# Apuntes sobre la vida de Enrico Fermi

J. J. Ruiz-Lorenzo

Dep. Física, Universidad de Extremadura
`http://www.unex.es/fisteor/juan`

Badajoz, 6 de Marzo de 2009

# Enrico Fermi (1901-1954)

# Enrico Fermi (1901-1954)

- Físico italo-americano. Roma (29/Sept/1901)-Chicago (28/Nov/1954)

# Enrico Fermi (1901-1954)

- Físico italo-americano. Roma (29/Sept/1901)-Chicago (28/Nov/1954)
- Se “laureó” en la Scuola Normale Superiore di Pisa.

# Enrico Fermi (1901-1954)

- Físico italo-americano. Roma (29/Sept/1901)-Chicago (28/Nov/1954)
- Se "laureó" en la Scuola Normale Superiore di Pisa.
- Profesor de la Universidad de Florencia (1925-1926).

# Enrico Fermi (1901-1954)

- Físico italo-americano. Roma (29/Sept/1901)-Chicago (28/Nov/1954)
- Se "laureó" en la Scuola Normale Superiore di Pisa.
- Profesor de la Universidad de Florencia (1925-1926).
- A los 24 años fue nombrado profesor de la Universidad de Roma.

# Enrico Fermi (1901-1954)

- Físico italo-americano. Roma (29/Sept/1901)-Chicago (28/Nov/1954)
- Se “laureó” en la Scuola Normale Superiore di Pisa.
- Profesor de la Universidad de Florencia (1925-1926).
- A los 24 años fue nombrado profesor de la Universidad de Roma.
- Estancias en el extranjero: Göttingen (con M. Born) y Leiden (con Eherenfest).

# Enrico Fermi (1901-1954)

- Físico italo-americano. Roma (29/Sept/1901)-Chicago (28/Nov/1954)
- Se “laureó” en la Scuola Normale Superiore di Pisa.
- Profesor de la Universidad de Florencia (1925-1926).
- A los 24 años fue nombrado profesor de la Universidad de Roma.
- Estancias en el extranjero: Göttingen (con M. Born) y Leiden (con Eherenfest).
- Tuvo como estudiante postdoctoral en Roma a Hans Bethe.

# Enrico Fermi (1901-1954)

- Físico italo-americano. Roma (29/Sept/1901)-Chicago (28/Nov/1954)
- Se "laureó" en la Scuola Normale Superiore di Pisa.
- Profesor de la Universidad de Florencia (1925-1926).
- A los 24 años fue nombrado profesor de la Universidad de Roma.
- Estancias en el extranjero: Göttingen (con M. Born) y Leiden (con Eherenfest).
- Tuvo como estudiante postdoctoral en Roma a Hans Bethe.
- Premio Nobel de Física 1938.
  *"for his demonstrations of the existence of new radioactive elements produced by neutron irradiation, and for his related discovery of nuclear reactions brought about by slow neutrons"*

# Enrico Fermi: Contribuciones científicas

- Estadística de Fermi-Dirac.

# Enrico Fermi: Contribuciones científicas

- Estadística de Fermi-Dirac.
- Teoria de la desintegración $\beta$.

# Enrico Fermi: Contribuciones científicas

- Estadística de Fermi-Dirac.
- Teoria de la desintegración $\beta$.
- Producción de nuevos elementos radiactivos usando fuentes de neutrones.

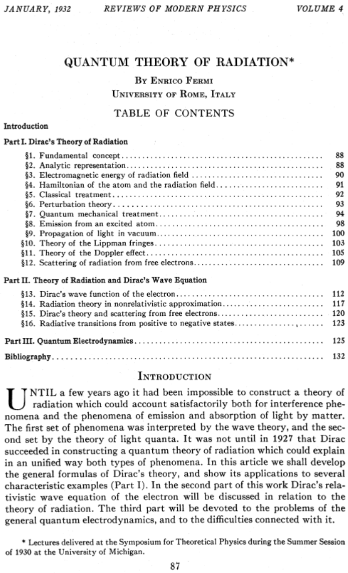
# Enrico Fermi: Contribuciones científicas

- Estadística de Fermi-Dirac.
- Teoria de la desintegración $\beta$.
- Producción de nuevos elementos radiactivos usando fuentes de neutrones.
- Primera reacción nuclear controlada.

# Enrico Fermi: Contribuciones científicas

- Estadística de Fermi-Dirac.
- Teoria de la desintegración $\beta$.
- Producción de nuevos elementos radiactivos usando fuentes de neutrones.
- Primera reacción nuclear controlada.
- Construcción de la primera bomba atómica.

