INTRODUCTION TO THE SUPERCONDUCTIVITY

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Electromagnetismo II (3º Física)

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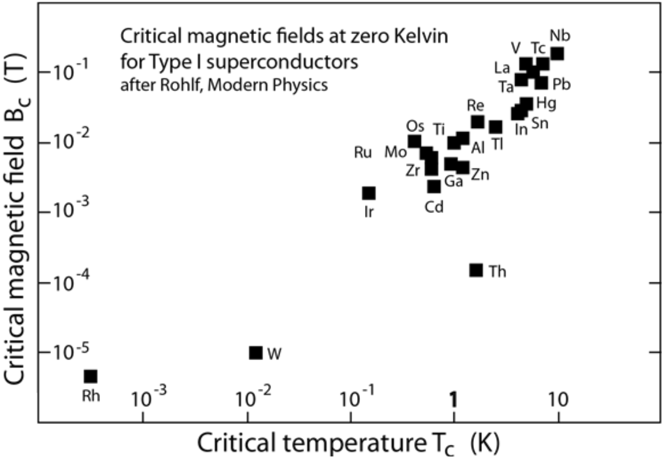
What is Superconductivity?

- Superconductivity is a low-temperature state of some materials characterized by:
  - Zero DC electrical resistance.
  - Expulsion of magnetic field from the bulk.
- It was discovered in 1911 by Heike Kamerlingh Onnes in mercury.
- A material becomes superconducting below a critical temperature T_c.
- It is not just a better conductor: it is a distinct thermodynamic phase.

Main question

How does a superconductor respond to electromagnetic fields, and why is this response different from that of an ordinary metal?

Experimental data



J. W. Rohlf, *Modern Physics from a to Z* (Wiley 1994)

Electromagnetic Waves in a Metal

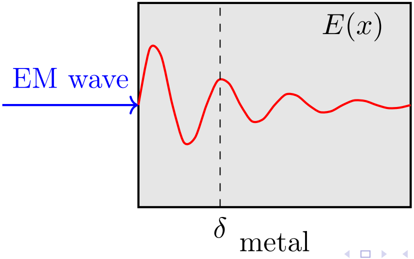
Consider a metal with finite conductivity σ exposed to an EM wave. From Maxwell equations and Ohm's law,

$$\boldsymbol{J} = \sigma \boldsymbol{E},$$

one finds that the EM field does not propagate freely inside the conductor. Instead it decays exponentially:

$$E(x) = E_0 e^{-x/\delta}, \text{ with } \delta = \sqrt{\frac{2}{\mu_0 \sigma \omega}},$$

δ is the skin depth.



Perfect Conductors vs Superconductors

Perfect conductor:

- If $\rho = 0$, then in the stationary regime

$$\boldsymbol{E} = \rho \boldsymbol{J} = 0.$$

- From Faraday's law,

$$\nabla \times \boldsymbol{E} = -\frac{\partial \boldsymbol{B}}{\partial t},$$

  one gets

$$\frac{\partial \boldsymbol{B}}{\partial t} = 0.$$

So, in a perfect conductor, the magnetic field is frozen in: whatever field was present initially remains trapped.

Superconductor:

- In equilibrium, the material expels magnetic field from the bulk:

$$\boldsymbol{B} \approx 0.$$

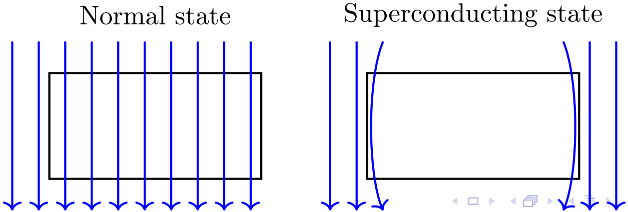
The Meissner Effect

Meissner effect

When a material enters the superconducting state, magnetic flux is expelled from its interior.

