Extreme violation of equipartition in mixtures

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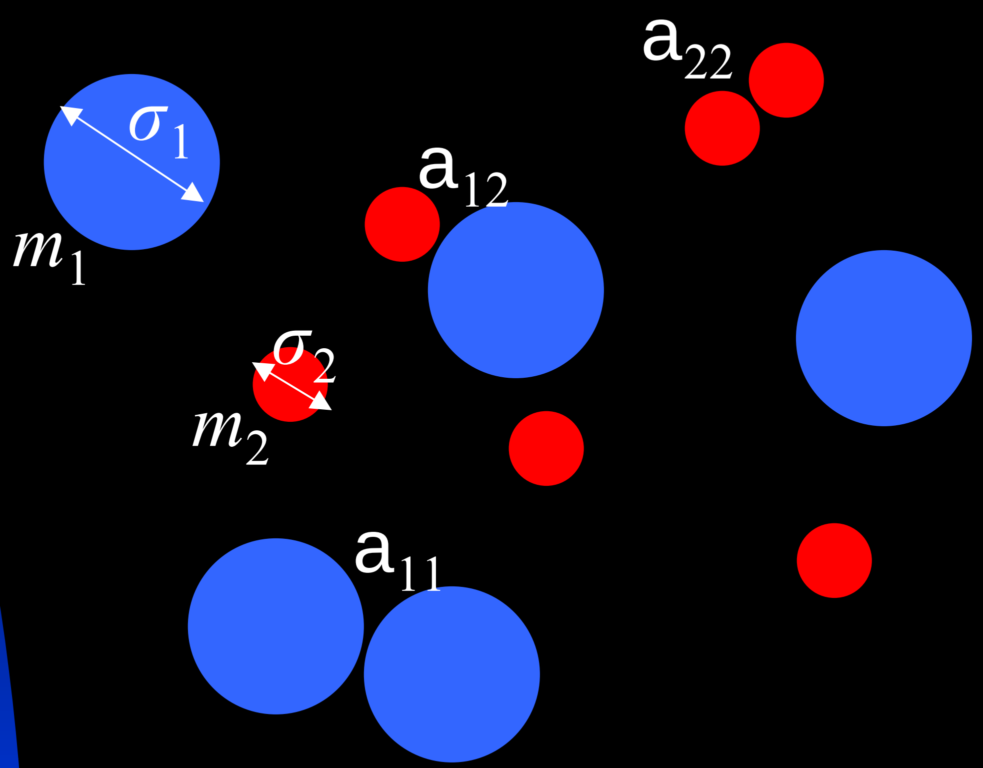
Florida-Paris Workshop on Granular Fluids

Outline

Binary granular mixture



VOLUME 88, NUMBER 19 PHYSICAL REVIEW LETTERS 13 MAY 2002

Breakdown of Energy Equipartition in a 2D Binary Vibrated Granular Gas

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TABLE I. Some material properties of the spheres used in the experiment.

Particle	Mass [mg]	Effective inelasticity[a]	Mass ratio w/glass
Glass	5.24	0.17	–
Aluminum	5.80	0.31	0.92
Steel	15.80	0.21	0.33
Brass	18.00	0.39	0.28

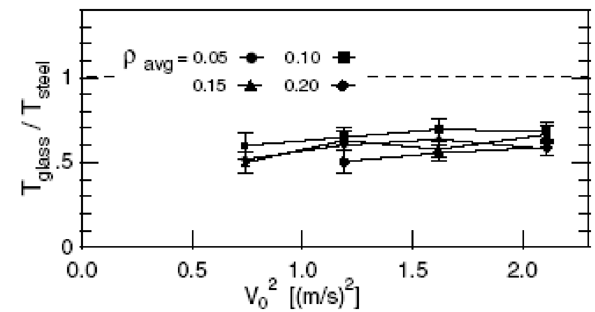


FIG. 4. Temperature ratio, $\gamma = T_{\text{glass}}/T_{\text{steel}}$, in a steel-glass mixture plotted against squared vibration velocity, v_0^2. Different markers represent different number densities of the mixture. The number fraction is fixed at $x = 1/2$. The horizontal dashed line represents equipartition ($\gamma = 1$).

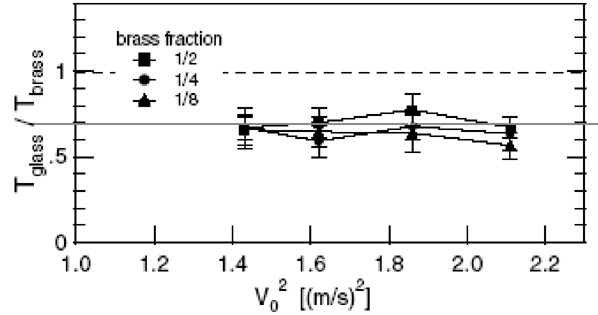


FIG. 5. Temperature ratio, $\gamma = T_{\text{glass}}/T_{\text{brass}}$, in a brass-glass mixture versus the squared vibration velocity of the cell, v_0^2. Different markers represent different number fractions of brass for the same total number of particles ($\rho_{\text{avg}} = 0.049$). The horizontal dashed line represents equipartition ($\gamma = 1$).

