Mixtures of inelastic rough hard spheres

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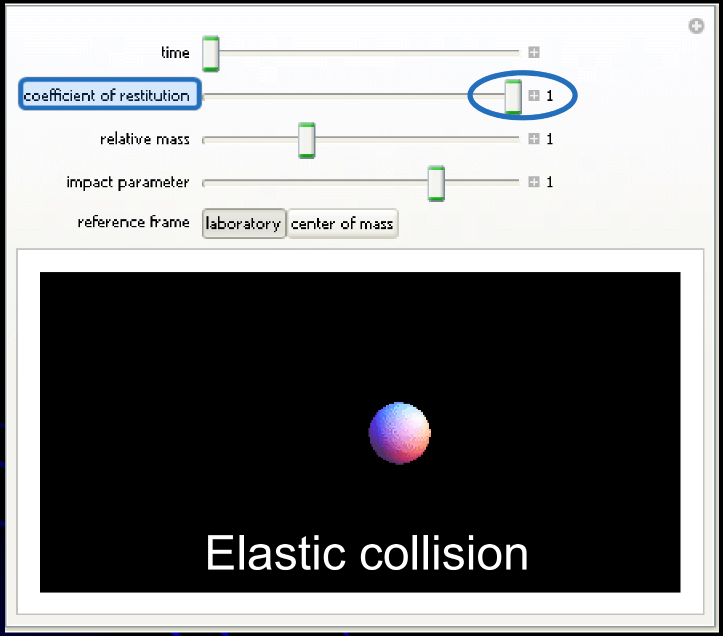
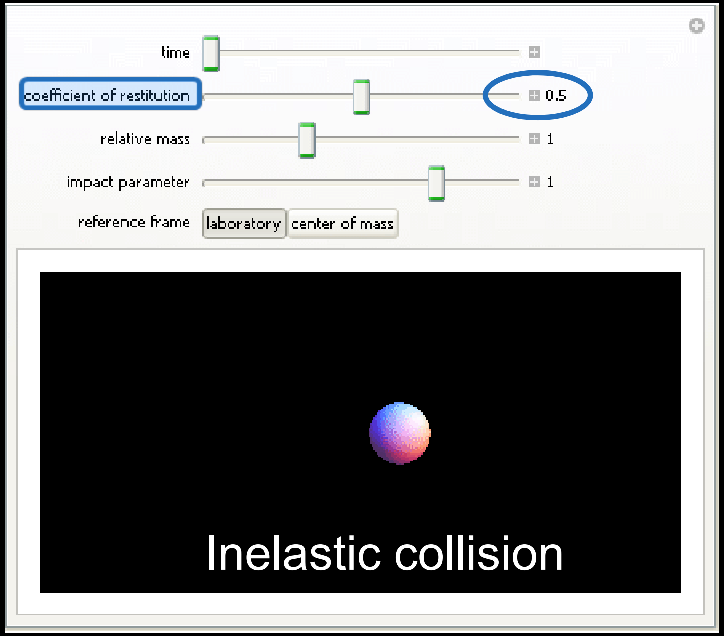
Outline

- Mixture of inelastic rough hard spheres. The Boltzmann equation.
- Collisional thermal rates. Equilibration and cooling rates
- Application to the homogeneous cooling state. Non-equipartition of energy.
- Simple kinetic model for monodisperse systems. Application to the USF.
- Conclusions and outlook.

Minimal model of a granular gas: A gas of *identical smooth inelastic* hard spheres