- This happens even if the field was present before cooling below T_c.
- Therefore superconductors are not simply perfect conductors.
- The superconducting state is characterized by a new constitutive electrodynamics.

$$\boldsymbol{B}(x) \to 0 \qquad \text{deep inside the sample.}$$



Idea of the London Model

- In a superconductor, some electrons form a dissipationless fluid.
- Let n_s be the density of superconducting carriers.
- For an electron of charge $-e$ and mass m, neglect resistive forces and write Newton's law:

$$m\frac{d\boldsymbol{v}}{dt} = -e\boldsymbol{E}.$$

- The superconducting current density is

$$\boldsymbol{J}_s = -n_s e\,\boldsymbol{v}.$$

Goal

Use these two equations to derive the London equations.

Derivation of the First London Equation

Start from

$$\boldsymbol{J}_s = -n_s e\,\boldsymbol{v}.$$

Differentiate with respect to time:

$$\frac{\partial \boldsymbol{J}_s}{\partial t} = -n_s e\,\frac{d\boldsymbol{v}}{dt}.$$

Using Newton's law,

$$m\frac{d\boldsymbol{v}}{dt} = -e\boldsymbol{E} \qquad \Rightarrow \qquad \frac{d\boldsymbol{v}}{dt} = -\frac{e}{m}\boldsymbol{E},$$

we obtain

$$\frac{\partial \boldsymbol{J}_s}{\partial t} = -n_s e\left(-\frac{e}{m}\boldsymbol{E}\right).$$

Therefore

$$\boxed{\frac{\partial \boldsymbol{J}_s}{\partial t} = \frac{n_s e^2}{m}\boldsymbol{E}}$$

This is the first London equation.

Physical Meaning of the First London Equation

$$\frac{\partial \boldsymbol{J}_s}{\partial t} = \frac{n_s e^2}{m} \boldsymbol{E}$$

- In an ordinary conductor, Ohm's law gives

$$\boldsymbol{J} = \sigma \boldsymbol{E}.$$

- In a superconductor, a constant electric field does not produce a constant current.
- Instead, it produces an accelerating supercurrent.

Interpretation

The first London equation is the phenomenological expression of zero resistance.

Derivation of the Second London Equation

Start from the first London equation:

$$\frac{\partial \boldsymbol{J}_s}{\partial t} = \frac{n_s e^2}{m}\boldsymbol{E}.$$

Take the curl:

$$\frac{\partial}{\partial t}(\nabla \times \boldsymbol{J}_s) = \frac{n_s e^2}{m}\nabla \times \boldsymbol{E}.$$

Use Faraday's law:

$$\nabla \times \boldsymbol{E} = -\frac{\partial \boldsymbol{B}}{\partial t}.$$

Then

$$\frac{\partial}{\partial t}\left(\nabla \times \boldsymbol{J}_s + \frac{n_s e^2}{m}\boldsymbol{B}\right) = 0.$$

Assuming the integration constant is zero in equilibrium, we get

$$\boxed{\nabla \times \boldsymbol{J}_s = -\frac{n_s e^2}{m}\boldsymbol{B}}$$

This is the second London equation.

Consequence: Magnetic Field Screening ⇒ Meissner Screening

Combine the second London equation

$$\nabla \times \boldsymbol{J}_s = -\frac{n_s e^2}{m} \boldsymbol{B}$$

with Ampère's law

$$\nabla \times \boldsymbol{B} = \mu_0 \boldsymbol{J}_s.$$

Taking the curl of Ampère's law,

$$\nabla \times (\nabla \times \boldsymbol{B}) = \mu_0 \nabla \times \boldsymbol{J}_s.$$

Using $\nabla \cdot \boldsymbol{B} = 0$,

$$-\nabla^2 \boldsymbol{B} = -\mu_0 \frac{n_s e^2}{m} \boldsymbol{B}.$$

Therefore

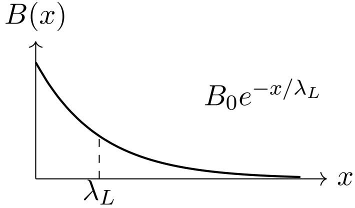
$$\nabla^2 \boldsymbol{B} = \frac{1}{\lambda_L^2} \boldsymbol{B}, \qquad \lambda_L = \sqrt{\frac{m}{\mu_0 n_s e^2}}.$$