Formulation of the problem

- Binary mixture of smooth inelastic hard spheres
  - ✓ Heavy species (1):

m_1, σ_1, $x_1=n_1/n$, a_{11}, a_{12}.

  - ✓ Light species (2):

m_2, σ_2, $x_2=n_2/n=1\text{-}x_1$, a_{22}, $a_{21}= a_{12}$.

- In the homogeneous cooling state,

m_1 à m_2 ï ,v_1^{2}Ú/,v_2^{2}Ú=?

Enskog-Boltzmann equation

$$\partial_t f_1(v) = J_{11}[f_1, f_1] + J_{12}[f_1, f_2]$$

$$\partial_t f_2(v) = J_{21}[f_2, f_1] + J_{22}[f_2, f_2]$$

$$\partial_t \langle v_1^2 \rangle = -\zeta_1 \langle v_1^2 \rangle, \quad \partial_t \langle v_2^2 \rangle = -\zeta_2 \langle v_2^2 \rangle$$

Rates of change $\zeta_1 = \nu \left(\xi_{11} + \xi_{12} \right), \quad \zeta_2 = \nu \left(\xi_{21} + \xi_{22} \right)$

Cooling rates

Thermalization rates

$$\nu = \frac{8\pi}{3} n g_{12} \sigma_{12}^2 \langle v_2 \rangle \frac{1+\alpha_{12}}{2} \frac{m_2}{m_1 + m_2}$$

Effective collision frequency

“Order” parameter

$$\phi \equiv \frac{\langle v_1^2 \rangle}{\langle v_2^2 \rangle}$$

$$\partial_t \phi = -(\zeta_1 - \zeta_2)\,\phi$$

Condition for HCS: $\zeta_1 = \zeta_2$

Maxwellian approximation

PHYSICAL REVIEW E VOLUME 60, NUMBER 5 NOVEMBER 1999

Homogeneous cooling state for a granular mixture

Vicente Garzó* and James Dufty

$$f_i(v) = n_i \left(\frac{m_i}{2\pi T_i}\right)^{3/2} \exp\left(-\frac{m_i v^2}{2T_i}\right), \quad \langle v_i^2 \rangle = \frac{3}{2}\frac{2T_i}{m_i}$$

$$h_2 \sim \frac{m_2}{m_1} \ll 1$$

$$\xi_{11}(\phi) \to x_1 \sqrt{\phi}\beta_1, \quad \xi_{12}(\phi) \to x_2 \sqrt{1+\phi}\left(1 - \frac{h_2}{\phi}\right),$$

cooling rates | thermalization rates

$$\xi_{22}(\phi) \to x_2 \beta_2, \quad \xi_{21}(\phi) \to x_1 \sqrt{1+\phi} h_1^2 \left(1 + \frac{\phi_0 - \phi}{h_2}\right)$$

$$\beta_1 \sim \frac{1-\alpha_{11}^2}{h_2}, \quad \beta_2 \sim \frac{1-\alpha_{22}^2}{h_2}\left(\frac{\sigma_2}{\sigma_1}\right)^2$$

$$\phi_0 = \frac{1-\alpha_{12}}{1+\alpha_{12}}, \quad h_1 = \frac{1+\alpha_{12}}{2}$$

Elastic collisions: $\phi = h_2$ ï $T_1/T_2 = 1$ Energy equipartition!

A few representative cases

- 1. Quasi-elastic cross collisions:

$\alpha_{11} = \alpha_{22} = 1$, $1 - \alpha_{12} = O(h_2)$

$$\beta_1 = \beta_2 = 0, \quad \phi_0 \sim h_2$$

$$\xi_{11}(\phi) = 0, \quad \xi_{12}(\phi) \to x_2 \left(1 - \frac{h_2}{\phi}\right),$$

$$\xi_{22}(\phi) = 0, \quad \xi_{21}(\phi) \to x_1 \left(1 + \frac{\phi_0 - \phi}{h_2}\right)$$

$$h_2 < \phi < h_2 + \phi_0$$

Weak breakdown of energy equipartition

f $\sim h_2$, $T_1 \sim T_2$

"Normal" state

- 2. Inelastic cross collisions:

$\alpha_{11}=\alpha_{22}=1,\ 1-\alpha_{12}=O(1)$

$$\beta_1=\beta_2=0, \quad \phi_0 \lesssim 1$$

$$\xi_{11}(\phi)=0, \quad \xi_{12}(\phi)\to x_2,$$

$$\xi_{22}(\phi)=0, \quad \xi_{21}(\phi)\to x_1\sqrt{1+\phi}\,h_1^2\,\frac{\phi_0-\phi}{h_2}$$

$$\phi=\phi_0=\frac{1-\alpha_{12}}{1+\alpha_{12}}$$

Regardless of the concentrations î

- No Brownian dynamics (x_1 ö 0)
- No Lorentz gas (x_2 ö 0)

Strong breakdown of energy equipartition
f ~1, T_1/T_2 ö ¶

"Ordered" state

3. Inelastic heavy-heavy collisions:

$\alpha_{12} = \alpha_{22} = 1,\ 1 - \alpha_{11} = O(1)$

$$\beta_2 = \phi_0 = 0, \quad \beta_1 \sim h_2^{-1}$$

$$\xi_{11}(\phi) \to x_1 \sqrt{\phi}\beta_1, \quad \xi_{12}(\phi) \to -x_2 \frac{h_2}{\phi}$$

$$\xi_{22}(\phi) = 0, \quad \xi_{21}(\phi) \to x_1$$

$$\phi \sim h_2^{4/3} \to 0, \quad T_1/T_2 \sim h_2^{1/3} \to 0$$

Again, **strong** breakdown of energy equipartition

"Sub-normal" state

- 4. Inelastic light-light collisions + disparate sizes:

