

SPIN GLASSES WARS

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Plan of the Talk

- What are spin glasses?

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- Different Theories and Models.

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 - The overlaps: spin and link overlap (D. Yllanes' talk).
 - Equilibrium Studies (D. Yllanes' talk).

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 - Equilibrium Studies (D. Yllanes' talk).
- Some Open (in field) Problems.

What are Spin glasses

- Materials with disorder and frustration.
- Quenched disorder (similar to the Born-Oppenheimer in Molecular Physics).
- Canonical Spin Glass: Metallic host (Cu) with magnetic impurities (Mn).
- RKKY interaction between magnetic moments: $J(r) \sim \frac{\cos(2k_F r)}{r^3}$.
- Role of anisotropy: Ag:Mn at 2.5% (Heisenberg like), CdCr_{1.7}In_{0.3}S₄ (also Heisenberg like) and Fe_{0.5}Mn_{0.5}TiO₃ (Ising like).

Some equations

- Edwards-Anderson Hamiltonian:

$$\mathcal{H} = - \sum_{\langle ij \rangle} J_{ij} \sigma_i \sigma_j$$

J_{ij} are random quenched variables with zero mean and unit variance, $\sigma = \pm 1$ are Ising spins.

- The order parameter is:

$$q_{\text{EA}} = \overline{\langle \sigma_i \rangle^2}$$

Using two real replicas:

$$\mathcal{H} = - \sum_{\langle ij \rangle} J_{ij} (\sigma_i \sigma_j + \tau_i \tau_j)$$

Let $q_i = \sigma_i \tau_i$ be the normal overlap, then: $q_{\text{EA}} = \overline{\langle \sigma_i \tau_i \rangle}$.

We also define the link overlap: $q'_{i,\mu} = q_i q_{i+\mu}$.

Dedicated Computers: Janus.



Some figures

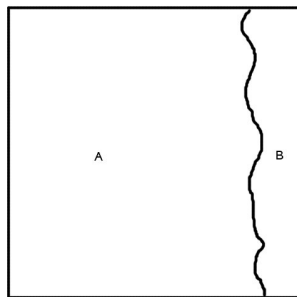
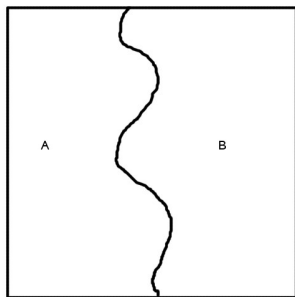
- Dedicated computer optimized to simulate a wide variety of spin models.
- 16 boards of 16 FPGA's each.
- Performance. For Ising models: Janus is equivalent to 10000 PC.
- Previous numerical simulations simulated the 10^{-5} sec region (SSUE).
- Janus allows us to simulate in the 0.1 second time region. Note: Experimental times range from 1 sec to 3000 sec.

The Droplet Model.

- Based on the Migdal-Kadanoff implementation (approximate) of the Renormalization Group (exact in $D = 1$).
- *Disguised Ferromagnet*: Only two pure states with order parameter $\pm q_{EA}$ (related by spin flip).
- Compact Excitations of fractal dimension d_f . The energy of an excitation of linear size L grows as L^θ . The free energy barriers (in the dynamics) grow as L^ψ . $\psi < \theta < (D - 1)/2 < D - 1 < d_f < D$.
- Any amount of magnetic field destroys the spin glass phase (even for Heisenberg spin glasses).
- Trivial probability distributions of the overlaps (both normal overlap and link one).

The Trivial Non Trivial (TNT) Model.

- *Disguised Ferromagnet with Anti Periodic Boundary conditions.*
- Trivial probability distributions for the link overlap (the interface has no effect) but Non Trivial probability distribution for the normal one (induced by the interface).



Replica Symmetry Breaking (RSB) Theory.

- Exact in $D = \infty$.
- Infinite number of phases (pure states) not related by any kind of symmetry.
- These (pure) states are organized in a ultrametric fashion.
- The spin glass phase is stable under (small) magnetic field. Phase transition in field: the de Almeida-Thouless line.
- The excitations of the ground state are space filling: e.g. the interphase between two pure states is space filling.
- Overlap equivalence: All the definitions of the overlap are equivalent.

Note: In a pure state, α , the clustering property holds:

$$\langle S_i S_j \rangle_\alpha - \langle S_i \rangle_\alpha \langle S_j \rangle_\alpha \rightarrow 0 \text{ as } |i - j| \rightarrow \infty.$$

Different Theories and Models (III cont.).

