Can a Spin Glass Phase Survive in an External Magnetic Field?

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With Janus Collaboration (Rome-Ferrara-Zaragoza-Madrid-Extremadura)
(Proc. Natl. Acad. Sci. USA 109, 6452 (2012) and in prep.(2013))
and
La Sapienza Rome Group (L. Leuzzi, G. Parisi and F. Ricci-Tersenghi)
(Phys. Rev. Lett. 103, 267201 (2009))

Plan of the Talk

- What are spin glasses?
- Different Theories: Droplet/Scaling and RSB.
- Results for the one dimensional Edwards-Anderson (diluted) long range model in field.
- Experiments.
- Results for the four dimensional Edwards-Anderson in a field.
- Some (new) results for the three dimensional Edwards-Anderson in a field.
- Conclusions.

What are Spin glasses

- Materials with disorder and frustration.
- Quenched disorder (similar to the Born-Oppenheimer in Molecular Physics).
- Canonical Spin Glass: Metallic host (Cu) with magnetic impurities (Mn).
- RKKY interaction between magnetic moments: $J(r) \sim \frac{\cos(2k_F r)}{r^3}$.
- Role of anisotropy: Ag:Mn at 2.5% (Heisenberg like), $CdCr_{1.7}IN_{0.3}S_4$ (also Heisenberg like) and $Fe_{0.5}Mn_{0.5}TiO_3$ (Ising like).

Some equations

- Edwards-Anderson Hamiltonian:

$$\mathcal{H} = -\sum_{<ij>} J_{ij}\sigma_i\sigma_j$$

  J_{ij} are random quenched variables with zero mean and unit variance, $\sigma = \pm 1$ are Ising spins.

- The order parameter is:

$$q_{\mathrm{EA}} = \overline{\langle \sigma_i \rangle^2}$$

  Using two real replicas:

$$\mathcal{H} = -\sum_{<ij>} J_{ij}\left(\sigma_i\sigma_j + \tau_i\tau_j\right)$$

  Let $q_i = \sigma_i\tau_i$ be the normal overlap, then: $q_{\mathrm{EA}} = \overline{\langle \sigma_i\tau_i \rangle}$.
  We also define the link overlap: $q^l_{i,\mu} = q_i q_{i+\mu}$.

Different Theories.

The Droplet/Scaling Theory.

- Based on the Migdal-Kadanoff implementation (approximate) of the Renormalization Group (exact in $D = 1$).
- *Disguised Ferromagnet*: Only two pure states with order parameter $\pm q_{\mathrm{EA}}$ (related by spin flip).
- Compact Excitations of fractal dimension d_f. The energy of a excitation of linear size L grows as L^θ. The free energy barriers (in the dynamics) grow as L^ψ. $\theta < (D-1)/2 < D-1 < d_f < D$ and $\psi \geq \theta$.
- Any amount of magnetic field destroys the spin glass phase (even for Heisenberg spin glasses).
- Trivial probability distributions of the overlaps (both normal overlap and link one).

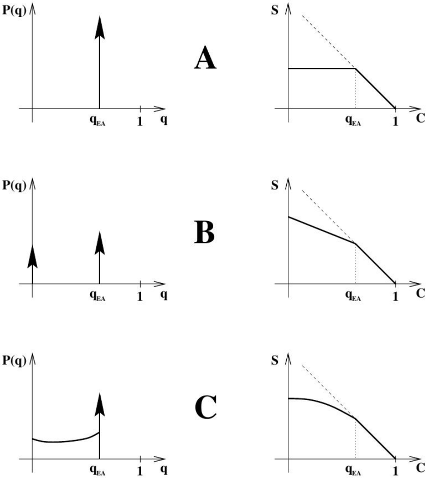
Different Theories.

Replica Symmetry Breaking (RSB) Theory.

- Exact in $D = \infty$.
- Infinite number of phases (pure states) not related by any kind of symmetry.
- These (pure) states are organized in a ultrametric fashion.
- The spin glass phase is stable under (small) magnetic field. Phase transition in field: the de Almeida-Thouless line.
- The excitations of the ground state are space filling: e.g. the interface between two pure states is space filling.
- Overlap equivalence: All the definitions of the overlap are equivalent.