$\alpha_{11}=\alpha_{12}=1,\ 1-\alpha_{22}=O(1),\ s_i \sim m_i^{1/3}$

$$\beta_1 = \phi_0 = 0, \quad \beta_2 \sim h_2^{-1/3}$$

$$\xi_{11}(\phi) = 0, \quad \xi_{12}(\phi) \to x_2$$

$$\xi_{22}(\phi) \to x_2\beta_2, \quad \xi_{21}(\phi) \to -x_1 \frac{\phi}{h_2}$$

$$\phi \sim h_2^{2/3} \to 0, \quad T_1/T_2 \sim h_2^{-1/3} \to \infty$$

Intermediate between normal and ordered states

"Sub-ordered" (or "Super-normal") state

- 5. Inelastic light-light collisions + Brownian limit:

$\alpha_{11} = \alpha_{12} = 1,\ 1 - \alpha_{22} = O(1),\ x_1 = O(h_2)$

$$\beta_1 = \phi_0 = 0, \quad \beta_2 \sim h_2^{-1}$$

$$\xi_{11}(\phi) = 0, \quad \xi_{12}(\phi) \to \phi^{1/2}$$

$$\xi_{22}(\phi) \to \beta_2, \quad \xi_{21}(\phi) \to -\frac{x_1}{h_2}\phi^{3/2}$$

$$\phi \sim h_2^{-2/3} \to \infty$$

Very strong breakdown of energy equipartition

“Super-ordered” state

Classification of states

$$h_2 \sim \frac{m_2}{m_1}, \quad \phi = \frac{\langle v_1^2 \rangle}{\langle v_2^2 \rangle} \sim h_2^{\eta}, \quad \frac{T_1}{T_2} \sim h_2^{\eta-1}$$

State	η	$\langle v_1^2 \rangle / \langle v_2^2 \rangle$	T_1/T_2	Example
Sub-normal	$\eta > 1$	0	0	$\alpha_{11} < 1$
Normal	$\eta = 1$	0	finite	$1 - \alpha_{12} \sim m_2/m_1$
Sub-ordered	$0 < \eta < 1$	0	∞	$\alpha_{12} < 1$
Ordered	$\eta = 0$	finite	∞	$\alpha_{22} < 1, \sigma_i \sim m_i^{1/3}$
Super-ordered	$\eta < 0$	∞	∞	$\alpha_{22} < 1, x_1 \sim m_2/m_1$

Scaling laws

$$1-\alpha_{11}\sim h_2^{a_1},\quad (1-\alpha_{22})(\sigma_2/\sigma_1)^2\sim h_2^{a_2},\quad 1-\alpha_{12}\sim h_2^{b}$$

a_1=0 ï Inelastic heavy-heavy collisions
a_1= ¶ ï Elastic heavy-heavy collisions

b=0 ï Inelastic cross collisions
b= ¶ ï Elastic cross collisions

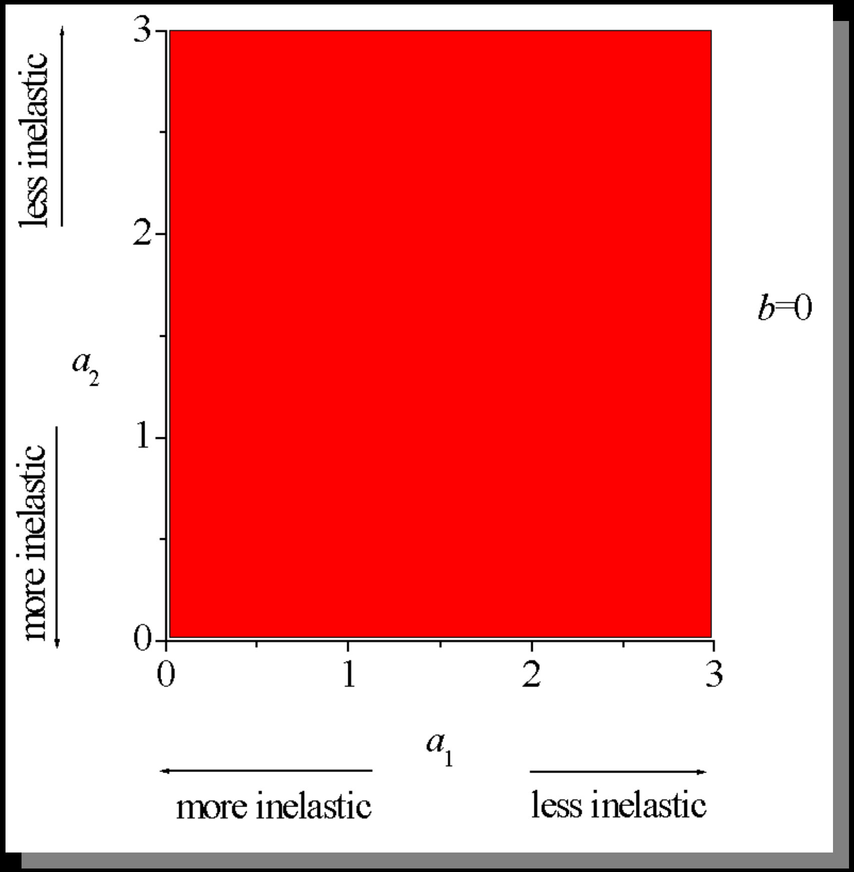
a_2=0 ï Inelastic light-light collisions + comparable sizes
a_2= ¶ ï Elastic light-light collisions