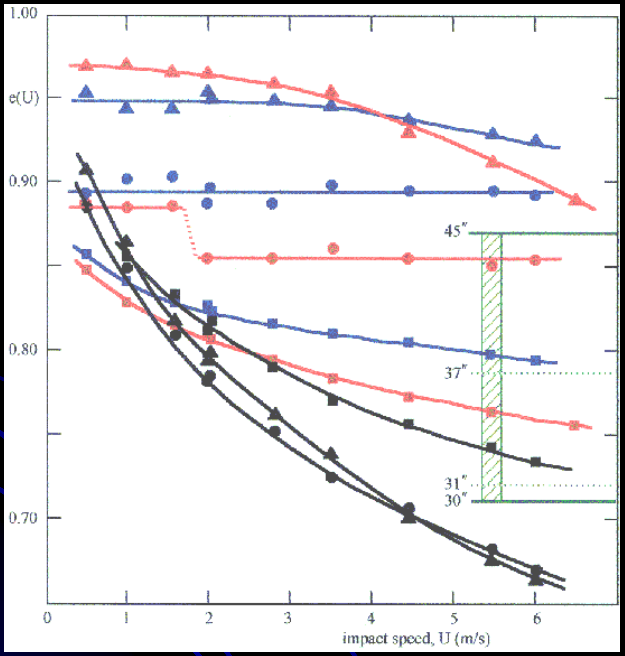


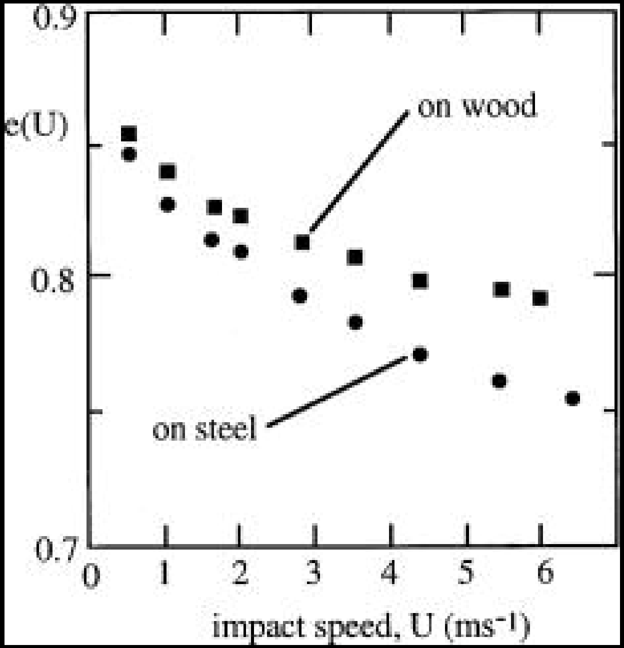


http://demonstrations.wolfram.com/InelasticCollisionsOfTwoSpheres/

But … real grains

Have a non-constant coefficient of restitution

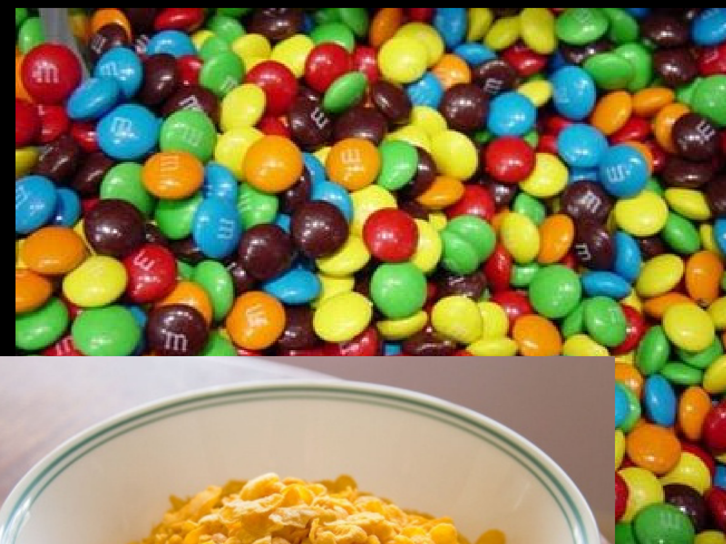




www.oxfordcroquet.com/tech/

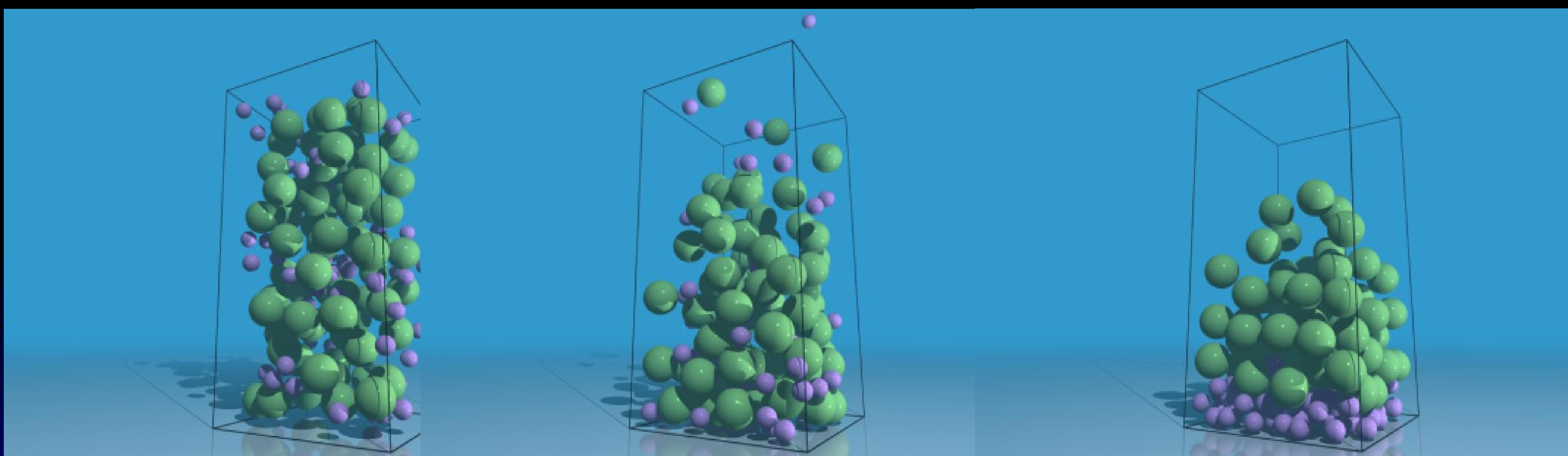
But … real grains

Are non-spherical



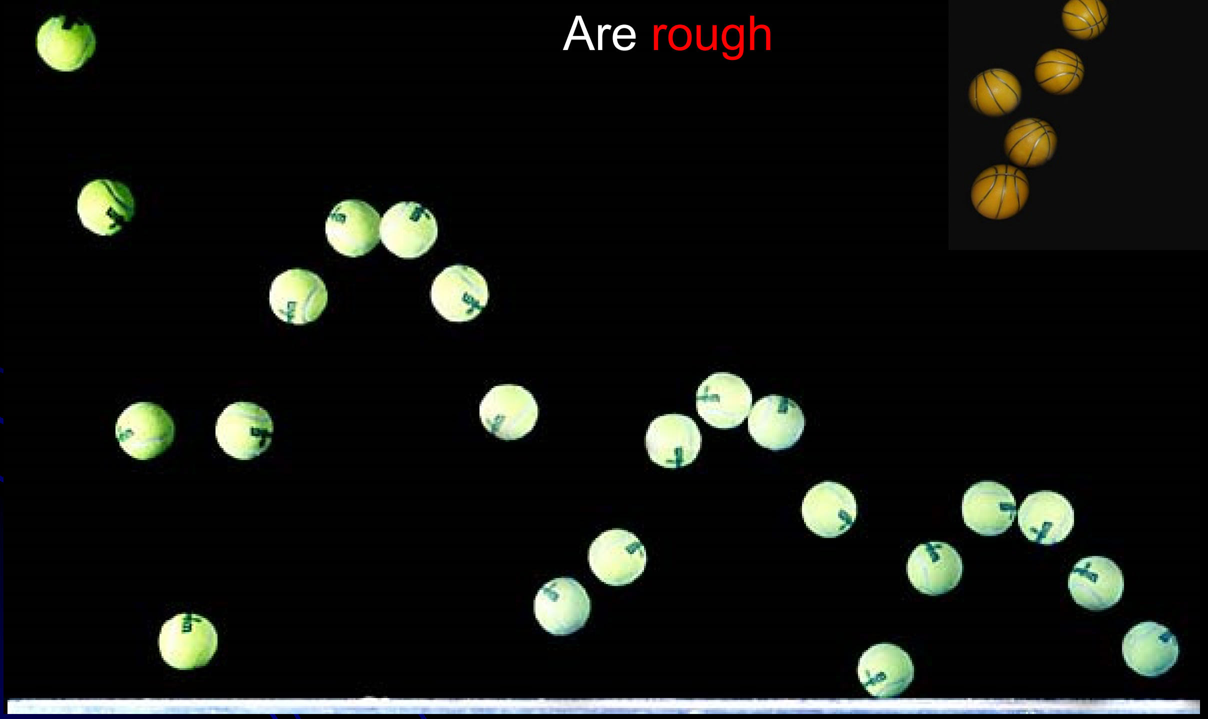
But … real grains

Are polydisperse



http://www.cmt.york.ac.uk/~ajm143/nuts.html

But … real grains



Are rough

Model of a granular gas:
A *mixture* of *inelastic rough* hard spheres

This model unveils an inherent breakdown of energy equipartition in granular fluids, even in homogeneous and isotropic states

Several circles
(Kandinsky, 1926)

Some previous works

- Monodisperse inelastic *rough* spheres
  - ✓ Jenkins & Richman (1985): Shear flow
  - ✓ Lun (1991): Quasi-smooth, shear flow
  - ✓ Goldshtein & Shapiro (1995): Rates of change, HCS