# Enrico Fermi (Publicaciones)



*JANUARY, 1932* *REVIEWS OF MODERN PHYSICS* *VOLUME 4*

## QUANTUM THEORY OF RADIATION*

By Enrico Fermi
University of Rome, Italy

### TABLE OF CONTENTS

**Introduction**

**Part I. Dirac's Theory of Radiation**

§1. Fundamental concept .......... 88
§2. Analytic representation .......... 88
§3. Electromagnetic energy of radiation field .......... 90
§4. Hamiltonian of the atom and the radiation field .......... 91
§5. Classical treatment .......... 92
§6. Perturbation theory .......... 93
§7. Quantum mechanical treatment .......... 94
§8. Emission from an excited atom .......... 98
§9. Propagation of light in vacuum .......... 100
§10. Theory of the Lippman fringes .......... 103
§11. Theory of the Doppler effect .......... 105
§12. Scattering of radiation from free electrons .......... 109

**Part II. Theory of Radiation and Dirac's Wave Equation**

§13. Dirac's wave function of the electron .......... 112
§14. Radiation theory in nonrelativistic approximation .......... 117
§15. Dirac's theory and scattering from free electrons .......... 120
§16. Radiative transitions from positive to negative states .......... 123

**Part III. Quantum Electrodynamics** .......... 125

**Bibliography** .......... 132

### INTRODUCTION

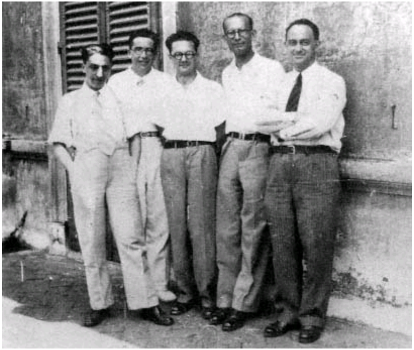
UNTIL a few years ago it had been impossible to construct a theory of radiation which could account satisfactorily both for interference phenomena and the phenomena of emission and absorption of light by matter. The first set of phenomena was interpreted by the wave theory, and the second set by the theory of light quanta. It was not until in 1927 that Dirac succeeded in constructing a quantum theory of radiation which could explain in an unified way both types of phenomena. In this article we shall develop the general formulas of Dirac's theory, and show its applications to several characteristic examples (Part I). In the second part of this work Dirac's relativistic wave equation of the electron will be discussed in relation to the theory of radiation. The third part will be devoted to the problems of the general quantum electrodynamics, and to the difficulties connected with it.

---

* Lectures delivered at the Symposium for Theoretical Physics during the Summer Session of 1930 at the University of Michigan.

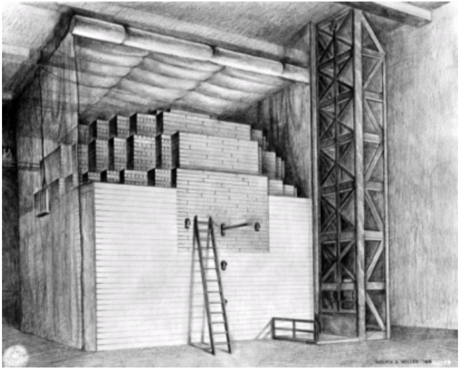
87

# Enrico Fermi (Italia)



# Enrico Fermi (USA)(I)

# Enrico Fermi (USA)(II)



# The Curve is Exponential(I)

"I couldn't see the instruments," said Weil. "I had to watch Fermi every second, waiting for orders. He face was motionless. His eyes darted from one dial to another. His expression was so calm it was hard to read. But suddenly, his whole face broke into a broad smile."
Fermi closed his slide rule—
"The reaction is self-sustaining," he announced quietly, happily. "The curve is exponential."
The group tensely watched for twenty-eight minutes while the world's first nuclear chain reactor operated.
The upward movement of the pen was leaving a straight line. There was no change in indicate a leveling off. This was it.
"O.K., 'Zip' in," called Fermi to Zinn who controlled that rod. The time was 3:53 p.m. Abruptly, the counters slowed down, the pen slid down across the paper. It was all over.

Man had initiated a self-sustaining nuclear reaction - and then stopped it. He had released the energy of the atom's nucleus and controlled that energy.

# The Curve is Exponential (II)

Right after Fermi ordered the reaction stopped, the Hungarian-born theoretical physicist Eugene Wigner presented him with a bottle of Chianti wine. All through the experiment Wigner had kept this wine hidden behind his back.

Fermi uncorked the wine bottle and sent out for paper cups so all could drink. He poured a little wine in all the cups, and silently, solemnly, without toasts, the scientists raised the cups to their lips - the Canadian Zinn, the Hungarians Szilard and Wigner, the Italian Fermi, the Americans Compton, Anderson, Hilberry, and a score of others. They drank to success - and to the hope they were the first to succeed.

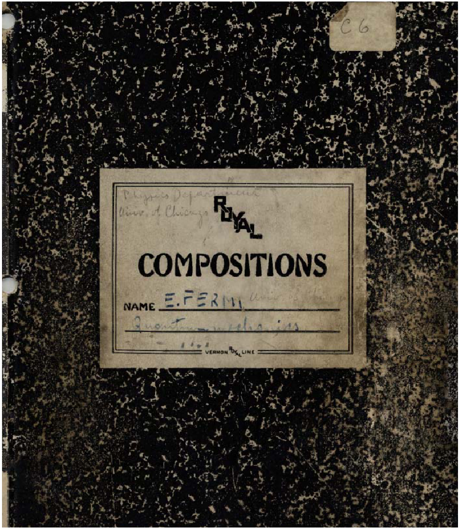
A small crew was left to straighten up, lock controls, and check all apparatus.