$$\phi\sim h_2^{\eta},\qquad \eta\stackrel{?}{=}\eta(a_1,a_2,b)$$

Phase diagram (finite concentrations)

Inelastic cross collisions

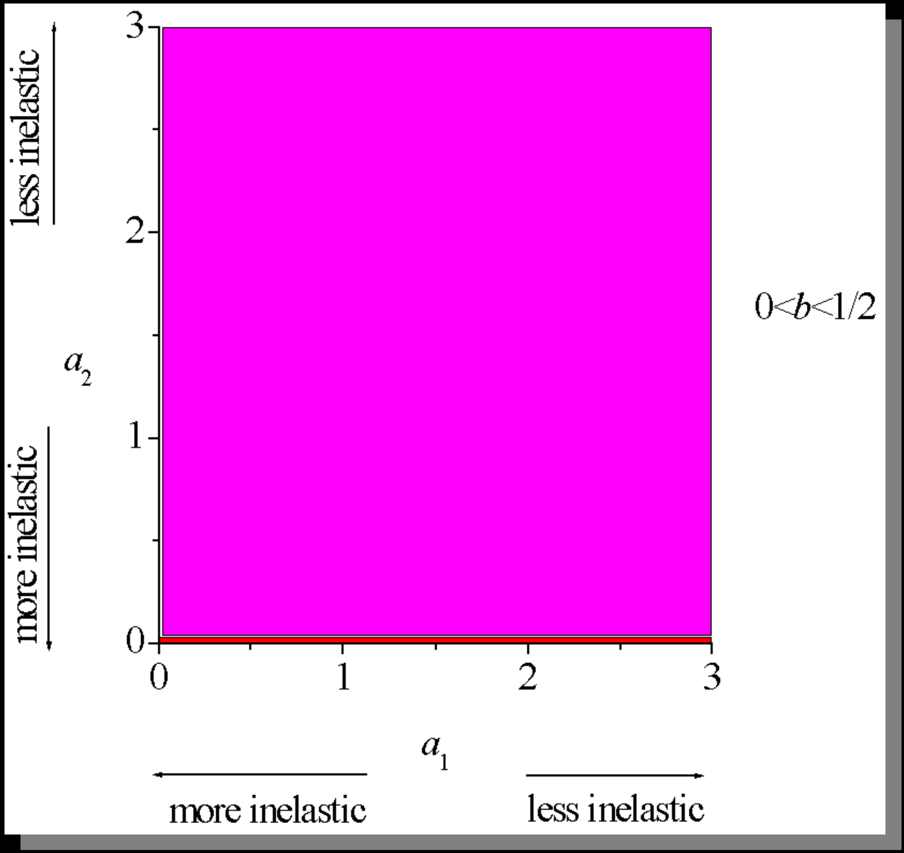
Sub-normal	
Normal	
Sub-ordered	
Ordered	
Super-ordered	



Phase diagram (finite concentrations)

Weakly quasi-elastic cross collisions

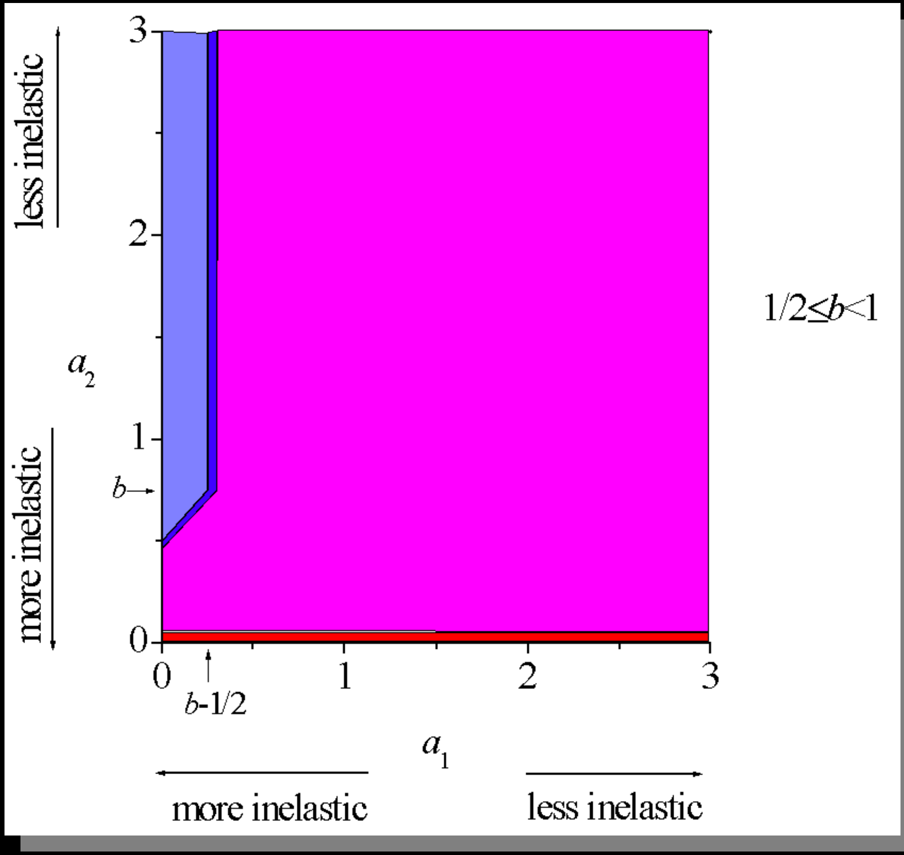
Sub-normal	
Normal	
Sub-ordered	
Ordered	
Super-ordered	