Penetration Depth

For a semi-infinite superconductor occupying $x > 0$, the solution is

$$B(x) = B_0 e^{-x/\lambda_L}.$$

- λ_L is the London penetration depth.
- It gives the distance over which the magnetic field penetrates into the material.
- Deep in the bulk, $B(x) \to 0$.



Typical values are

$$\lambda_L \sim 10^{-8} - 10^{-7}\,\text{m}.$$

Metal vs Superconductor: Electromagnetic Response

	Normal metal	Superconductor
Current law	$\boldsymbol{J} = \sigma \boldsymbol{E}$	London equations
Field penetration	Skin effect	Meissner screening
Characteristic length	$\delta = \sqrt{\dfrac{2}{\mu_0 \sigma \omega}}$	$\lambda_L = \sqrt{\dfrac{m}{\mu_0 n_s e^2}}$
Frequency dependence	depends on ω	approximately intrinsic
Bulk magnetic field	generally nonzero	expelled from bulk
Physical origin	dissipation/conductivity	coherent quantum state

Key message

A superconductor is not just a perfect conductor: its electromagnetic response is qualitatively different.

Type I and Type II Superconductors

Type I superconductors

- One critical field: H_c.
- Full Meissner state below H_c.
- Superconductivity is destroyed abruptly above H_c.

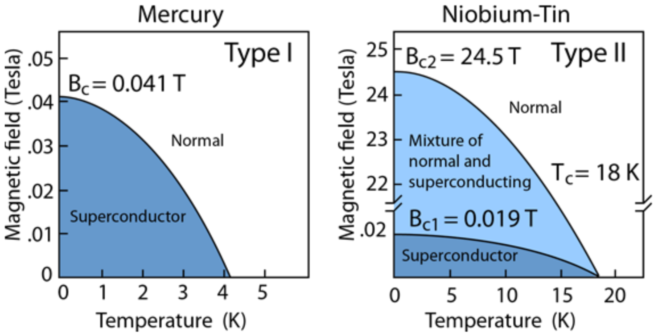
Type II superconductors

- Two critical fields: H_{c1} and H_{c2}.
- For $H < H_{c1}$: Meissner state.
- For $H_{c1} < H < H_{c2}$: mixed state.
- For $H > H_{c2}$: normal state.

Why important?

Most technologically useful superconductors are type II because they survive at much higher magnetic fields.

Type I vs Type II superconductors



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Flux Quantization and Vortices

The superconducting condensate has a macroscopic wavefunction whose phase must be single-valued. This leads to flux quantization:

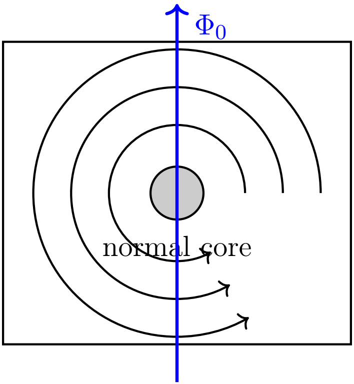
$$\Phi = n\Phi_0, \qquad \Phi_0 = \frac{h}{2e}, \qquad n \in \mathbb{Z}.$$

- Φ_0 is the flux quantum.
- The factor $2e$ reflects the charge of a Cooper pair.

In type II superconductors, flux enters as vortices:

- the core is locally normal,
- supercurrents circulate around it,
- each vortex carries one quantum of flux.

Sketch of a Vortex in a Type II Superconductor



- Outside the core, the material remains superconducting.
- Vortex motion can lead to dissipation.
- Vortex pinning is therefore important in applications.

From Electrodynamics to Microscopic Theory

The London equations describe superconductivity at a macroscopic level, but they do not explain its microscopic origin.

