

Ultrametric predictions for small-field chaos in spin glasses: theory and simulations

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Outline of the Talk

- Replica Symmetry Breaking
- Finite Dimensions
- Vanishing fields
- Methodology: numerical simulations and the generation of the tree
- Results
- Conclusions

Solution of the Mean Field approximation: ∞ -step RSB (G. Parisi, 1979)

Physical Interpretation: 1979-1986

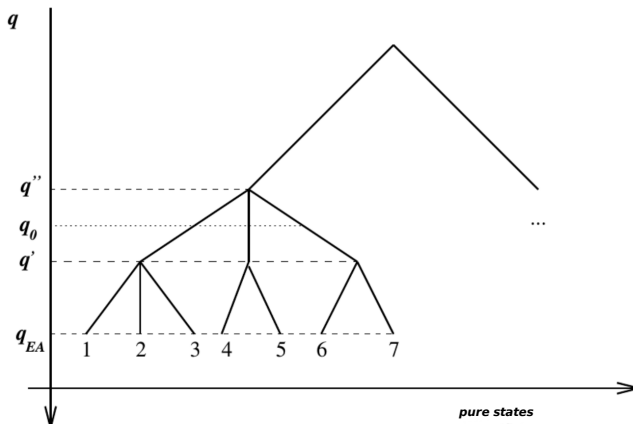
- The solution **is exact in infinite dimension**. It has been rigorously proved by F. Guerra (2003), M. Talagrand (2011) and D. Panchenko (2013).
- The solution presents an **infinite number of pure states not related by spin flip symmetry**.

$$\langle(\cdots)\rangle_J = \sum_{\alpha} w_{\alpha} \langle(\cdots)\rangle_{\alpha}, \quad w_{\alpha} = \exp(-\beta F_{\alpha}) / \sum_{\gamma} \exp(-\beta F_{\gamma}).$$

- The infinite number of pure states are organized following an **ultrametric structure**.

Giorgio Parisi's plenary talk yesterday.

Ultrametricity

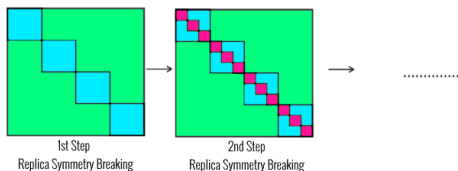


In spin glasses, 1/4 of the triangles are equilateral and 3/4 are isosceles.

$$d(\alpha, \beta) = \frac{1}{2} (q_{EA} - q_{\alpha\beta}) \quad \text{with} \quad q_{EA} = q_{\alpha\alpha}.$$

Finite Dimensions

Mean Field: Parisi's solution



Finite dimensions: open question



Some definitions

- The Edwards-Anderson Spin-Glass Hamiltonian:

$$\mathcal{H} = - \sum_{\langle i,j \rangle} J_{ij} \sigma_i \sigma_j + H \sum_i \sigma_i$$

$J_{ij} = \pm 1$ with equal probability.

- The order parameter is:

$$q = \overline{\langle \sigma_i \rangle^2}$$

Using two real replicas:

$$\mathcal{H} = - \sum_{\langle i,j \rangle} J_{ij} (\sigma_i \sigma_j + \tau_i \tau_j) + H_\sigma \sum_i \sigma_i + H_\tau \sum_i \tau_i .$$

Let $q_i = \sigma_i \tau_i$ be the normal overlap, then the order parameter is:
 $q = \overline{\langle \sigma_i \tau_i \rangle}.$

Why a *vanishing* magnetic field?

For the spin glass, we introduce the *vanishing* field

$$H = \frac{h}{\sqrt{N}}, \quad h = O(1).$$

This is the crossover regime where the perturbation has a finite effect on state weights.

Effect on a state

State magnetizations scale as

$$m_\alpha = \frac{1}{\sqrt{N}} \sum_i \langle \sigma_i \rangle_\alpha = O(1),$$

so the field reweights the state as

$$w_\alpha \longrightarrow w_\alpha(h) \propto w_\alpha \cosh(\beta h m_\alpha).$$

Key separation

- Relative weights change by $O(1)$.
- Internal state properties remain unchanged.
- Therefore predictions depend only on the tree and $P(q)$.

Observables: what is compared with simulations?