Phase diagram (finite concentrations)

Quasi-elastic cross collisions

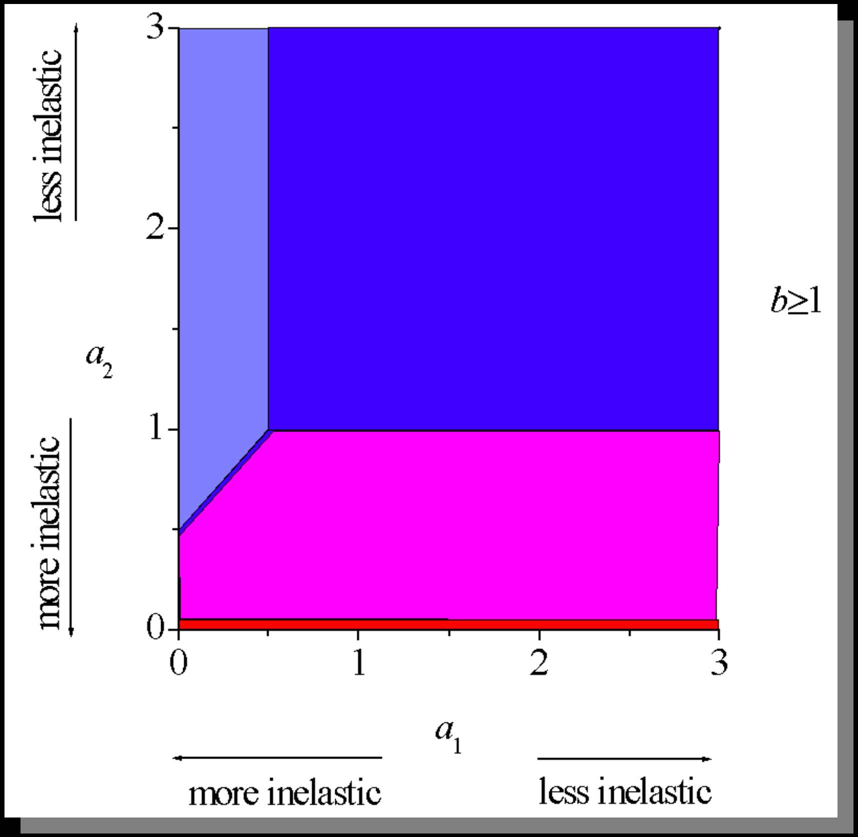
Sub-normal	
Normal	
Sub-ordered	
Ordered	
Super-ordered	



Phase diagram (finite concentrations)

Strongly quasi-elastic cross collisions

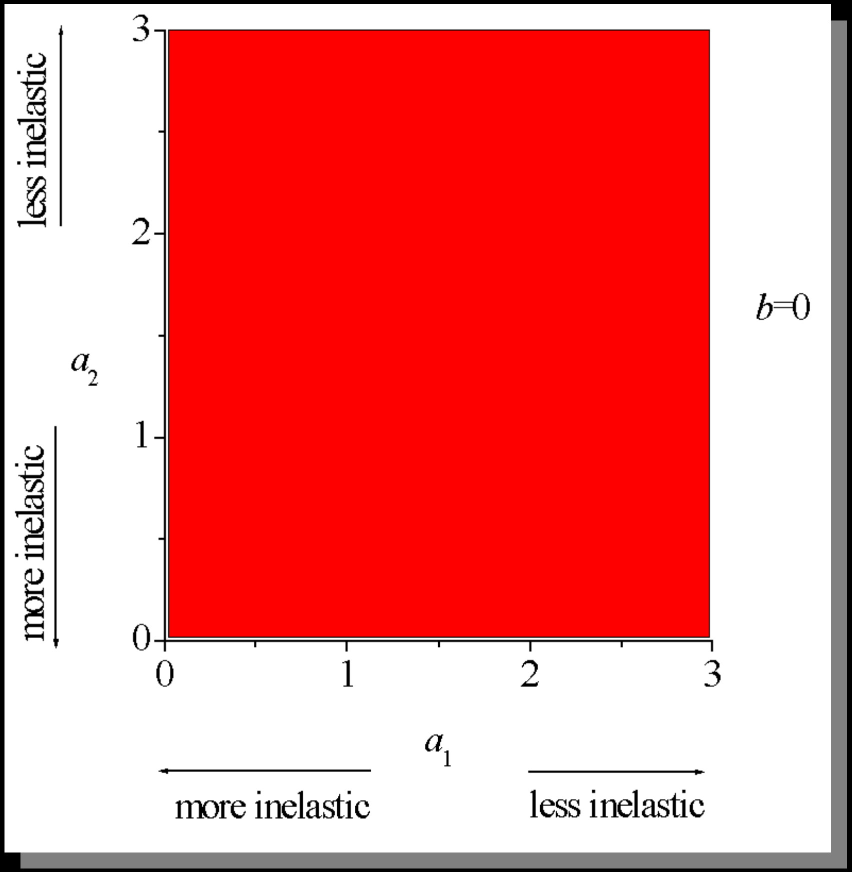
Sub-normal	
Normal	
Sub-ordered	
Ordered	
Super-ordered	