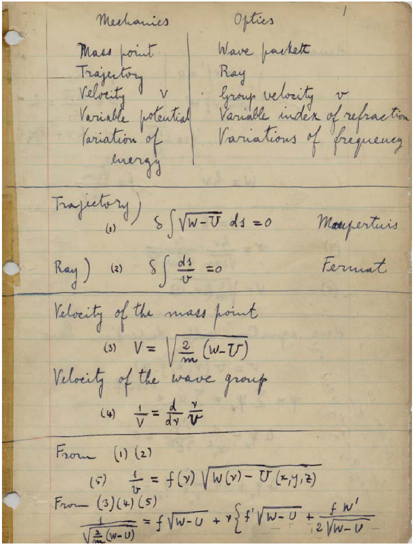
As the group filed from the West Stands, one of the guards asked Zinn: "What's going on, Doctor, something happen in there?"

The guard did not hear the message which Arthur Compton was giving James B. Conant at Harvard, by long-distance telephone. Their code was not prearranged.

"The Italian navigator has landed in the New World," said Compton. "How were the natives?" asked Conant. "Very friendly."

# Enrico Fermi: Profesor


C 6
Physics Department
Univ. of Chicago
ROYAL
COMPOSITIONS
NAME E. FERMI
Univ. of Chicago
Quantum mechanics
VERNON ROYAL LINE



1

| Mechanics | Optics |
|---|---|
| Mass point | Wave packett |
| Trajectory | Ray |
| Velocity $V$ | Group velocity $v$ |
| Variable potential | Variable index of refraction |
| Variation of energy | Variations of frequency |

Trajectory) (1) $\delta\int\sqrt{W-U}\,ds=0$ Maupertuis

Ray) (2) $\delta\int\frac{ds}{v}=0$ Fermat

Velocity of the mass point

$$V=\sqrt{\frac{2}{m}(W-U)} \quad (3)$$

Velocity of the wave group

$$\frac{1}{V}=\frac{d}{d\nu}\frac{\nu}{v} \quad (4)$$

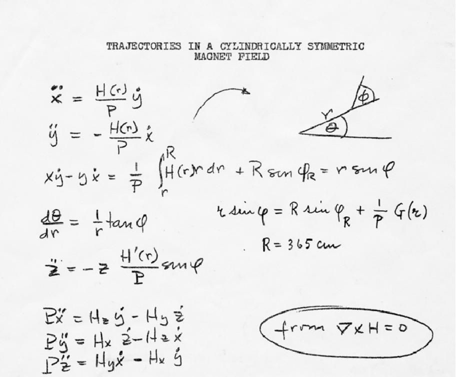
From (1) (2)

$$\frac{1}{v}=f(\nu)\sqrt{W(\nu)-U(x,y,z)} \quad (5)$$

From (3)(4)(5)

$$\frac{1}{\sqrt{\frac{2}{m}(W-U)}}=f\sqrt{W-U}+\nu\left\{f'\sqrt{W-U}+\frac{f\,W'}{2\sqrt{W-U}}\right.$$

# Enrico Fermi: Programador Informático (MANIAC)



TRAJECTORIES IN A CYLINDRICALLY SYMMETRIC MAGNET FIELD

$$\ddot{x} = \frac{H(r)}{P}\dot{y}$$

$$\ddot{y} = -\frac{H(r)}{P}\dot{x}$$

$$x\dot{y} - y\dot{x} = \frac{1}{P}\int_r^R H(r)\,r\,dr + R\sin\varphi_R = r\sin\varphi$$

$$\frac{d\theta}{dr} = \frac{1}{r}\tan\varphi$$

$$r\sin\varphi = R\sin\varphi_R + \frac{1}{P}G(r)$$

$R = 365$ cm

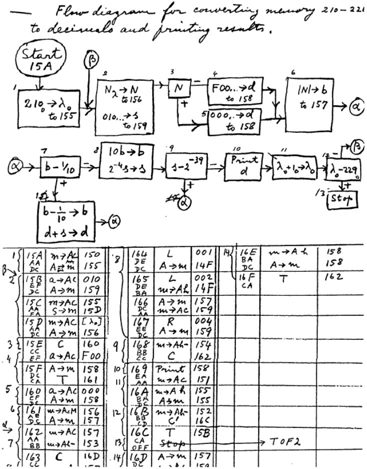
$$\ddot{z} = -z\frac{H'(r)}{P}\sin\varphi$$