Three overlap distributions are measured:

$$P^L(q) = \sum_{\alpha,\beta} w_\alpha(h=0)w_\beta(h=0)\delta(q - q_{\alpha\beta}),$$

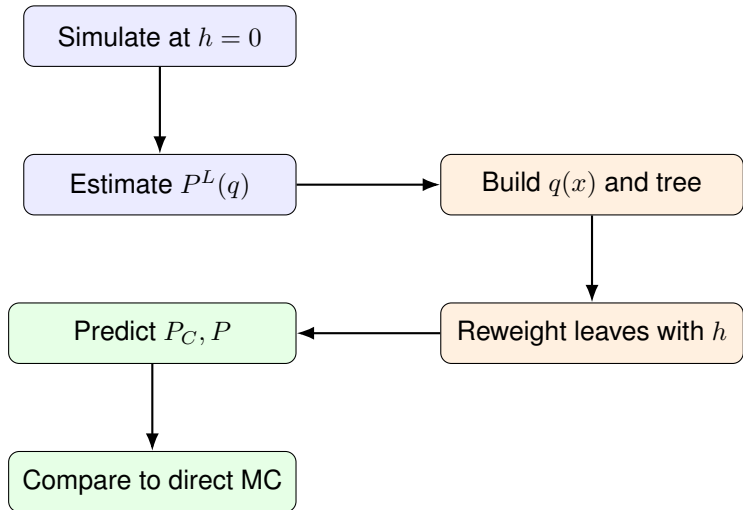
$$P_C^L(q; h) = \sum_{\alpha,\beta} w_\alpha(h=0)w_\beta(h)\delta(q - q_{\alpha\beta}),$$

$$P^L(q; h) = \sum_{\alpha,\beta} w_\alpha(h)w_\beta(h)\delta(q - q_{\alpha\beta}).$$

If RSB is correct, the $P(q)$'s generated from the (mean-field) ultrametric tree using only $P(q)$ should match direct simulations in a vanishing field.

- Numerical Simulations ($h = 0$ and $h \neq 0$):
 - 1 Metropolis.
 - 2 Parallel tempering.
 - 3 Multispin coding.
 - 4 Individual sample thermalization test based on the PT random walk.
- Building the ultrametric Tree:
 - 1 The theoretical predictions from RSB have been obtained by generating ultrametric trees with the proper statistics via the Bolthausen-Sznitman (BS) coalescent algorithm.
- Models:
 - 1 Bethe lattice.
 - 2 4D Edwards-Anderson model.
 - 3 3D Edwards-Anderson model.