Phase diagram (finite concentrations)

Inelastic cross collisions

Sub-normal	
Normal	
Sub-ordered	
Ordered	
Super-ordered	



Phase diagram (Brownian limit)

Inelastic cross collisions

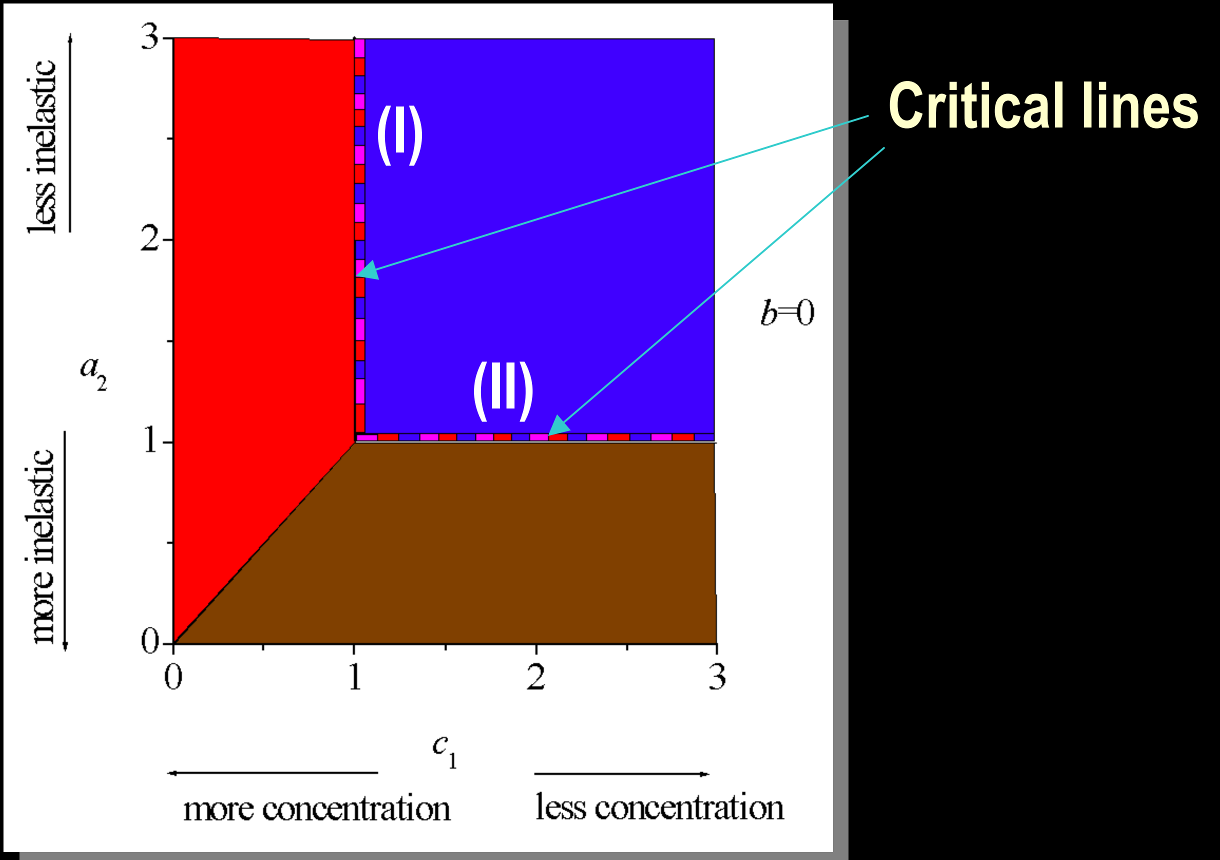
$$1-\alpha_{12} = \mathcal{O}(1)$$

$$1-\alpha_{11} \sim h_2^{a_1}$$

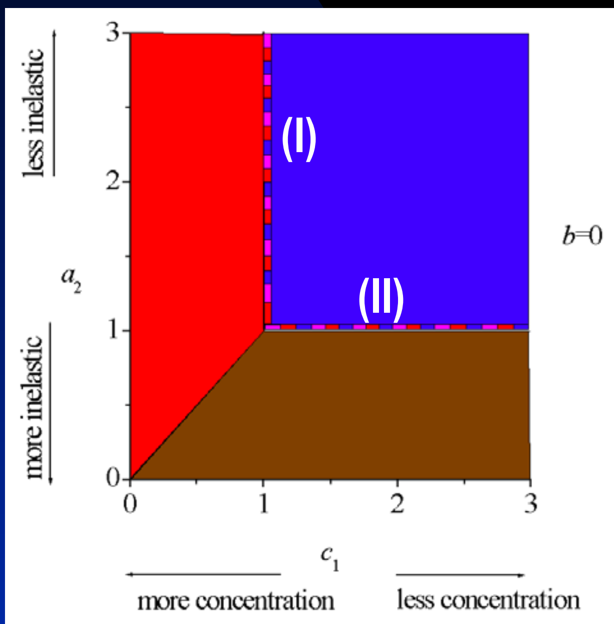
$$(1-\alpha_{22})(\sigma_2/\sigma_1)^2 \sim h_2^{a_2}$$