Note: In a pure state, α, the clustering property holds:
$\langle S_i S_j \rangle_\alpha - \langle S_i \rangle_\alpha \langle S_j \rangle_\alpha \to 0$ as $|i - j| \to \infty$.

Different Theories (Comparison).



Different Theories

- Field Theory of the Replica Symmetry Breaking (RSB) Theory.
- Hamiltonian (Action)

$$S_n = H_n = \int d^D x \left[(\partial_\mu Q_{ab})^2 + \tau \mathrm{Tr} Q^2 + g_3 \mathrm{Tr}(Q^3) + \lambda \sum Q_{ab}^4 \right]$$

$a, b = 1, ..., n$. At the end, $n \to 0$! (The replica trick)

- Propagator ($T > T_c$):

$$G(p) = \frac{1}{p^2 + m^2}$$

- Propagator ($T = T_c$, λ is irrelevant: ϕ^3 theory, and the upper critical dimensions is $D = 6$):

$$G(p) = \frac{1}{p^{2-\eta}}$$

Different Theories.

- Propagators (Parisi Matrix) ($T < T_c$ and λ is relevant):

$$G_q(r) \simeq q^2 + A(q) r^{-\theta(q)}$$

where

  - $\theta(q_M) = D - 2$. This result may be exact (some kind of Goldstone theorem).
  - $\theta(q_M) = D - 3$ for $q_M > q > q_m$. This result should be modified below $D = 6$.
  - $\theta(q_m) = D - 4$ for $q_m = 0$. For $D < 6$ this result should be modified to:

$$\theta(q = 0) = \frac{D - 2 + \eta}{2}$$

- In the droplet/scaling Theory:

$$G(r) \simeq q_{EA}^2 + A r^{-\theta}$$

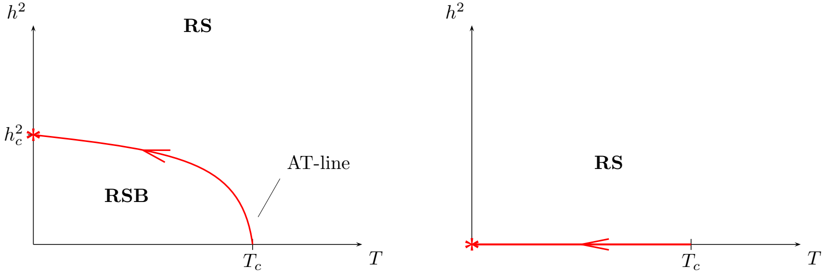
where θ is the standard droplet exponent.

Different Theories: External Magnetic Field

RG from the paramagnetic phase:

- The upper critical dimension in a field is still six (Bray and Moore).
- Due to a dangerous irrelevant variable, some observables change behavior at eight dimensions (Fisher and Sompolinsky).
- Projecting the theory (replicon mode) no fixed points were found (Bray and Roberts).
- However, starting with the most general Hamiltonian of the RS phase and relaxing the $n = 0$ condition a stable fixed point below six dimensions was found (Dominicis, Temesvári, Kondor and Pimentel)
- Temesvári is able to build the dAT slightly below $D = 6$ (but Bray and Moore, Temesvári and Parisi, Moore,...)

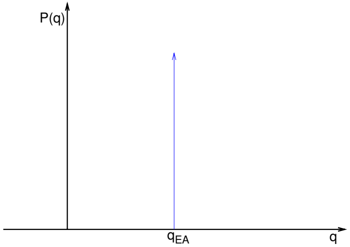
Renormalization group predictions (from Temesvári and Parisi):

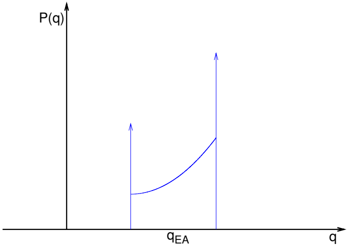


(a) (b)

Different Theories: External Magnetic Field

Different behavior of $P(q)$ in a magnetic field:





Long Range Interactions

- $J(r) \simeq r^{-\rho/2}$.
- Hamiltonian (Action) for the long range model:

$$S_n = H_n \propto \int d^d k \left(k^{\rho-d} + \tau\right) \mathrm{Tr} Q^2 + \int d^d x \left[g_3 \mathrm{Tr}(Q^3) + \lambda \sum Q_{ab}^4\right]$$

- $\dim_p(g_3) = d - \frac{3}{4}\rho$. In MF: $\eta = d + 2 - \rho$ (holds in IRD!) and $1/\nu = \rho - d$.
- It is possible to show (equivalence D-SR and $1d$-LR):

$$\frac{2 - \eta(D)}{D} = \rho - 1 \; ; \; \rho = 1.8 \to D = 3$$

- Be aware in presence of a magnetic field!! ($\rho > 1.5$) (RFIM: Leuzzi and Parisi): In $1d$ LR RFIM $\rho_c(h = 0) = 2$ and $\rho_c(h) = 1.5$.

Long Range Interactions

Hence, the Mean Field and Infrared region are ($d = 1$):

ρ	transition type
≤ 1	Bethe lattice like
$(1, 4/3]$	2^{nd} order, MF
$(4/3, 2]$	2^{nd} order, non-MF
2	Kosterlitz-Thouless or $T = 0$-like
> 2	none

Numerical Simulations and Observables

Numerical Simulations

- The spins live on a finite connectivity network ($z = 6$) with periodic boundary conditions: $J_{ij} = 0, \pm 1$ with $P(J_{ij} \neq 0) \propto r_{ij}^{-\rho}$. With this choice one has $\overline{J_{ij}^2} \propto r_{ij}^{-\rho}$.
- We have implemented the Parallel Tempering Method.
- We have used multispin coding (64 bits) on a C++ code.
- We have simulated a Gaussian magnetic field and only two replicas.
- We have run on PC's Clusters.

Some Observables

- The spin glass correlation function:

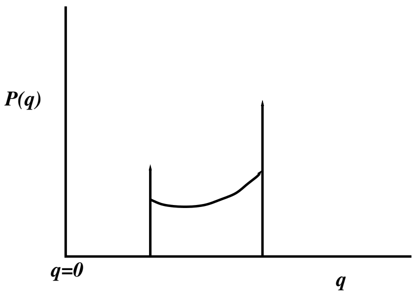
$$C(x) = \sum_{i=1}^{L} \overline{\left(\langle \sigma_i \sigma_{i+x} \rangle - \langle \sigma_i \rangle \langle \sigma_{i+x} \rangle\right)^2}$$

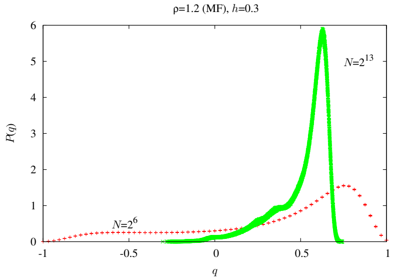
- The associated spin glass correlation length:

$$\xi \equiv \frac{1}{2\sin(\pi/L)} \left[\frac{\tilde{C}(0)}{\tilde{C}(2\pi/L)} - 1 \right]^{\frac{1}{\rho-1}}$$

The negative overlap problem

- $P(q)$ in a magnetic field: SK results and numerical ones.





- The negative overlap region induces large corrections in $\tilde{C}(0)$!!

Numerical Analysis of the Correlation function

- We will avoid the $k = 0$ value by fitting ($k > 0$):

$$\left(\frac{1}{\tilde{C}_4(k)}\right)^{\text{fit}} = A(L,T) + B(L,T)[\sin(k/2)/\pi]^{\rho-1}$$

- We can analyze the L and T dependence of

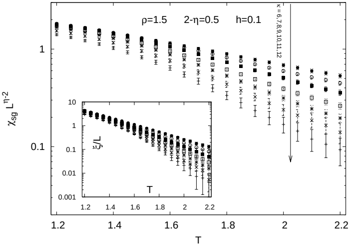
$$A(L,T) \equiv \lim_{k\to 0} \frac{1}{\tilde{C}_4(k)}$$