  - ✓ Luding, Huthmann, McNamara & Zippelius (1998): Evolution HCS, MD
  - ✓ Hayakawa, Mitarai & Nakanishi (2002): Micropolar fluid model
  - ✓ Goldhirsch, Noskowicz & Bar-Lev (2005): Quasi-elastic, quasi-smooth, constitutive equations
  - ✓ …

- *Polydisperse* inelastic smooth spheres
  - ✓ Garzó & Dufty (1999): Rates of change, HCS
  - ✓ Barrat & Trizac (2002): WN driving
  - ✓ …

Mechanical parameters:

- X components (i=1, …, X)
- Masses m_i
- Diameters σ_i
- Moments of inertia I_i
- Coefficients of normal restitution α_{ij}
- Coefficients of tangential restitution β_{ij}
- $\alpha_{ij}=1$ for elastic particles
- $\beta_{ij}=-1$ for smooth particles
- $\beta_{ij}=+1$ for totally rough particles

Collision rules:

Translational velocities: $\mathbf{v}_i' = \mathbf{v}_i - \frac{1}{m_i}\mathbf{Q}_{ij}, \quad \mathbf{v}_j' = \mathbf{v}_j + \frac{1}{m_j}\mathbf{Q}_{ij}$

Angular velocities: $\boldsymbol{\omega}_i' = \boldsymbol{\omega}_i + \frac{\sigma_i}{2I_i}\widehat{\boldsymbol{\sigma}} \times \mathbf{Q}_{ij}, \quad \boldsymbol{\omega}_j' = \boldsymbol{\omega}_j + \frac{\sigma_j}{2I_j}\widehat{\boldsymbol{\sigma}} \times \mathbf{Q}_{ij}$

Smooth spheres

Impulse exerted by i on j:

$$\mathbf{Q}_{ij} = \overline{\beta}_{ij}\left[\mathbf{v}_{ij} - (\mathbf{v}_{ij}\cdot\widehat{\boldsymbol{\sigma}})\widehat{\boldsymbol{\sigma}} + \frac{1}{2}\widehat{\boldsymbol{\sigma}} \times (\sigma_i\boldsymbol{\omega}_i + \sigma_j\boldsymbol{\omega}_j)\right] + \overline{\alpha}_{ij}(\mathbf{v}_{ij}\cdot\widehat{\boldsymbol{\sigma}})\widehat{\boldsymbol{\sigma}}$$

$$\mathbf{v}_{ij} \equiv \mathbf{v}_i - \mathbf{v}_j, \quad \overline{\alpha}_{ij} \equiv m_{ij}\,(1+\alpha_{ij})\,, \quad \overline{\beta}_{ij} \equiv \frac{m_{ij}\kappa_{ij}}{1+\kappa_{ij}}\,(1+\beta_{ij})$$

$$m_{ij} \equiv \frac{m_i m_j}{m_i + m_j}, \quad \kappa_{ij} \equiv \kappa_i\kappa_j\frac{m_i + m_j}{\kappa_i m_i + m_j\kappa_j}, \quad \kappa_i \equiv \frac{I_i}{m_i(\sigma_i/2)^2}$$