$$x_1 \sim h_2^{c_1}$$

Sub-normal	
Normal	
Sub-ordered	
Ordered	
Super-ordered	



Critical lines (Brownian limit)



(I) $x_1 \sim h_2$, b$_2$ ö 0

$$\frac{x_1}{h_2}\,\frac{1-\alpha_{12}^2}{4}\begin{cases} <1 \Rightarrow & \phi \sim h_2 \\ =1 \Rightarrow & \phi \sim h_2^{1/2} \\ >1 \Rightarrow & \phi \sim 1 \end{cases}$$

normal

sub-ordered

ordered

(II) 1-a$_{22}$ $\sim h_2$, x_1/h_2 ö 0

$$\beta_2 \begin{cases} <1 \Rightarrow & \phi \sim h_2 \\ =1 \Rightarrow & \phi \sim h_2^{1/2} \\ >1 \Rightarrow & \phi \sim 1 \end{cases}$$

normal

sub-ordered

ordered

Case (II) $1\text{-}a_{22} \sim h_2$, x_1/h_2 ö 0

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Critical Behavior of a Heavy Particle in a Granular Fluid

Andrés Santos* and James W. Dufty†

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Nonequilibrium phase transition for a heavy particle in a granular fluid

Andrés Santos* and James W. Dufty†

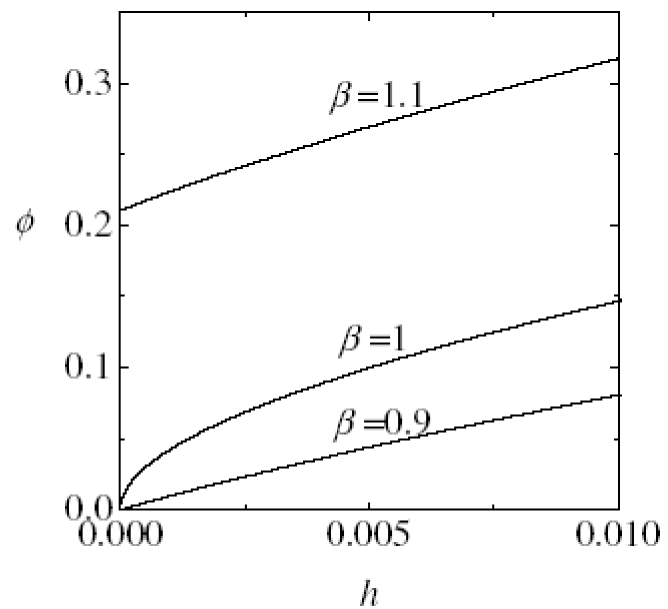

FIG. 1. Ratio of mean square velocities, ϕ, as a function of the mass ratio parameter h for $\beta = 0.9$, 1, and 1.1.

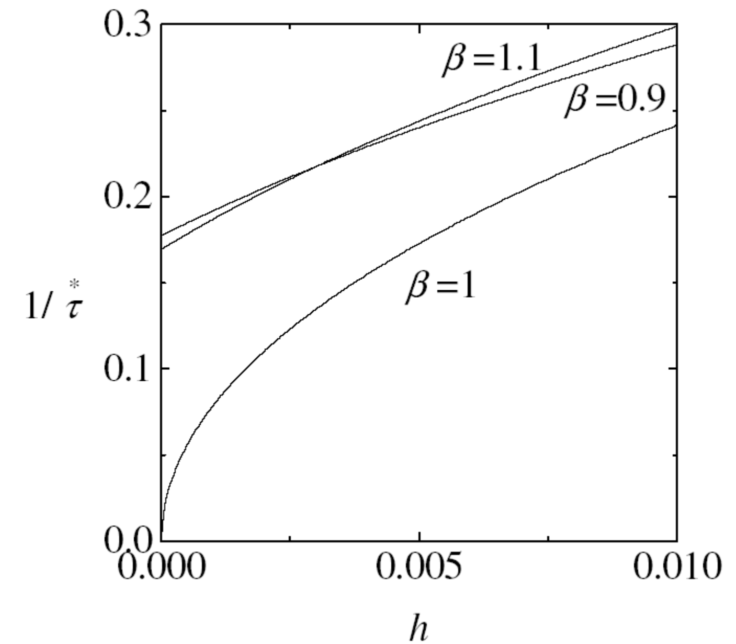

FIG. 3. Inverse characteristic time $\tau^{*-1} \equiv (\nu^* \tau)^{-1}$ as a function of the mass ratio parameter h for $\beta = 0.9$, 1, and 1.1.