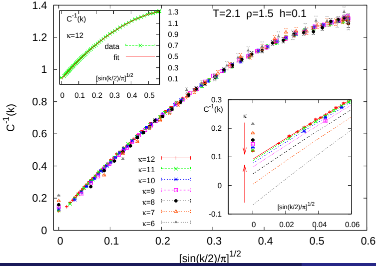
- We fix the L-dependent critical temperature by means:

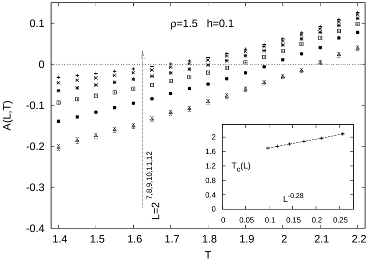
$$A(L, T_c(L)) = 0$$

Numerical Analysis of the Correlation function ($h \neq 0$)

- $h = 0.1$ and $\rho = 1.5$.





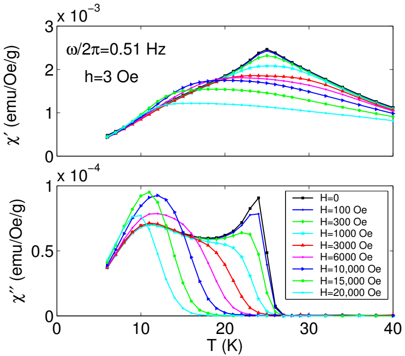


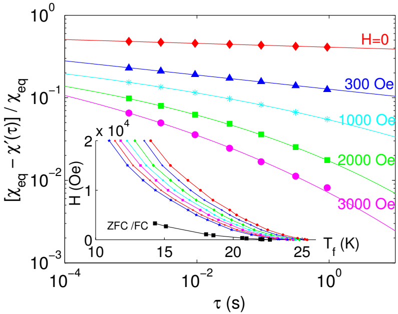
Characterization of the de Almeida-Thouless line

	ρ	h	T_c from $\tilde{C}(0)$	T_c from $A(L,T)$
MF	1.2	0.0	2.24(1)	2.34(3)
	1.2	0.1	2.02(2)	1.9(2)
	1.2	0.2	1.67(3)	1.4(2)
	1.2	0.3	1.46(3)	1.5(4)
	1.25	0.0	2.191(5)	2.23(2)
IRD	1.4	0.0	1.954(3)	1.970(2)
	1.4	0.1	~ 1.5	1.67(7)
	1.4	0.2	~ 1.1	1.2(2)
	1.5	0.0	1.758(4)	1.770(5)
	1.5	0.1	—	1.46(3)
	1.5	0.15	—	1.20(7)

Experiments

- Experimental data from $Fe_{0.5}Mn_{0.5}TiO_3$ (Jönsson et al.).

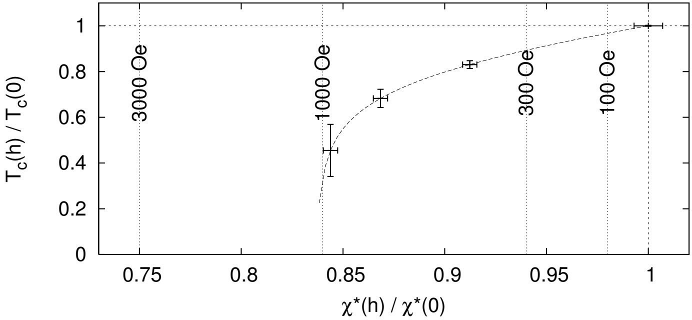




- $q(t) \simeq 1/t^x$ clear signature of a Spin Glass Phase (Ogileski).

Experiments

- Relative decrease of $T_c(h)/T_c(0)$ with increase field for $\rho = 1.5$ and $h = 0$, 0.1, 0.15 and 0.2 versus the relative decrease of χ^* (ZFC susceptibility). Experimental data from $Fe_{0.5}Mn_{0.5}TiO_3$ (Jönsson et al.).



- Hence, the critical field should be $H_c < 1000$ Oe.

Dedicated Computers: Janus.