$$P\ddot{x} = H_z\dot{y} - H_y\dot{z}$$

$$P\ddot{y} = H_x\dot{z} - H_z\dot{x}$$

$$P\ddot{z} = H_y\dot{x} - H_x\dot{y}$$

from $\nabla \times H = 0$

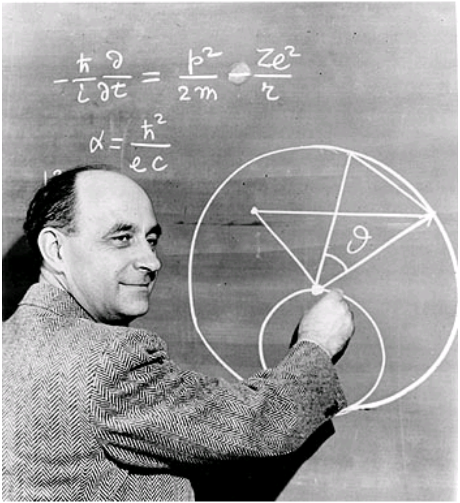


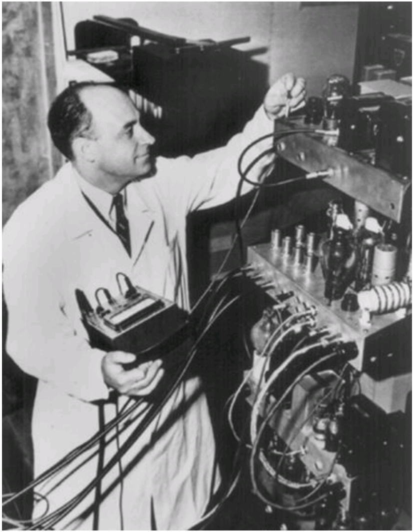
— Flow diagram for converting memory 210–221 to decimals and printing results.

Start 15A
β
1 $210_0 \to \lambda_0$ to 155
2 $N_\lambda \to N$ to 156
010... → s to 159
3 N
−
+
4 F00... → d to 158
5 000... → d to 158
6 |N| → b to 157
α
7 b − 1/10
8 10b → b
$2^{-4}s \to s$
9 $s - 2^{-39}$
10 Print d
11 $\lambda_0 + 1_0 \to \lambda_0$
12 $\lambda_0 - 229_0$
13 Stop
14 $b - \frac{1}{10} \to b$
d + s → d

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 15A<br>AA<br>DC | m→Ac<br>A⇄m | 150<br>155 | 8 | 164<br>DE<br>DC | L<br>A→m | 001<br>14F | 14 | 16E<br>BA<br>DC | m→Ah<br>A→m | 158<br>158 |
| β→ 2 | 15B<br>EF<br>DC | a→Ac<br>A→m | 010<br>159 | | 165<br>DE<br>BA | L<br>m→Ah | 002<br>14F | | 16F<br>CA | T | 162 |
| | 15C<br>AA<br>FA | m→AC<br>S→m | 155<br>15D | | 166<br>DC<br>AA | A→m<br>m→AC | 157<br>159 | | | | |
| | 15D<br>AA<br>DC | m→AC<br>A→m | [$\lambda_0$]<br>156 | | 167<br>EE<br>DC | R<br>A→m | 004<br>159 | | | | |
| 3<br>4 | 15E<br>CC<br>EF | C<br>a→Ac | 160<br>F00 | 9 | 168<br>BB<br>CC | m→Ah−<br>C | 154<br>162 | | | | |
| | 15F<br>DC<br>CA | A→m<br>T | 158<br>161 | 10<br>11 | 169<br>EA<br>AA | Print<br>m→Ac | 158<br>151 | | | | |
| 5 | 160<br>EF<br>DC | a→AC<br>A→m | 000<br>158 | | 16A<br>BA<br>DC | m→Ah<br>A→m | 155<br>155 | | | | |
| 6 | 161<br>AE<br>DC | m→AcM<br>A→m | 156<br>157 | 12 | 16B<br>BB<br>CD | m→Ah−<br>C′ | 152<br>16C | | | | |
| α→ 7 | 162<br>AA<br>BB | m→AC<br>m→Ah− | 157<br>153 | 13 | 16C<br>CA<br>OFF | T<br>~~Stop~~ | 15B | | | → TOF2 | |
| | 163<br>CC | C | 16D | 14 | 16D<br>DC | A→m | 157 | | | | |

# Enrico Fermi: El último Físico Completo

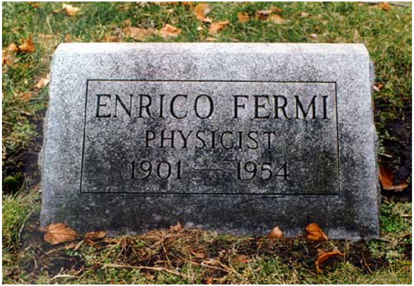

$-\frac{\hbar}{i}\frac{\partial}{\partial t} = \frac{p^2}{2m} - \frac{Ze^2}{r}$
$\alpha = \frac{\hbar^2}{ec}$
$\vartheta$



# Enrico Fermi (tenista)

# Enrico Fermi: Epitafio

ENRICO·FERMI
1901-1954


ENRICO FERMI
PHYSICIST
1901 — 1954

# Estadística de Fermi-Dirac (Fermi)

- Interesado en problemas en Mecánica Estadística Cuántica (1923-1925), e.g.:
  - “Sopra la teoria di Stern della costante assoluta dell’entropia in un gas perfetto monoatomico” (1923).
  - “Sulla probabilitàdegli stati quantici” (1923).
  - “Considerazioni sulla quantizzazione dei sistemi che contengono degli elementi identici” (1924).
  - “Sull’equilibrio termico di ionizazione” (1924).
  - “Sopra la teoria dei corpi solidi” (1925).

# Estadística de Fermi-Dirac (Fermi)

- Interesado en problemas en Mecánica Estadística Cuántica (1923-1925), e.g.:
  - “Sopra la teoria di Stern della costante assoluta dell’entropia in un gas perfetto monoatomico” (1923).
  - “Sulla probabilitàdegli stati quantici” (1923).
  - “Considerazioni sulla quantizzazione dei sistemi che contengono degli elementi identici” (1924).
  - “Sull’equilibrio termico di ionizazione” (1924).
  - “Sopra la teoria dei corpi solidi” (1925).
- Pauli publica su principio de exclusión (1925).