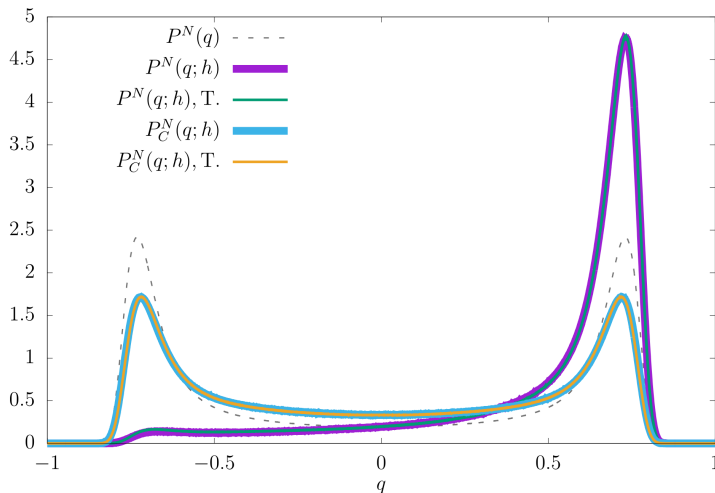
The mixed analytical–numerical pipeline



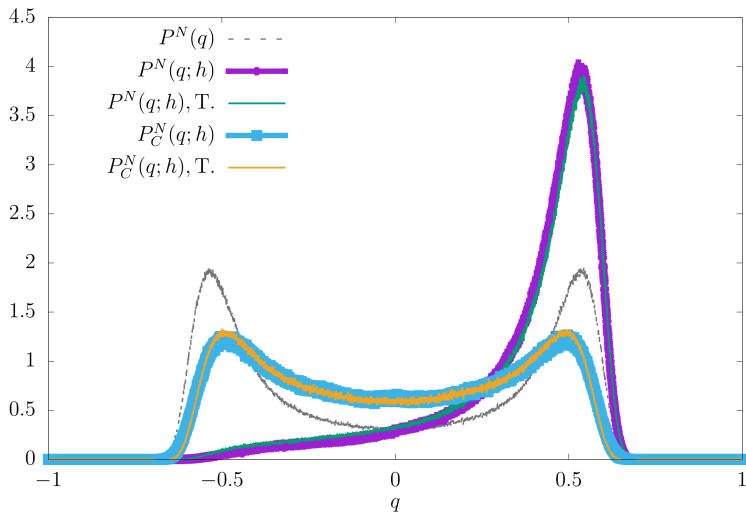
The only model-specific input is the zero-field overlap distribution $P(q)$.

Bethe lattice: direct simulations vs tree predictions

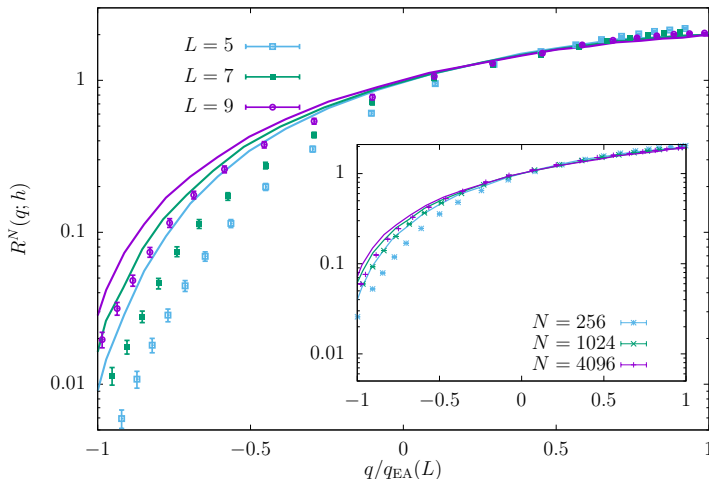
$N = 4096$, $h = 4$, $T = 0.5T_c$



4D Edwards-Anderson: direct simulations vs tree prediction $L = 9, T = 0.7T_c$

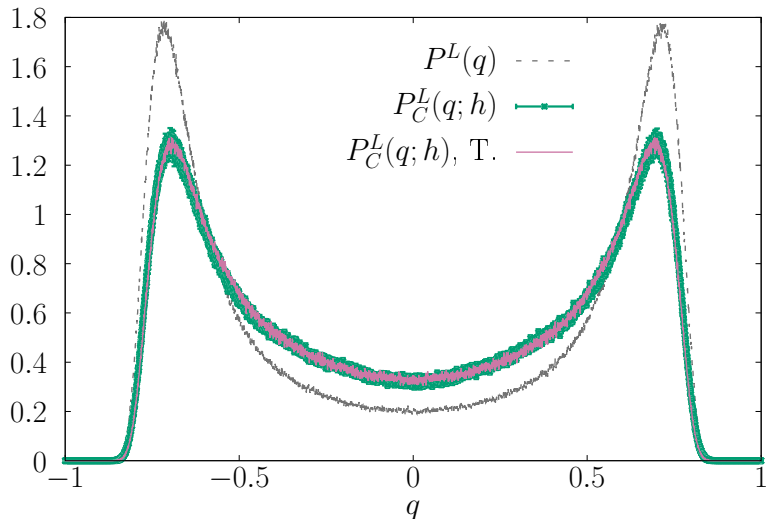


4D Edwards-Anderson: direct simulations vs tree prediction, Finite Effects (Full), $h = 10$, $T = 0.7T_c$