Energy collisional loss

$$E_{ij} = \frac{1}{2}m_i v_i^2 + \frac{1}{2}m_j v_j^2 + \frac{1}{2}I_i\omega_i^2 + \frac{1}{2}I_j\omega_j^2$$

$$\begin{aligned} E'_{ij} - E_{ij} &= -(1-\alpha_{ij}^2)\times\cdots \\ &\quad -(1-\beta_{ij}^2)\times\cdots \end{aligned}$$

Energy is conserved *only* if the spheres are

- elastic ($\alpha_{ij}=1$) and
- either
    - smooth ($\beta_{ij}=-1$) or
    - perfectly rough ($\beta_{ij}=+1$)

Partial (granular) temperatures

$$\text{Translational temperatures: } T_i^{\text{tr}} = \frac{m_i}{3}\langle v_i^2\rangle$$

$$\text{Rotational temperatures: } T_i^{\text{rot}} = \frac{I_i}{3}\langle \omega_i^2\rangle = \frac{m_i\kappa_i}{12}\sigma_i^2\langle \omega_i^2\rangle$$

$$\text{Total temperature: } T = \sum_i \frac{n_i}{2n}\left(T_i^{\text{tr}} + T_i^{\text{rot}}\right)$$

Collisional rates of change for temperatures

Thermal rates:

$$\xi_i^{\mathrm{tr}} = -\frac{1}{T_i^{\mathrm{tr}}}\left(\frac{\partial T_i^{\mathrm{tr}}}{\partial t}\right)_{\mathrm{coll}}, \quad \xi_i^{\mathrm{tr}} = \sum_j \xi_{ij}^{\mathrm{tr}}$$

$$\xi_i^{\mathrm{rot}} = -\frac{1}{T_i^{\mathrm{rot}}}\left(\frac{\partial T_i^{\mathrm{rot}}}{\partial t}\right)_{\mathrm{coll}}, \quad \xi_i^{\mathrm{rot}} = \sum_j \xi_{ij}^{\mathrm{rot}}$$

Net *cooling* rate:

$$\zeta = -\frac{1}{T}\left(\frac{\partial T}{\partial t}\right)_{\mathrm{coll}}, \quad \zeta = \sum_i \frac{n_i}{2nT}\left(\xi_i^{\mathrm{tr}} T_i^{\mathrm{tr}} + \xi_i^{\mathrm{rot}} T_i^{\mathrm{rot}}\right)$$