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Some History: Design of Dedicated Computers in our group

- Reconfigurable Transputer Network (RTN): Triviality issue in U(1)-Higgs model (lattice field theory) and spin glasses. (1989)
- Spin Update Engine (SUE). (2000)
- Super Spin Update Engine (SSUE). (2003)
- Janus. (2008)
- Janus II. (2013)

Dedicated Computers: Janus.

Some figures

- Built in 2008.
- Dedicated computer optimized to simulate a wide variety of spin models.
- 16 boards of 16 FPGA's each (Virtex 4).
- Performance. For Ising models: Janus is equivalent to 10000 PC.
- Parallelization inside the boards.
- Previous numerical simulations simulated the 10^{-5} sec region (SSUE).
- Janus allows us to simulate in the 0.1 second time region. Note: Experimental times range from 1 sec to 3000 sec.

Dedicated Computers: The new Janus II.

Some figures

- It will start to work next July.
- 0.5 million euros.
- ~ 5 times most powerful than Janus.
- Still a dedicated computer optimized to simulate a wide variety of spin models.
- More flexible topology.
- 16 boards of 16 FPGA's each (one IOP and PC integrated on each board) (Virtex 7).
- Janus II will allow us to simulate in the 1 second time region.

$D = 4$ ($h \neq 0$): Observables

- We have simulated using the JANUS computer.
- $L = 5, 6, 8, 10, 12$ and 16.
- Three (uniform) magnetic Fields: $h = 0.075, 0.150$ and 0.3.
- Parallel Tempering in Temperature (e.g. 32 temperatures in $L = 16$)
- Single sample thermalization protocol.
- We avoid the mode $\boldsymbol{k} = 0$ in the analysis.

- Correlation Functions

$$
\begin{aligned}
G_1(\boldsymbol{r}) &= \frac{1}{L^4} \sum_{\boldsymbol{x}} \overline{\left(\langle S_{\boldsymbol{x}} S_{\boldsymbol{x}+\boldsymbol{r}} \rangle - \langle S_{\boldsymbol{x}} \rangle \langle S_{\boldsymbol{x}+\boldsymbol{r}} \rangle \right)^2}\,, \\
G_2(\boldsymbol{r}) &= \frac{1}{L^4} \sum_{\boldsymbol{x}} \overline{\left(\langle S_{\boldsymbol{x}} S_{\boldsymbol{x}+\boldsymbol{r}} \rangle^2 - \langle S_{\boldsymbol{x}} \rangle^2 \langle S_{\boldsymbol{x}+\boldsymbol{r}} \rangle^2 \right)}\,.
\end{aligned}
$$

- Correlation Length:

$$
\xi_2 = \frac{1}{2 \sin(\pi/L)} \left(\frac{\hat{G}(0)}{\hat{G}(\boldsymbol{k}_1)} - 1 \right)^{1/2},
$$

where $\boldsymbol{k}_1 = (2\pi/L, 0, 0, 0)$ (and two perm.)

$D = 4$ $(h \neq 0)$: Observables

- R_{12}:

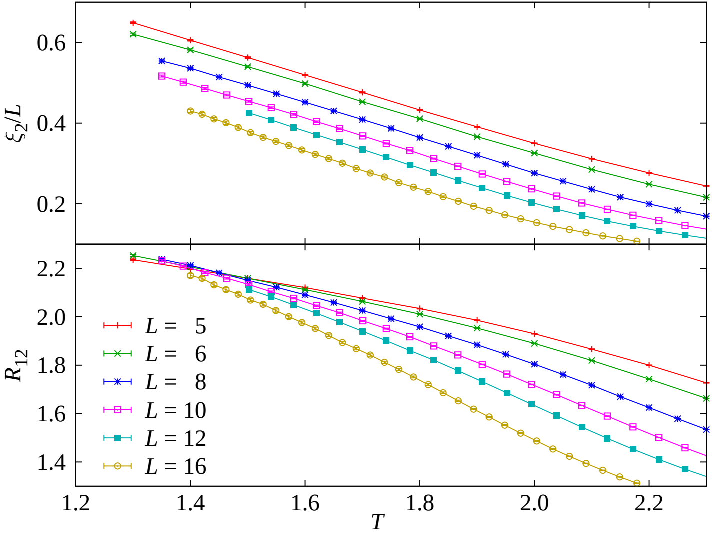
$$R_{12} = \frac{\hat{G}(\boldsymbol{k}_1)}{\hat{G}(\boldsymbol{k}_2)},$$