# Estadística de Fermi-Dirac (Fermi)

- Interesado en problemas en Mecánica Estadística Cuántica (1923-1925), e.g.:
  - “Sopra la teoria di Stern della costante assoluta dell’entropia in un gas perfetto monoatomico” (1923).
  - “Sulla probabilitàdegli stati quantici” (1923).
  - “Considerazioni sulla quantizzazione dei sistemi che contengono degli elementi identici” (1924).
  - “Sull’equilibrio termico di ionizazione” (1924).
  - “Sopra la teoria dei corpi solidi” (1925).
- Pauli publica su principio de exclusión (1925).
- Inmediatamente Fermi publica dos artículos:
  - Uno largo: “Zur Quantelung Des Ideales Einatomigen Gas” (1926)
  - Uno corto (5 páginas): “Sulla quantizzazione del gas perfetto monoatomico” (1926).

# Estadística de Fermi-Dirac (Fermi)

- Usa un potencial armónico (en vez de un pozo cuadrado infinito) para confinar el gas.

# Estadística de Fermi-Dirac (Fermi)

- Usa un potencial armónico (en vez de un pozo cuadrado infinito) para confinar el gas.
- Asume que solo puede haber un “átomo” en cada nivel cuántico.

# Estadística de Fermi-Dirac (Fermi)

- Usa un potencial armónico (en vez de un pozo cuadrado infinito) para confinar el gas.
- Asume que solo puede haber un “átomo” en cada nivel cuántico.
- Calcula correctamente los niveles de energía del oscilador armónico (mecánica cuántica antigua).

# Estadística de Fermi-Dirac (Fermi)

- Usa un potencial armónico (en vez de un pozo cuadrado infinito) para confinar el gas.
- Asume que solo puede haber un “átomo” en cada nivel cuántico.
- Calcula correctamente los niveles de energía del oscilador armónico (mecánica cuántica antigua).
- A altas temperaturas recupera la formulación clásica. Ecuación de Stein y Tretode.

# Estadística de Fermi-Dirac (Fermi)

- Usa un potencial armónico (en vez de un pozo cuadrado infinito) para confinar el gas.
- Asume que solo puede haber un “átomo” en cada nivel cuántico.
- Calcula correctamente los niveles de energía del oscilador armónico (mecánica cuántica antigua).
- A altas temperaturas recupera la formulación clásica. Ecuación de Stein y Tretode.
- $S(T=0)=0$.

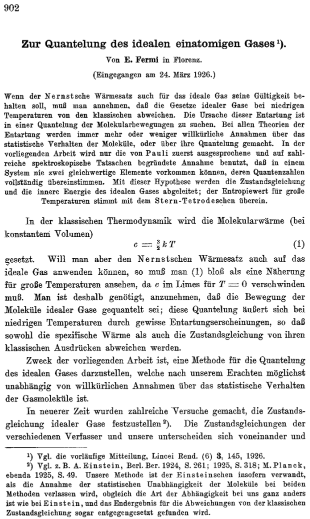
# Estadística de Fermi-Dirac (Fermi)

- Usa un potencial armónico (en vez de un pozo cuadrado infinito) para confinar el gas.
- Asume que solo puede haber un “átomo” en cada nivel cuántico.
- Calcula correctamente los niveles de energía del oscilador armónico (mecánica cuántica antigua).
- A altas temperaturas recupera la formulación clásica. Ecuación de Stein y Tretode.
- $S(T=0)=0$.
- Fermi asume que todos los átomos obedecen el principio de exclusión de Pauli: ahora conocemos que no es cierto.

# Estadística de Fermi-Dirac (Fermi)

- Usa un potencial armónico (en vez de un pozo cuadrado infinito) para confinar el gas.
- Asume que solo puede haber un “átomo” en cada nivel cuántico.
- Calcula correctamente los niveles de energía del oscilador armónico (mecánica cuántica antigua).
- A altas temperaturas recupera la formulación clásica. Ecuación de Stein y Tretode.
- $S(T = 0) = 0$.
- Fermi asume que todos los átomos obedecen el principio de exclusión de Pauli: ahora conocemos que no es cierto.
- Fermi lo aplica al átomo de Helio que es un bosón.

# Estadística de Fermi-Dirac (Fermi) (III)



## Zur Quantelung des idealen einatomigen Gases[1]).

Von **E. Fermi** in Florenz.

(Eingegangen am 24. März 1926.)

Wenn der Nernstsche Wärmesatz auch für das ideale Gas seine Gültigkeit behalten soll, muß man annehmen, daß die Gesetze idealer Gase bei niedrigen Temperaturen von den klassischen abweichen. Die Ursache dieser Entartung ist in einer Quantelung der Molekularbewegungen zu suchen. Bei allen Theorien der Entartung werden immer mehr oder weniger willkürliche Annahmen über das statistische Verhalten der Moleküle, oder über ihre Quantelung gemacht. In der vorliegenden Arbeit wird nur die von Pauli zuerst ausgesprochene und auf zahlreiche spektroskopische Tatsachen begründete Annahme benutzt, daß in einem System nie zwei gleichwertige Elemente vorkommen können, deren Quantenzahlen vollständig übereinstimmen. Mit dieser Hypothese werden die Zustandsgleichung und die innere Energie des idealen Gases abgeleitet; der Entropiewert für große Temperaturen stimmt mit dem Stern-Tetrodeschen überein.