The microscopic explanation was developed in 1957 by

Bardeen, Cooper, and Schrieffer (BCS)

Key idea:

- Electrons can form weakly bound pairs called Cooper pairs.
- The pairing is mediated by interactions with the lattice (phonons).
- These pairs condense into a coherent quantum state.

Physical consequence

A collective quantum state forms, allowing current to flow without dissipation.

Bardeen-Cooper-Schriffer

Cooper Pairs and the BCS Ground State

Consider electrons near the Fermi surface.

Even a weak attractive interaction can produce bound pairs:

$$(\boldsymbol{k}, \uparrow) + (-\boldsymbol{k}, \downarrow) \longrightarrow \text{Cooper pair.}$$

The BCS ground state is a coherent superposition of paired states:

$$|\Psi_{\text{BCS}}\rangle = \prod_{\boldsymbol{k}} \left(u_k + v_k \, c^\dagger_{\boldsymbol{k}\uparrow} c^\dagger_{-\boldsymbol{k}\downarrow} \right) |0\rangle.$$

- The coefficients u_k and v_k describe the occupation of paired states.
- The ground state has long-range phase coherence.

Energy Gap and Quasiparticles

A central prediction of BCS theory is the appearance of an energy gap in the excitation spectrum.

The quasiparticle dispersion becomes

$$E_k = \sqrt{(\epsilon_k - \mu)^2 + \Delta^2},$$

where

- ϵ_k is the normal-state electron energy,
- μ is the chemical potential,
- Δ is the superconducting energy gap.

Consequence

To create excitations one must break Cooper pairs, which costs a finite energy.

At zero temperature,

$$\Delta(0) \approx 1.76\, k_B T_c.$$

The BCS Hamiltonian

The BCS model describes electrons in a metal with an effective attractive interaction.

$$H = \sum_{\boldsymbol{k},\sigma} \epsilon_{\boldsymbol{k}} c^{\dagger}_{\boldsymbol{k}\sigma} c_{\boldsymbol{k}\sigma} - \sum_{\boldsymbol{k},\boldsymbol{k'}} V_{\boldsymbol{k}\boldsymbol{k'}} c^{\dagger}_{\boldsymbol{k}\uparrow} c^{\dagger}_{-\boldsymbol{k}\downarrow} c_{-\boldsymbol{k'}\downarrow} c_{\boldsymbol{k'}\uparrow}$$

- First term: kinetic energy of electrons in the metal.
- Second term: effective attractive interaction between electrons.
- The interaction scatters pairs of electrons.

Key idea

Electrons near the Fermi surface form correlated pairs called Cooper pairs.

Physical Meaning of the Interaction Term

The interaction term is

$$-\sum_{\boldsymbol{k},\boldsymbol{k}'} V_{\boldsymbol{k}\boldsymbol{k}'} c^\dagger_{\boldsymbol{k}\uparrow} c^\dagger_{-\boldsymbol{k}\downarrow} c_{-\boldsymbol{k}'\downarrow} c_{\boldsymbol{k}'\uparrow}$$

It describes the scattering process

$$(\boldsymbol{k}', \uparrow), (-\boldsymbol{k}', \downarrow) \rightarrow (\boldsymbol{k}, \uparrow), (-\boldsymbol{k}, \downarrow)$$

- Two electrons with opposite momenta form a pair.
- The pair is scattered into another pair state.
- Total momentum remains zero.

Origin of attraction

The effective attraction comes from electron–phonon interaction.

Take-Home Messages

- Superconductivity is defined by two key properties:
  - zero resistance,
  - Meissner effect.
- Its electrodynamics is described macroscopically by the London equations.
- Magnetic fields penetrate only over the London length λ_L.
- Type II superconductors admit quantized vortices carrying flux Φ_0.
- Microscopically, superconductivity arises from Cooper pairing and the BCS condensate.
- The superconducting state is protected by an energy gap Δ.