Our main goal

To obtain the binary thermal rates

ξ_{ij}^{tr} and ξ_{ij}^{rot}

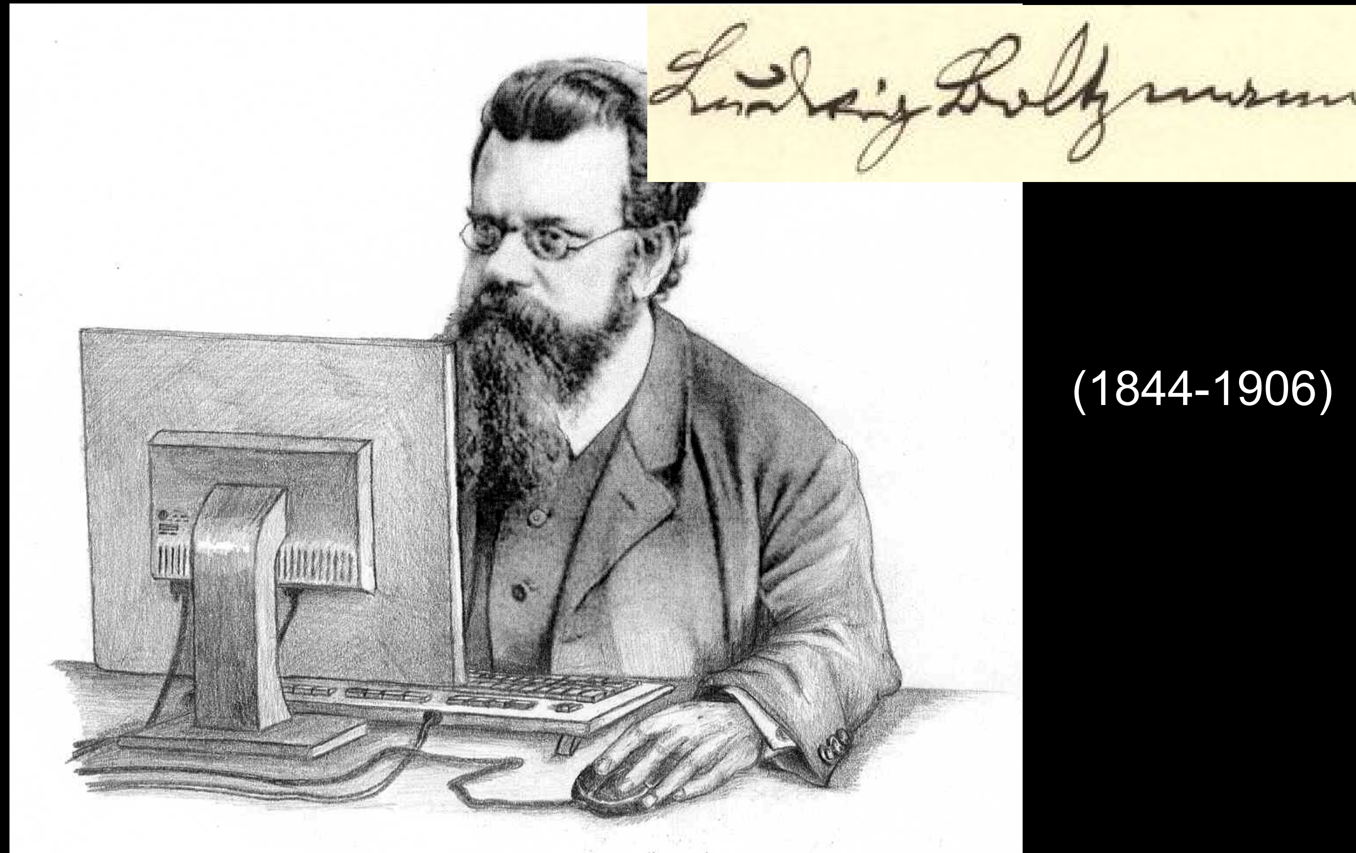
in terms of

$T_i^{\mathrm{tr}}, T_j^{\mathrm{tr}}, T_i^{\mathrm{rot}}, T_j^{\mathrm{rot}}, n_i, n_j$

and the mechanical parameters

$m_i, m_j, \sigma_i, \sigma_j, \kappa_i, \kappa_j, \alpha_{ij}, \beta_{ij}$

(Cartoon by Bernhard Reischl, University of Vienna)



(1844-1906)

Boltzmann equation:

$$\partial_t f_i(\mathbf{r},\mathbf{v}_i,\boldsymbol{\omega}_i,t) + \mathbf{v}_i\cdot\nabla f_i(\mathbf{r},\mathbf{v}_i,\boldsymbol{\omega}_i,t) = \sum_j \underbrace{J_{ij}[\mathbf{r},\mathbf{v}_i,\boldsymbol{\omega}_i,t|f_i,f_j]}$$

Binary collisions

"Exact" results

$$\xi_{ij}^{\mathrm{tr}} = \frac{n_j \sigma_{ij}^2 \pi}{3T_i^{\mathrm{tr}}} \left[\left(\overline{\alpha}_{ij} + \overline{\beta}_{ij}\right) \langle v_{ij} \mathbf{v}_i \cdot \mathbf{v}_{ij} \rangle + \frac{2\overline{\beta}_{ij}}{3} \langle (\sigma_i \boldsymbol{\omega}_i + \sigma_j \boldsymbol{\omega}_j) \cdot (\mathbf{v}_i \times \mathbf{v}_j) \rangle \right.$$

$$\left. - \frac{\overline{\alpha}_{ij}^2 + \overline{\beta}_{ij}^2}{2m_i} \langle v_{ij}^3 \rangle - \frac{\overline{\beta}_{ij}^2}{16 m_i} \langle v_{ij}^{-1} \left[\mathbf{v}_{ij} \cdot (\sigma_i \boldsymbol{\omega}_i + \sigma_j \boldsymbol{\omega}_j)\right]^2 \rangle - \frac{3\overline{\beta}_{ij}^2}{16 m_i} \langle v_{ij} \left(\sigma_i \boldsymbol{\omega}_i + \sigma_j \boldsymbol{\omega}_j\right)^2 \rangle \right]$$