In der klassischen Thermodynamik wird die Molekularwärme (bei konstantem Volumen)

$$c = \tfrac{3}{2} k T \qquad (1)$$

gesetzt. Will man aber den Nernstschen Wärmesatz auch auf das ideale Gas anwenden können, so muß man (1) bloß als eine Näherung für große Temperaturen ansehen, da $c$ im Limes für $T = 0$ verschwinden muß. Man ist deshalb genötigt, anzunehmen, daß die Bewegung der Moleküle idealer Gase gequantelt sei; diese Quantelung äußert sich bei niedrigen Temperaturen durch gewisse Entartungserscheinungen, so daß sowohl die spezifische Wärme als auch die Zustandsgleichung von ihren klassischen Ausdrücken abweichen werden.

Zweck der vorliegenden Arbeit ist, eine Methode für die Quantelung des idealen Gases darzustellen, welche nach unserem Erachten möglichst unabhängig von willkürlichen Annahmen über das statistische Verhalten der Gasmoleküle ist.

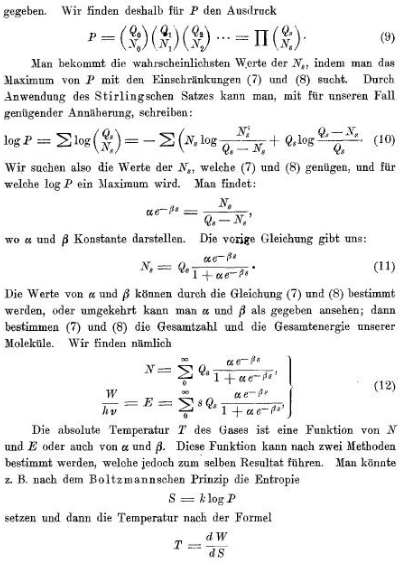
In neuerer Zeit wurden zahlreiche Versuche gemacht, die Zustandsgleichung idealer Gase festzustellen[2]). Die Zustandsgleichungen der verschiedenen Verfasser und unsere unterscheiden sich voneinander und

---

[1]) Vgl. die vorläufige Mitteilung, Lincei Rend. (6) **3**, 145, 1926.

[2]) Vgl. z. B. A. Einstein, Berl. Ber. 1924, S. 261; 1925, S. 318; M. Planck, ebenda 1925, S. 49. Unsere Methode ist der Einsteinschen insofern verwandt, als die Annahme der statistischen Unabhängigkeit der Moleküle bei beiden Methoden verlassen wird, obgleich die Art der Abhängigkeit bei uns ganz anders ist wie bei Einstein, und das Endergebnis für die Abweichungen von der klassischen Zustandsgleichung sogar entgegengesetzt gefunden wird.

# Estadística de Fermi-Dirac (Fermi) (IV)



gegeben. Wir finden deshalb für $P$ den Ausdruck

$$P = \binom{Q_0}{N_0}\binom{Q_1}{N_1}\binom{Q_2}{N_2}\cdots = \prod \binom{Q_s}{N_s}. \tag{9}$$

Man bekommt die wahrscheinlichsten Werte der $N_s$, indem man das Maximum von $P$ mit den Einschränkungen (7) und (8) sucht. Durch Anwendung des Stirlingschen Satzes kann man, mit für unseren Fall genügender Annäherung, schreiben:

$$\log P = \sum \log \binom{Q_s}{N_s} = -\sum \Big( N_s \log \frac{N_s}{Q_s - N_s} + Q_s \log \frac{Q_s - N_s}{Q_s}. \tag{10}$$

Wir suchen also die Werte der $N_s$, welche (7) und (8) genügen, und für welche $\log P$ ein Maximum wird. Man findet:

$$\alpha e^{-\beta s} = \frac{N_s}{Q_s - N_s},$$

wo $\alpha$ und $\beta$ Konstante darstellen. Die vorige Gleichung gibt uns:

$$N_s = Q_s \frac{\alpha e^{-\beta s}}{1 + \alpha e^{-\beta s}}. \tag{11}$$

Die Werte von $\alpha$ und $\beta$ können durch die Gleichung (7) und (8) bestimmt werden, oder umgekehrt kann man $\alpha$ und $\beta$ als gegeben ansehen; dann bestimmen (7) und (8) die Gesamtzahl und die Gesamtenergie unserer Moleküle. Wir finden nämlich

$$\left.\begin{aligned} N &= \sum_0^\infty Q_s \frac{\alpha e^{-\beta s}}{1 + \alpha e^{-\beta s}}, \\ \frac{W}{h\nu} = E &= \sum_0^\infty s\, Q_s \frac{\alpha e^{-\beta s}}{1 + \alpha e^{-\beta s}}, \end{aligned}\right\} \tag{12}$$