Case (II) $1\text{-}a_{22} \sim h_2, x_1/h_2$ ö 0

How good is the Maxwellian approximation?

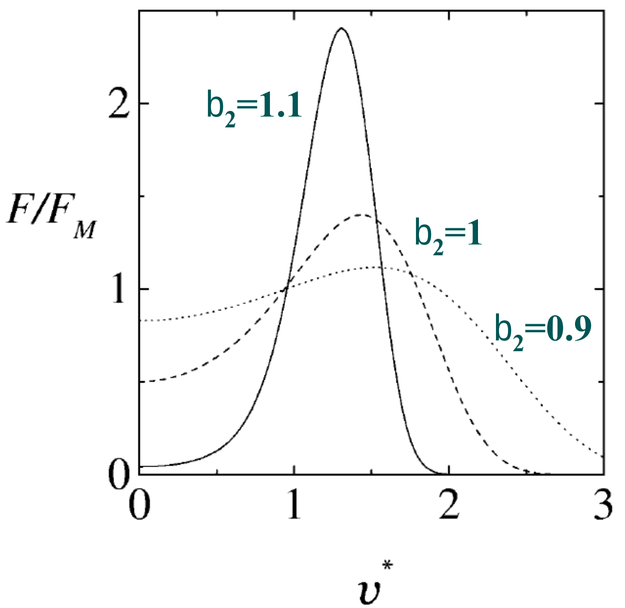

FIG. 4. Velocity distribution function of the impurity particle, F, relative to the Maxwellian, F_M, for $h = 10^{-2}$ and $\beta = 0.9$ (dotted line), $\beta = 1$ (dashed line), and $\beta = 1.1$ (solid line).

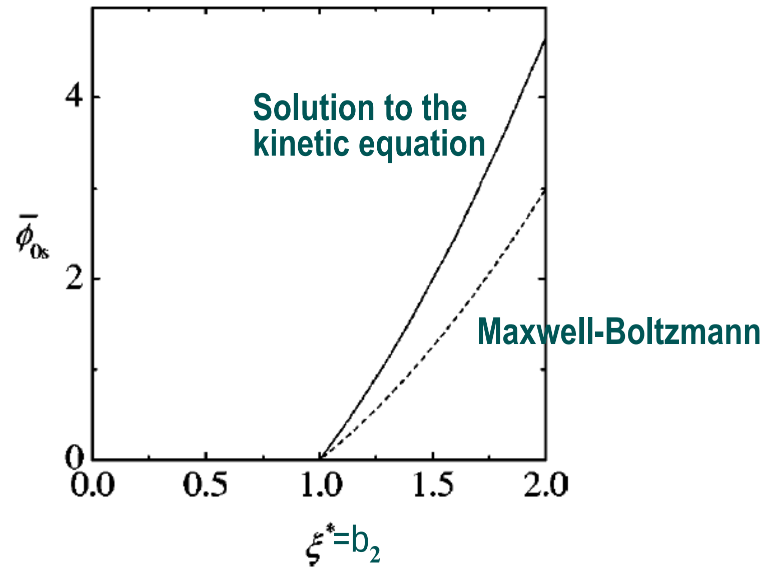

FIG. 9. Plot of the order parameter in the deterministic limit, $\bar{\phi}_{0s}$, as a function of ξ^*. The dashed line is the maximum entropy estimate $\bar{\phi}_s = \xi^{*2} - 1$ of Sec. II.

And if the system is heated?

- **Gaussian thermostat**

$$\partial_t f_i(v) \to \partial_t f_i(v) + \gamma \frac{\partial}{\partial \mathbf{v}} \cdot \mathbf{v} f_i(v)$$

NESS: $z_1 = z_2$ NO CHANGES

- **White noise**

$$\partial_t f_i(v) \to \partial_t f_i(v) - D \frac{\partial^2}{\partial v^2} f_i(v)$$

NESS: $z_1 \; f = z_2$ MINOR CHANGES

Conclusions

- Depending on the control parameters (coefficients of restitution, size ratio, and concentrations), the mean square velocity ratio $\langle v_1^2\rangle/\langle v_2^2\rangle$ (and the temperature ratio T_1/T_2) in a free cooling granular mixture exhibit a rich diversity of scaling behaviors in the disparate-mass limit $m_1/m_2 \to \infty$, ranging from the "sub-normal" state $(T_1/T_2 \to 0)$ to the "super-ordered" state ($\langle v_1^2\rangle/\langle v_2^2\rangle \to \infty$).

- If the cross collisions are **inelastic** ($\alpha_{12}<1$), the state is always "ordered" ($\langle v_1^2\rangle/\langle v_2^2\rangle \sim 1$). As a consequence, in this case there is neither Brownian dynamics (when $x_1 \to 0$) nor Lorentz gas (when $x_2 \to 0$).

Conclusions

- A “normal” state (T_1/T_2~1) is only possible if the three types of collisions are sufficiently quasi-elastic.

- A “super-ordered” state is only possible in the Brownian limit (when x_1 ö 0). There is no “sub-normal” state in that case.

- In the Brownian limit, there exist critical lines in the phase diagram where the state can be normal, ordered or sub-ordered.

- The same scenario as for free cooling mixtures holds essentially in the heated case.

THANKS!