- Field Theory of the Replica Symmetry Breaking (RSB) Theory.
- Hamiltonian (Action)

$$S_n = H_n = \int d^D x \left[(\partial_\mu Q_{ab})^2 + \tau \text{Tr} Q^2 + g_3 \text{Tr}(Q^3) + \lambda \sum Q_{ab}^4 \right]$$

$a, b = 1, \dots, n$. At the end, $n \rightarrow 0!$ (The replica trick)

- Propagator ($T > T_c$):

$$G(p) = \frac{1}{p^2 + m^2}$$

- Propagator ($T = T_c$, λ is irrelevant, ϕ^3 theory, upper critical dimensions $D = 6$):

$$G(p) = \frac{1}{p^{2-\eta}}$$

- Propagators (Parisi Matrix) ($T < T_c$ and λ is relevant):

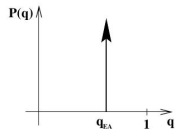
$$G_q(r) \simeq q^2 + A(q)r^{-\theta(q)}$$

where

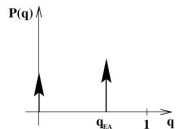
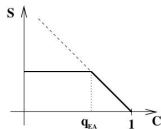
- $\theta(q_M) = D - 2$. This result may be exact (some kind of Goldstone theorem).
- $\theta(q_M) = D - 3$ for $q_M > q > q_m$. This result should be modified below $D=6$.
- $\theta(q_m) = D - 4$ for $q_m = 0$. For $D < 6$ this result should be modified to:

$$\theta(q = 0) = \frac{D - 2 + \eta}{2}$$

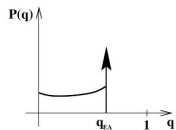
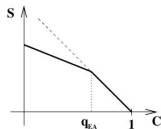
Different Theories and Models (Comparison).



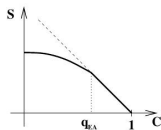
A



B



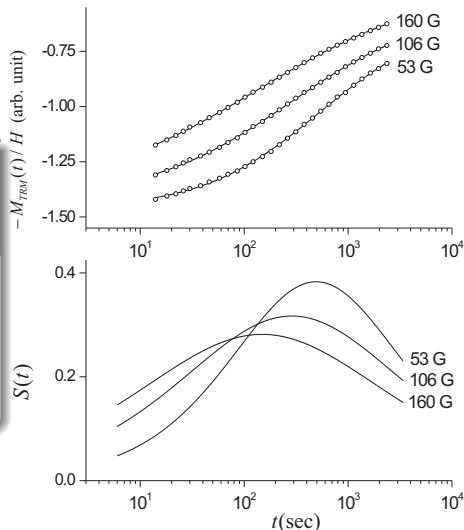
C



On the dynamical critical exponent z below and at the critical Temperature.

Experimental Approach based in Zeeman Energies (I) (Joh et al.)

- Measure of the TRM magnetization: $M_{\text{TRM}}(t, t_w)$.
- Computation of $S(t)$ defined as: $S(t) = -\frac{dM_{\text{TRM}}(t, t_w)}{H d \log t}$.



On the dynamical critical exponent z below and at the critical Temperature.

Experimental Approach based in Zeeman Energies (II)

- In absence of field, the free energy barriers scales as:

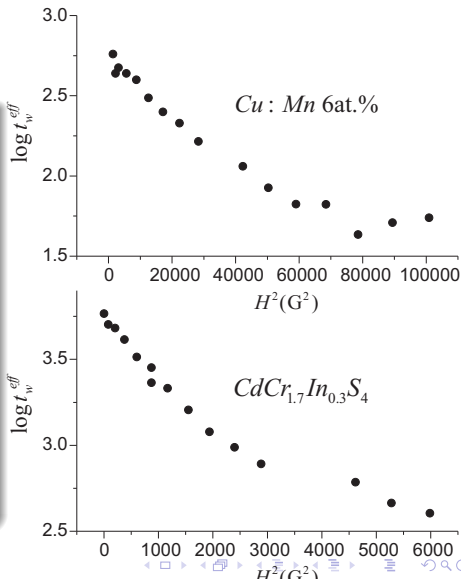
$$\Delta(t, t_w) = k_B T (\log t_w - \log \tau_0).$$