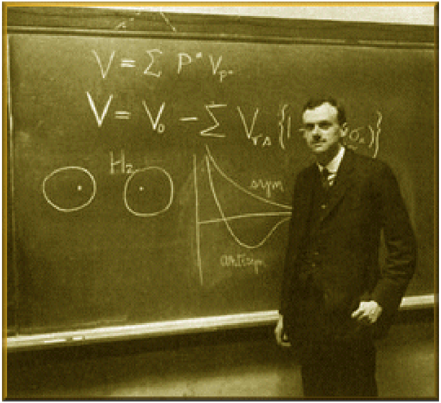
Die absolute Temperatur $T$ des Gases ist eine Funktion von $N$ und $E$ oder auch von $\alpha$ und $\beta$. Diese Funktion kann nach zwei Methoden bestimmt werden, welche jedoch zum selben Resultat führen. Man könnte z. B. nach dem Boltzmannschen Prinzip die Entropie

$$S = k \log P$$

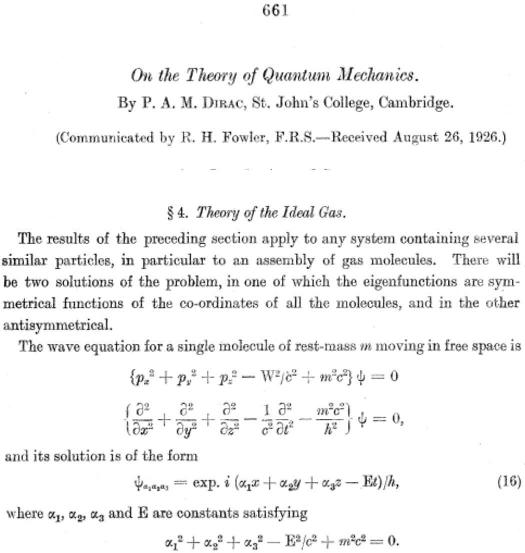
setzen und dann die Temperatur nach der Formel

$$T = \frac{dW}{dS}$$

# Paul Adrien Maurice Dirac (1902-1984)


H2
sym
antisym

# Estadística de Fermi-Dirac (Dirac) (I)



661

*On the Theory of Quantum Mechanics.*

By P. A. M. Dirac, St. John's College, Cambridge.

(Communicated by R. H. Fowler, F.R.S.—Received August 26, 1926.)

§ 4. *Theory of the Ideal Gas.*

The results of the preceding section apply to any system containing several similar particles, in particular to an assembly of gas molecules. There will be two solutions of the problem, in one of which the eigenfunctions are symmetrical functions of the co-ordinates of all the molecules, and in the other antisymmetrical.

The wave equation for a single molecule of rest-mass $m$ moving in free space is

$$\{p_x^2 + p_y^2 + p_z^2 - W^2/c^2 + m^2c^2\}\,\psi = 0$$

$$\left\{\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} - \frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \frac{m^2c^2}{h^2}\right\}\psi = 0,$$

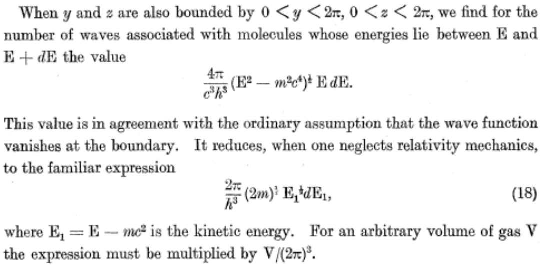
and its solution is of the form

$$\psi_{\alpha_1\alpha_2\alpha_3} = \exp.\, i\,(\alpha_1 x + \alpha_2 y + \alpha_3 z - \mathrm{E}t)/h, \qquad (16)$$

where $\alpha_1$, $\alpha_2$, $\alpha_3$ and E are constants satisfying

$$\alpha_1^2 + \alpha_2^2 + \alpha_3^2 - \mathrm{E}^2/c^2 + m^2c^2 = 0.$$

# Estadística de Fermi-Dirac (Dirac) (II)



When $y$ and $z$ are also bounded by $0 < y < 2\pi$, $0 < z < 2\pi$, we find for the number of waves associated with molecules whose energies lie between E and $\mathrm{E} + d\mathrm{E}$ the value

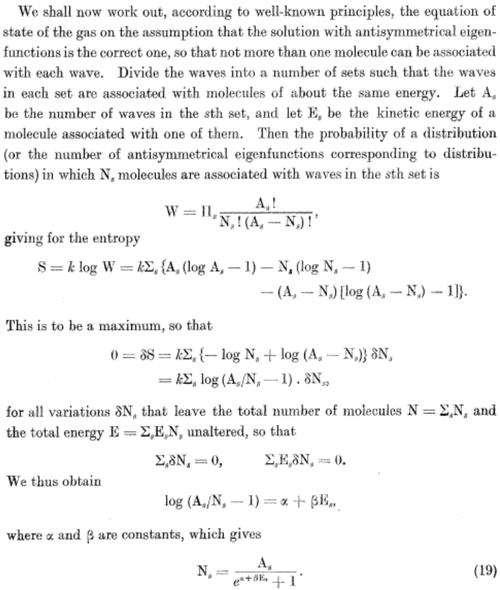
$$\frac{4\pi}{c^3 h^3}(\mathrm{E}^2 - m^2c^4)^{\frac{1}{2}}\,\mathrm{E}\,d\mathrm{E}.$$

This value is in agreement with the ordinary assumption that the wave function vanishes at the boundary. It reduces, when one neglects relativity mechanics, to the familiar expression

$$\frac{2\pi}{h^3}(2m)^{\frac{3}{2}}\,\mathrm{E}_1^{\frac{1}{2}}d\mathrm{E}_1, \tag{18}$$

where $\mathrm{E}_1 = \mathrm{E} - mc^2$ is the kinetic energy. For an arbitrary volume of gas V the expression must be multiplied by $\mathrm{V}/(2\pi)^3$.