  where $\boldsymbol{k}_1 = (2\pi/L, 0, 0, 0)$, $\boldsymbol{k}_2 = (2\pi/L, 2\pi/L, 0, 0)$ (and permutations)
- We have checked the behavior of this observable in the EA model in $4d$ (h=0).
- And in the two dimensional (ordered) Ising model. We have been able to compute its value at criticality using Conformal Field Theory:

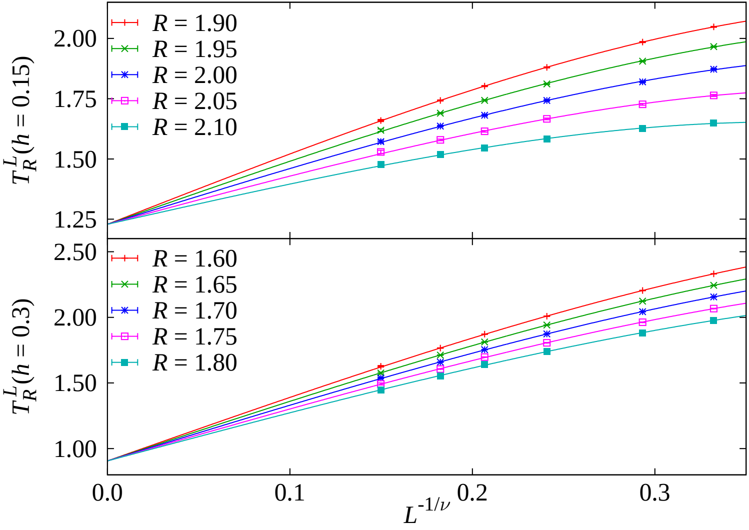
$$R_{12} = 1.694\ 024...$$

- In a paramagnetic phase, for large L: $R_{12} \to 1$.
- For a well behaved propagator $R_{12} \leq 2$.

$D = 4$ $(h = 0.15)$

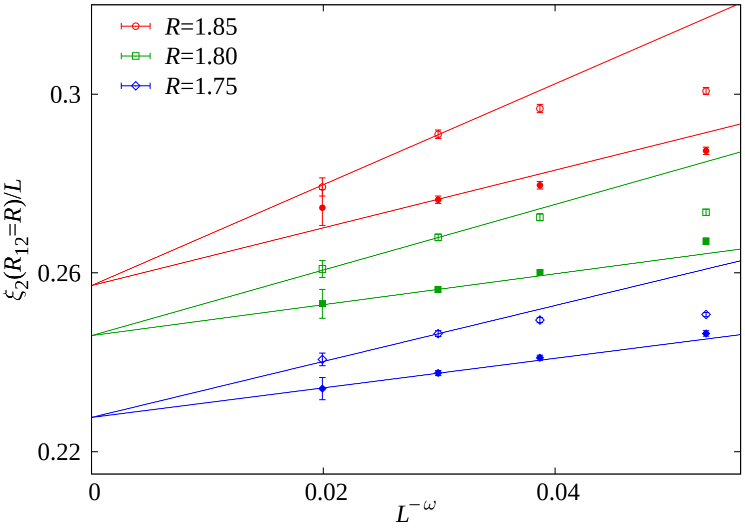

0.6
0.4
0.2
ξ2/L
2.2
2.0
1.8
1.6
1.4
R12
L = 5
L = 6
L = 8
L = 10
L = 12
L = 16
1.2
1.4
1.6
1.8
2.0
2.2
T

$D = 4\ (h \neq 0)$



R = 1.90
R = 1.95
R = 2.00
R = 2.05
R = 2.10
2.00
1.75
1.50
1.25
T_R^L(h = 0.15)
R = 1.60
R = 1.65
R = 1.70
R = 1.75
R = 1.80
2.50
2.00
1.50
1.00
T_R^L(h = 0.3)
0.0
0.1
0.2
0.3
L^-1/ν