- In presence of field, we obtain:

$$\Delta(t, t_w) - E_Z = k_B T (\log t_w^{\text{eff}} - \log \tau_0).$$

- The Zeeman Energy is:

$$E_Z = N_S \chi_{fc} H^2.$$



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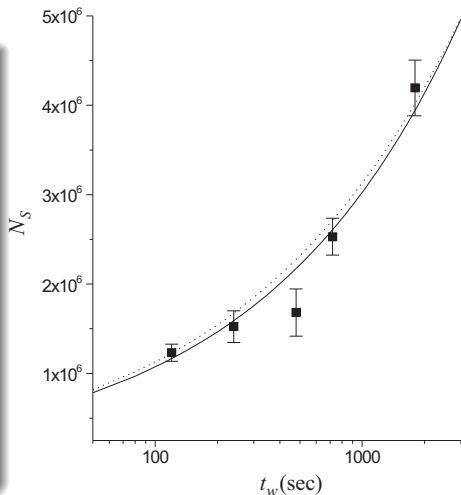
Experimental Approach based in Zeeman Energies (III)

- They extract the dynamical correlation length as:

$$N_s(t_w^{\text{eff}}) = \xi(t_w^{\text{eff}})^3.$$

- $\xi(t_w, T) = 0.653 (t_w/\tau_0)^{0.169 T/T_g}$.
 $\tau_0^{-1} = 4.1 \times 10^{12} \text{ s}^{-1}$.

- $\xi(t_w, T) = 10^{-5} \left[\frac{T}{T_g} \log\left(\frac{t_w}{\tau_0}\right) \right]^{1/0.21}$.
Numerically: $\psi = 0.7!$.



Theoretical Basis supporting $z(T)$.

- Assuming the dynamical free energy barriers scales as $\log L$ (i.e. $\psi = 0$) (Rieger).
- In the Sine-Gordon model with phase disorder. Near T_c , $z(T) = 2 + 2e^{\gamma E}(T - T_c)/T_c + O(\tau^2)$, and in the low T phase: $z(T) \simeq 1/T$. (Rieger and Schehr).
- A particle in a one dimensional disordered potential with logarithmic barriers in the marginal glassy phase ($\theta = 0$) (Le Doussal, Carpertier and Le Doussal).

The Quest for the Replicon.

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The replicon in $D = 6$.

- We assume $z(T) = 4 \frac{T_c}{T}$
- χ , computed off equilibrium, should grow following the propagator $p^{-2}(r^{-4})$ for $T = T_c$ and the replicon mode $p^{-4}(r^{-2})$ for $T < T_c$.
- Hence $\chi(t) \simeq t^{h(T)}$ where:
 - $h(T_c) = 1/2$.
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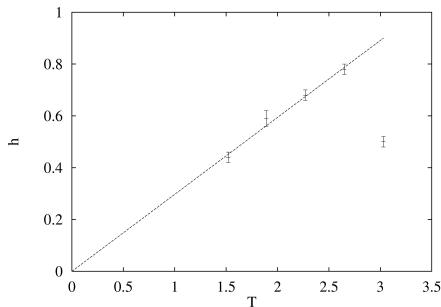
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- Note: $\chi(t) = \int^{\xi(t)} d^D x G(x)$. (Parisi, Ranieri, Ricci-Tersenghi and JJRL)

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- The replica-replica correlation function is:

$$C_4(x, t_w) = \frac{1}{L^3} \overline{\sum_i q_i(t_w) q_{i+x}(t_w)}.$$

$T < T_c$ $C_4(r, t_w) \simeq r^{-a} e^{-[r/\xi(t_w)]^b}$ (Parisi, Marinari and JJRL)

- In equilibrium at the critical point: $a = 1 + \eta = 0.625(10)$.
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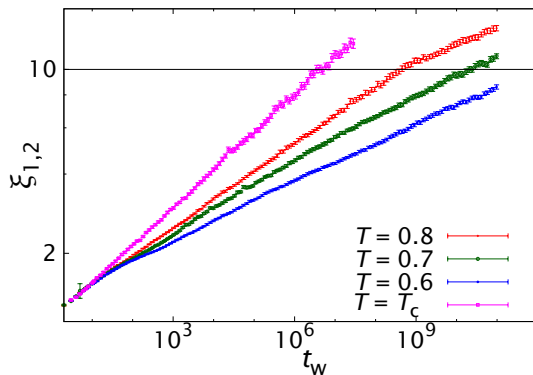
- In equilibrium at the critical point: $a = 1 + \eta = 0.625(10)$. (Hasenbusch et al.)
- We have proposed an Ansatz-independent determination of ξ and a (Janus collaboration):

