.0
1.8
1.6
1.4
R_{12}
$L = 5$
$L = 6$
$L = 8$
$L = 10$
$L = 12$
$L = 16$
1.2
1.4
1.6
1.8
2.0
2.2
T

$D = 4$ $(h \neq 0)$

Parameter	$h = 0.3$	$h = 0.15$	$h = 0.075$
$T_c(h)$	0.906(40)[3]	1.229(30)[2]	1.50(7)
ν	1.46(7)[6]		—
η	−0.30(4)[1]		—
ω	1.43(37)		—

For reference ($h = 0$):
$T_c^{(0)} = 2.03(3)$, $\nu^{(0)} = 1.025(15)$, $\eta^{(0)} = -0.275(25)$

$D = 3$ $(h \neq 0)$

- $L = 80$.
- Gaussian magnetic Fields (using Gauss-Hermite quadrature).
- Dynamical Studies (Fast and Slow annealing procedures):
  - Equilibrium dynamical studies in the high temperature region.
  - Out-of-equilibrium studies for the lower temperatures.

$D = 3\ (h \neq 0)$

Observables:

- $q_{\boldsymbol{x}}(t) = \sigma_{\boldsymbol{x}}^{(1)}(t)\sigma_{\boldsymbol{x}}^{(2)}(t)$
- $q(t) = \frac{1}{V}\overline{\sum_{\boldsymbol{x}} q_{\boldsymbol{x}(t)}}$
- $E_{\mathrm{mag}}(t) = \frac{1}{V}\overline{\sum_{\boldsymbol{x}} h_{\boldsymbol{x}}\sigma_{\boldsymbol{x}}(t)}$
- $W(t) = 1 - TE_{\mathrm{mag}}(t)/H^2$
- $W = \overline{\langle q \rangle}$

- Droplet prediction: $W = q_{\mathrm{EA}}$ and $q(t) \to q_{\mathrm{EA}}$, so

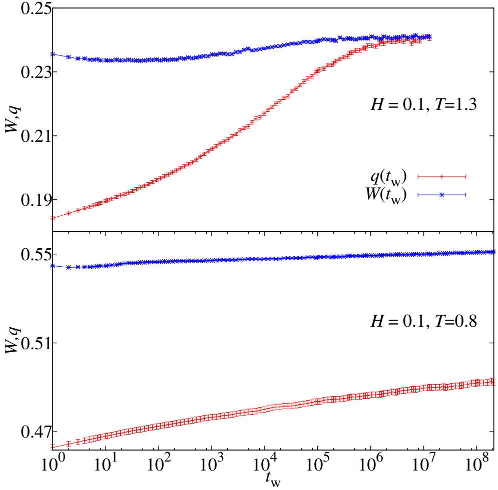
$$W - q \to 0$$

- RSB prediction: $W = \overline{\langle q \rangle}$ and $q(t) \to q_{\min}$, so

$$W - q \to \overline{\langle q \rangle} - q_{\min} > 0$$

$D = 3$ $(h \neq 0)$

Equilibrium and out-of-equilibrium regimes:

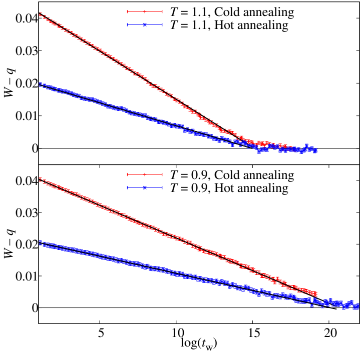


$D = 3\ (h \neq 0)$

The equilibrium data (obtained at high T) follow a stretched exponential behavior:

$$W - q = \frac{b}{t^x} \exp\left[-\left(t/\tau'\right)^{\beta}\right]$$

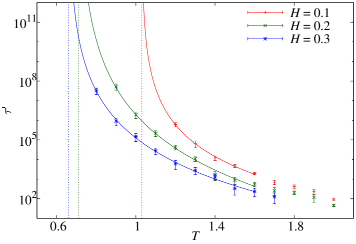
Another computation:



$D = 3\ (h \neq 0)$

On a Second Order Phase Transition:

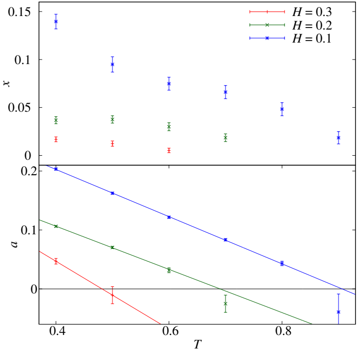
$$\tau = \tau_0 (T - T_c(H))^{-\nu z}$$



$D = 3\ (h \neq 0)$

In a spin glass phase:

$$W - q = a + \frac{b}{t^x}$$

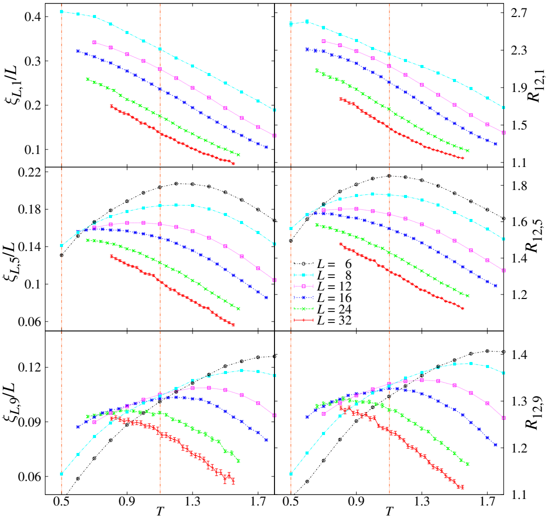


$D = 3$ $(h \neq 0)$

Scenarios:

- RSB with a non zero magnetic field fixed point: critical dynamics for τ'.
- RSB with a zero magnetic field fixed point: activated dynamics for τ'.
- A dynamical transition at which "apparently" diverges τ' and then a thermodynamical phase transition (RSB?) (Mode Coupling Theory, supercooled liquids).
- A $T = 0$ phase transition.
- Our data do not follow the droplet predictions.

$D = 3$ $(h \neq 0)$ Equilibrium



Conclusions

- We have shown strong numerical evidences which support a dAT line below the upper critical dimension ($D = 4$).
- Results in three dimensions start to show signatures of the phase transiton using both non-equilibrium and equilibrium approaches.
- Lower critical dimensions for the Ising spin glass in a field $D_l \geq 3$ or $D_l < 3$?