$D = 4\ (h \neq 0)$



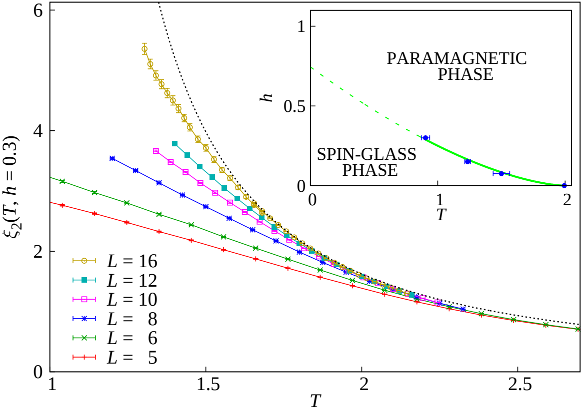
$D = 4$ $(h \neq 0)$

Parameter	$h = 0.3$	$h = 0.15$	$h = 0.075$
$T_c(h)$	0.906(40)[3]	1.229(30)[2]	1.50(7)
ν	1.46(7)[6]		—
η	−0.30(4)[1]		—
ω	1.43(37)		—

For reference $(h = 0)$:
$T_c^{(0)} = 2.03(3)$, $\nu^{(0)} = 1.025(15)$, $\eta^{(0)} = -0.275(25)$

$D = 4\ (h \neq 0)$

Summary:



Fisher-Sompolinsky relation: $h^2(T) \simeq A|T - T_c^{(0)}|^{\beta^{(0)}+\gamma^{(0)}}$

$D = 3$ ($h \neq 0$)

- We have simulated using the JANUS computer.
- $L = 80$.
- Gaussian magnetic Fields (using Gauss-Hermite quadrature).
- Dynamical Studies (Fast and Slow annealing procedures):
  - Equilibrium dynamical studies in the high temperature region.
  - Out-of-equilibrium studies for the lower temperatures.

$D = 3$ $(h \neq 0)$

Observables:

- $q_{\boldsymbol{x}}(t) = \sigma_{\boldsymbol{x}}^{(1)}(t)\sigma_{\boldsymbol{x}}^{(2)}(t)$
- $q(t) = \frac{1}{V}\overline{\sum_{\boldsymbol{x}} q_{\boldsymbol{x}(t)}}$
- $E_{\mathrm{mag}}(t) = \frac{1}{V}\overline{\sum_{\boldsymbol{x}} h_{\boldsymbol{x}}\sigma_{\boldsymbol{x}}(t)}$
- $W(t) = 1 - TE_{\mathrm{mag}}(t)/H^2$
- $W = \overline{\langle q \rangle}$