$$I_k(t_w) = \int_0^\infty dr r^k C_4(r, t_w)$$

then, if $C_4 \simeq r^{-a} f(r/\xi)$,

$$\xi_{k,k+1}(t_w) = I_{k+1}(t_w)/I_k(t_w) \propto \xi(t_w), \quad (\xi \ll L)$$

Our $z(T)$



$$\xi(t_w) = A(T)t_w^{1/z(T)}$$

$$z(T_c) = 6.86(16)$$

$$z(0.8) = 9.42(15)$$

$$z(0.7) = 11.84(22)$$

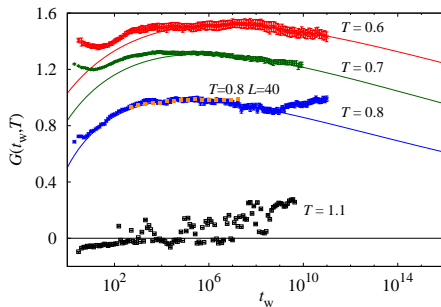
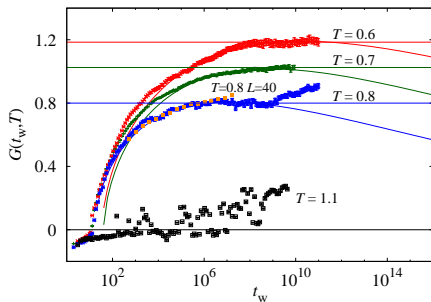
$$z(0.6) = 14.06(25)$$

$$z(T) \simeq z(T_c)T_c/T$$

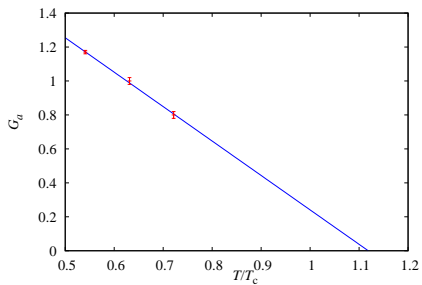
On the dynamical critical exponent z below T_c . Modified Droplet analysis.

$$t_w \sim \tau_0 \xi^{z_c} \exp\left(\frac{Y(T)\xi^\psi}{T}\right), \text{ (Bouchaud et al.)}$$

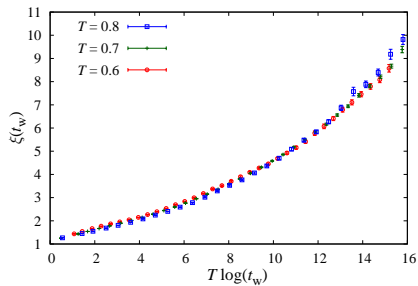
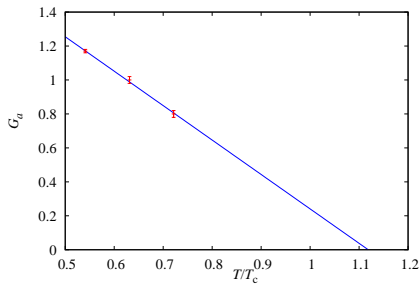
$$G(t_w, T) = \left(\frac{\log(t_w/\tau_0) - z_c \log \xi(t_w, T)}{\xi^\psi T_c/T}\right)^{\frac{1}{\psi\nu}}$$



$z(T)$ below T_c . Modified Droplet analysis.



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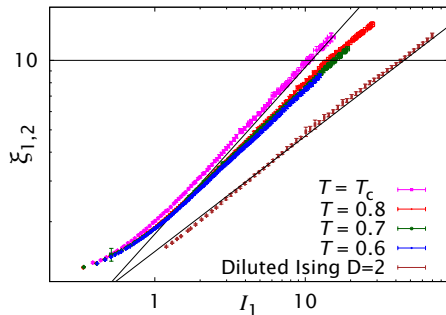


On the replicon.

- $C_4(r, t_w) \sim r^{-a} f(t/\xi(t_w)) \implies l_1(t_w) \propto \xi_{k,k+1}^{2-a}(t_w).$

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$$a(0.8) = 0.442(11)$$

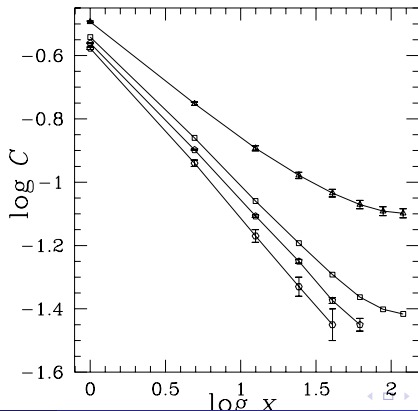
$$a(0.7) = 0.397(12)$$

$$a(0.6) = 0.359(13)$$

$$a(T_c) = 0.585(12)$$

The Quest for the Replicon.

- Equilibrium and out of equilibrium correlations in $D = 3$ (Marinari, Parisi and JJRL).



Some (in field) Open Problems.

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- Lower critical dimensions for the Ising spin glass in a field $D_l > 3$?