$$\xi_{ij}^{\mathrm{rot}} = \frac{n_j \sigma_{ij}^2 \pi}{24 T_i^{\mathrm{rot}}} \overline{\beta}_{ij} \left\{ 3\sigma_i \langle v_{ij} \left[\boldsymbol{\omega}_i \cdot (\sigma_i \boldsymbol{\omega}_i + \sigma_j \boldsymbol{\omega}_j)\right] \rangle - \sigma_i \langle v_{ij}^{-1} \left[\mathbf{v}_{ij} \cdot (\sigma_i \boldsymbol{\omega}_i + \sigma_j \boldsymbol{\omega}_j)\right] (\mathbf{v}_{ij} \cdot \boldsymbol{\omega}_i) \rangle \right.$$

$$\left. - \frac{\overline{\beta}_{ij}}{m_i \kappa_i} \left[4 \langle v_{ij}^3 \rangle + \frac{3}{2} \langle v_{ij} \left(\sigma_i \boldsymbol{\omega}_i + \sigma_j \boldsymbol{\omega}_j\right)^2 \rangle - \frac{1}{2} \langle v_{ij}^{-1} \left[\mathbf{v}_{ij} \cdot (\sigma_i \boldsymbol{\omega}_i + \sigma_j \boldsymbol{\omega}_j)\right]^2 \rangle \right] \right\}$$

Additional assumptions

1. No mutual diffusion, no chirality:

$$\langle \mathbf{v}_i \rangle = \langle \mathbf{v}_j \rangle, \quad \langle \boldsymbol{\omega}_i \rangle = \langle \boldsymbol{\omega}_j \rangle = \mathbf{0}$$

2. Translational and rotational degrees of freedom uncorrelated:

$$f_i(\mathbf{v}_i, \boldsymbol{\omega}_i) = f_i^{\mathrm{tr}}(\mathbf{v}_i) f_i^{\mathrm{rot}}(\boldsymbol{\omega}_i)$$

3. Maxwellian form:

$$f_i^{\mathrm{tr}}(\mathbf{v}_i) = n_i \left(\frac{m_i}{2\pi T_i^{\mathrm{tr}}} \right)^{3/2} \exp\left(-\frac{m_i v_i^2}{2T_i^{\mathrm{tr}}} \right)$$

Results

$$\xi_{ij}^{\rm tr} = \frac{\nu_{ij}}{m_i T_i^{\rm tr}}\Bigg[2\left(\overline{\alpha}_{ij}+\overline{\beta}_{ij}\right)T_i^{\rm tr} - \left(\overline{\alpha}_{ij}^2+\overline{\beta}_{ij}^2\right)\left(\frac{T_i^{\rm tr}}{m_i}+\frac{T_j^{\rm tr}}{m_j}\right)$$
$$-\overline{\beta}_{ij}^2\left(\frac{T_i^{\rm rot}}{m_i\kappa_i}+\frac{T_j^{\rm rot}}{m_j\kappa_j}\right)\Bigg]$$

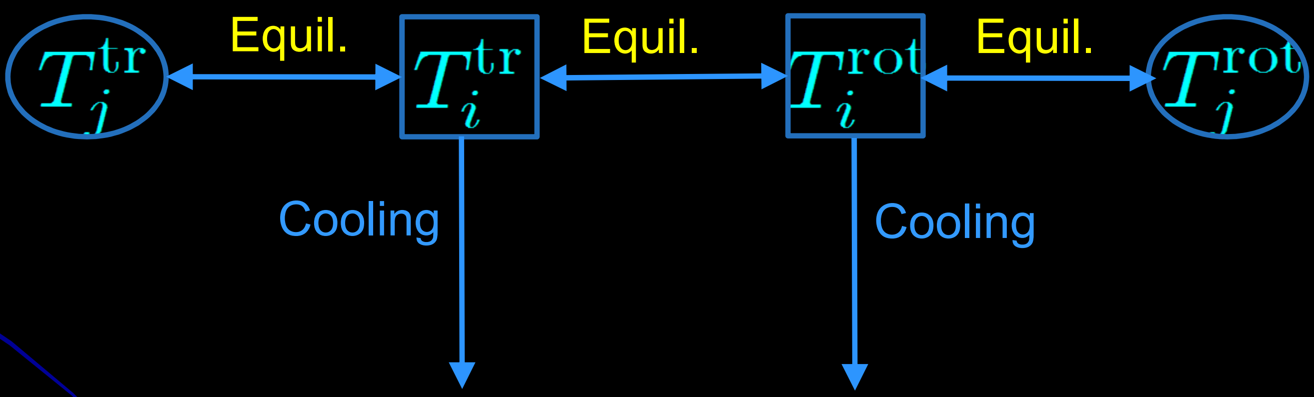
$$\xi_{ij}^{\rm rot} = \frac{\nu_{ij}}{m_i\kappa_i T_i^{\rm rot}}\overline{\beta}_{ij}\left[2T_i^{\rm rot} - \overline{\beta}_{ij}\left(\frac{T_i^{\rm tr}}{m_i}+\frac{T_j^{\rm tr}}{m_j}+\frac{T_i^{\rm rot}}{m_i\kappa_i}+\frac{T_j^{\rm rot}}{m_j\kappa_j}\right)\right]$$