- Droplet prediction: $W = q_{\mathrm{EA}}$ and $q(t) \to q_{\mathrm{EA}}$, so

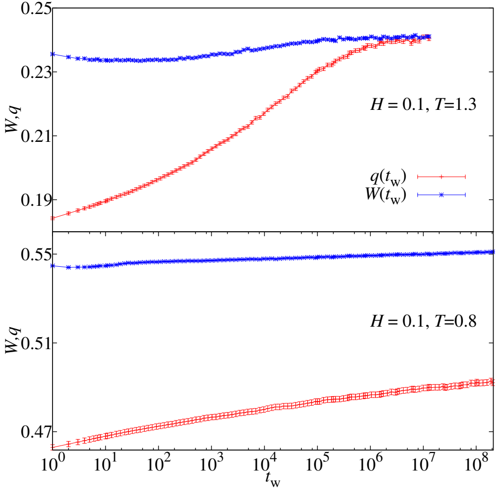
$$W - q \to 0$$

- RSB prediction: $W = \overline{\langle q \rangle}$ and $q(t) \to q_{\min}$, so

$$q - W \to \overline{\langle q \rangle} - q_{\min} > 0$$

$D = 3\ (h \neq 0)$

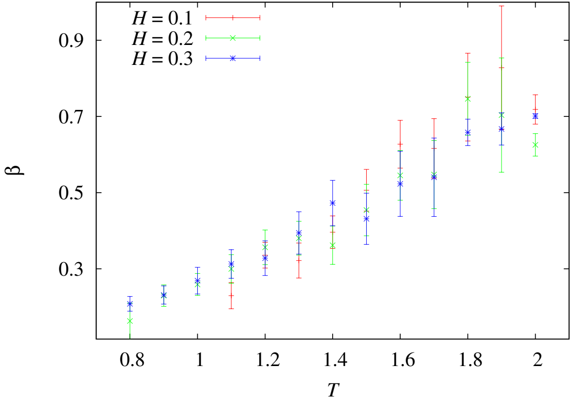
Equilibrium and out-of-equilibrium regimes:



$D = 3$ $(h \neq 0)$

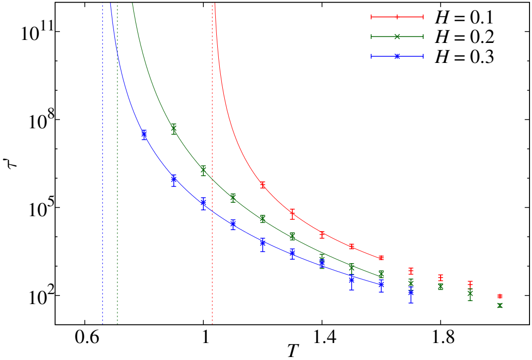
The equilibrium data (obtained at high T) follow a stretched exponential behavior:

$$W - q = \frac{b}{t^x} \exp\left[-\left(t/\tau'\right)^{\beta}\right]$$



$D = 3$ $(h \neq 0)$

Analysis of τ':



On a Second Order Phase Transition:

$$\tau = \tau_0 (T - T_2(H))^{-\nu z}$$

$D = 3$ $(h \neq 0)$

Analysis of τ':

- $H = 0.1$: $T_2 = 1.03(5)$ and $z\nu = 4.8(7)$. Using only $1.2 \leq T \leq 1.9$ $[\chi^2/\text{d.o.f.} = 1.47/5]$.
- $H = 0.2$: $T_2 = 0.71(5)$ and $z\nu = 7.5(1.1)$. Using only $0.9 \leq T \leq 1.6$ $[\chi^2/\text{d.o.f.} = 3.36/5]$.
- $H = 0.3$: $T_2 = 0.66(5)$ and $z\nu = 6.2(1.0)$. Using only $0.8 \leq T \leq 1.5$ $[\chi^2/\text{d.o.f.} = 1.74/5]$.

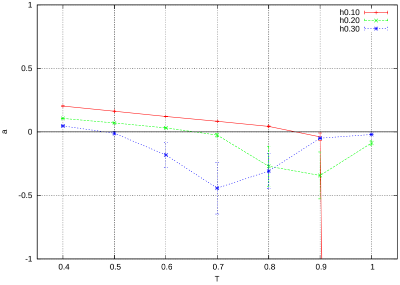
Remember $T_c(H = 0) = 1.109(10)$.

$D = 3\ (h \neq 0)$

In a spin glass phase we should expect:

$$W - q = a + \frac{b}{t^x}$$

with $a > 0$.



So: $T_{\rm oe}(H = 0.3) \sim 0.5$, $T_{\rm oe}(H = 0.2) \sim 0.65$ and $T_{\rm oe}(H = 0.1) \sim 0.85$

$D = 3$ $(h \neq 0)$

Scenarios:

- RSB with a non zero magnetic field fixed point: critical dynamics for τ'.
- RSB with a zero magnetic field fixed point: activated dynamics for τ'.
- A dynamical transition at which "apparently" diverges τ' and then a thermodynamical phase transition (RSB?) (Mode Coupling Theory, supercooled liquids).
- A $T = 0$ phase transition.
- Our data do not follow the droplet predictions.

Conclusions

- We have shown strong numerical evidences which support a dAT line below the upper critical dimension:
  - For $\rho = 1.5 > 4/3$ in the $1d$ LR spin glass.
  - In 4 dimensions for the EA model.
- The situation in 3 dimensions is not yet clear.
- Lower critical dimensions for the Ising spin glass in a field $D_l \geq 3$ or $D_l < 3$?