# Estadística de Fermi-Dirac (Dirac) (III)



We shall now work out, according to well-known principles, the equation of state of the gas on the assumption that the solution with antisymmetrical eigenfunctions is the correct one, so that not more than one molecule can be associated with each wave. Divide the waves into a number of sets such that the waves in each set are associated with molecules of about the same energy. Let $A_s$ be the number of waves in the $s$th set, and let $E_s$ be the kinetic energy of a molecule associated with one of them. Then the probability of a distribution (or the number of antisymmetrical eigenfunctions corresponding to distributions) in which $N_s$ molecules are associated with waves in the $s$th set is

$$W = \Pi_s \frac{A_s!}{N_s!\,(A_s - N_s)!},$$

giving for the entropy

$$S = k \log W = k\Sigma_s \{A_s (\log A_s - 1) - N_s (\log N_s - 1) \\ - (A_s - N_s)[\log (A_s - N_s) - 1]\}.$$

This is to be a maximum, so that

$$0 = \delta S = k\Sigma_s \{-\log N_s + \log (A_s - N_s)\}\, \delta N_s \\ = k\Sigma_s \log (A_s/N_s - 1)\,.\,\delta N_s,$$

for all variations $\delta N_s$ that leave the total number of molecules $N = \Sigma_s N_s$ and the total energy $E = \Sigma_s E_s N_s$ unaltered, so that

$$\Sigma_s \delta N_s = 0, \qquad \Sigma_s E_s \delta N_s = 0.$$

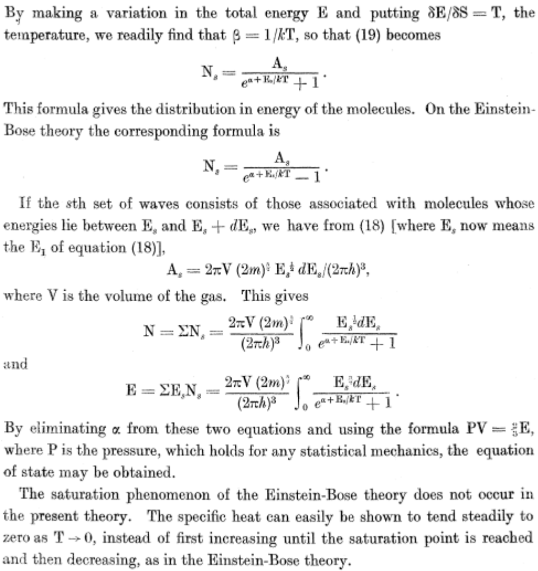
We thus obtain

$$\log (A_s/N_s - 1) = \alpha + \beta E_s,$$

where $\alpha$ and $\beta$ are constants, which gives

$$N_s = \frac{A_s}{e^{\alpha + \beta E_s} + 1}. \qquad (19)$$

# Estadística de Fermi-Dirac (Dirac) (IV)



By making a variation in the total energy E and putting $\delta E/\delta S = T$, the temperature, we readily find that $\beta = 1/kT$, so that (19) becomes

$$N_s = \frac{A_s}{e^{\alpha + E_s/kT} + 1}.$$

This formula gives the distribution in energy of the molecules. On the Einstein-Bose theory the corresponding formula is

$$N_s = \frac{A_s}{e^{\alpha + E_s/kT} - 1}.$$

If the *s*th set of waves consists of those associated with molecules whose energies lie between $E_s$ and $E_s + dE_s$, we have from (18) [where $E_s$ now means the $E_1$ of equation (18)],

$$A_s = 2\pi V\,(2m)^{\frac{3}{2}}\,E_s^{\frac{1}{2}}\,dE_s/(2\pi h)^3,$$

where V is the volume of the gas. This gives

$$N = \Sigma N_s = \frac{2\pi V\,(2m)^{\frac{3}{2}}}{(2\pi h)^3} \int_0^\infty \frac{E_s^{\frac{1}{2}} dE_s}{e^{\alpha + E_s/kT} + 1}$$

and

$$E = \Sigma E_s N_s = \frac{2\pi V\,(2m)^{\frac{3}{2}}}{(2\pi h)^3} \int_0^\infty \frac{E_s^{\frac{3}{2}} dE_s}{e^{\alpha + E_s/kT} + 1}.$$

By eliminating $\alpha$ from these two equations and using the formula $PV = \frac{2}{3}E$, where P is the pressure, which holds for any statistical mechanics, the equation of state may be obtained.

The saturation phenomenon of the Einstein-Bose theory does not occur in the present theory. The specific heat can easily be shown to tend steadily to zero as T → 0, instead of first increasing until the saturation point is reached and then decreasing, as in the Einstein-Bose theory.