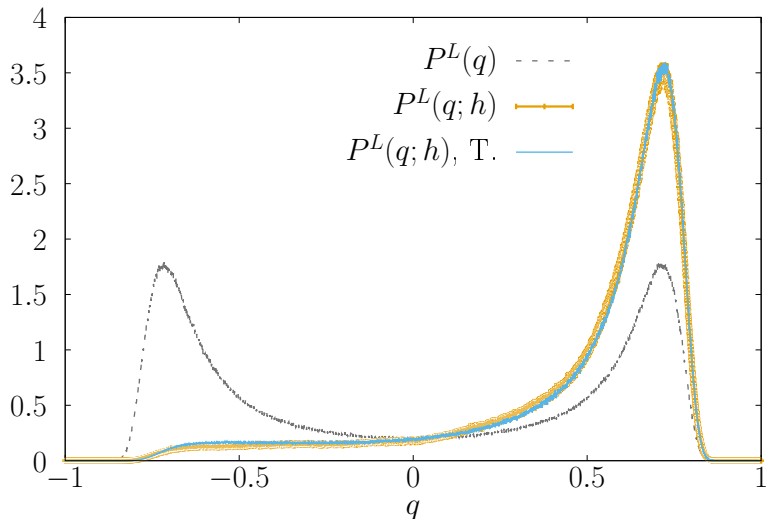


$$R^N(q, h) = \frac{P(q; h)}{P(q, 0)}$$

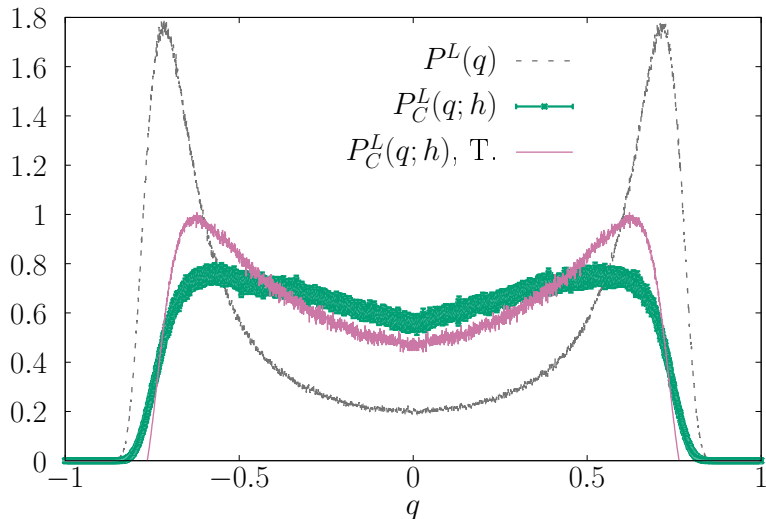
3D Edwards-Anderson: direct simulations vs tree prediction (Chaos) $L = 16, T = 0.73T_c, h = 4$



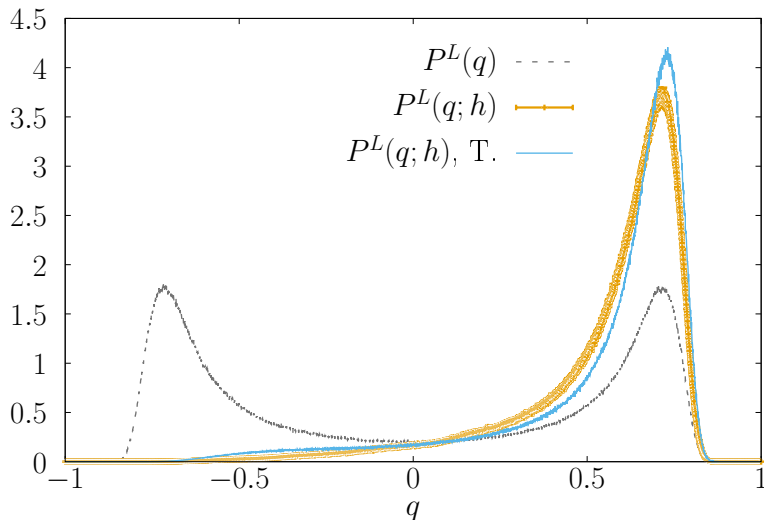
3D Edwards-Anderson: direct simulations vs tree prediction (Full) $L = 16$, $T = 0.73T_c$, $h = 4$



3D Edwards-Anderson: direct simulations vs tree prediction (Chaos) $L = 16, T = 0.73T_c, h = 8$



3D Edwards-Anderson: direct simulations vs tree prediction (Full) $L = 16$, $T = 0.73T_c$, $h = 8$



Conclusions

- The introduction of *vanishing* fields allows us to test the predictions of RSB in finite-dimensional spin-glass models.
- RSB gives parameter-free predictions once zero-field $P(q)$ is known.
- As a test, our approach works very well for a model defined on the Bethe lattice, where the RSB solution is exact.
- We have found very good agreement in the four- and three-dimensional Edwards–Anderson models.
- The *vanishing* fields approach could be useful for studying RSB properties experimentally.