$$\nu_{ij} \equiv \frac{4\sqrt{2\pi}}{3} n_j \sigma_{ij}^2 \sqrt{\frac{T_i^{\rm tr}}{m_i}+\frac{T_j^{\rm tr}}{m_j}}$$

Decomposition

Thermal rates = Equilibration rates + Cooling rates

Net cooling rate = Σ Cooling rates



Decomposition

$$\xi_{ij}^{\mathrm{tr}} = \xi_{ij}^{\mathrm{tr},\alpha} + \xi_{ij}^{\mathrm{tr},\beta} + \zeta_{ij}^{\mathrm{tr}} + \kappa_i \frac{T_i^{\mathrm{rot}}}{T_i^{\mathrm{tr}}} \xi_{ij}^{\mathrm{rot}}$$

$$\xi_{ij}^{\mathrm{rot}} = \xi_{ij}^{\mathrm{rot},\beta} + \zeta_{ij}^{\mathrm{rot}}$$

Thermal rates

$$\xi_{ij}^{\mathrm{tr},\alpha} \propto (1+\alpha_{ij})(T_i^{\mathrm{tr}} - T_j^{\mathrm{tr}})$$

$$\xi_{ij}^{\mathrm{tr},\beta} \propto (1+\beta_{ij})(T_i^{\mathrm{tr}} - T_i^{\mathrm{rot}})$$

$$\xi_{ij}^{\mathrm{rot},\beta} \propto (1+\beta_{ij}) \left\{T_i^{\mathrm{rot}} - T_j^{\mathrm{rot}}, T_i^{\mathrm{tr}} - T_j^{\mathrm{tr}}, T_i^{\mathrm{tr}} - T_i^{\mathrm{rot}}\right\}$$

Equilibration rates

$$\zeta_{ij}^{\mathrm{tr}} \propto (1-\alpha_{ij}^2)$$

$$\zeta_{ij}^{\mathrm{rot}} \propto (1-\beta_{ij}^2)$$

Cooling rates

Net cooling rate

$$\zeta = \sum_i \frac{n_i}{2nT}\left(\xi_i^{\rm tr} T_i^{\rm tr} + \xi_i^{\rm rot} T_i^{\rm rot}\right)$$

$$\zeta = \sum_{ij} \frac{n_i \nu_{ij}}{4nT} \frac{m_i m_j}{m_i + m_j}\left[(1-\alpha_{ij}^2)\left(\frac{T_i^{\rm tr}}{m_i} + \frac{T_j^{\rm tr}}{m_j}\right) + \frac{\kappa_{ij}}{1+\kappa_{ij}}(1-\beta_{ij}^2)\left(\frac{T_i^{\rm tr}}{m_i} + \frac{T_j^{\rm tr}}{m_j} + \frac{T_i^{\rm rot}}{m_i \kappa_i} + \frac{T_j^{\rm rot}}{m_j \kappa_j}\right)\right]$$

Simple application: The Homogeneous Cooling State (HCS)

$$\partial_t f_i(\mathbf{v}_i, \boldsymbol{\omega}_i, t) = \sum_j J_{ij}[\mathbf{v}_i, \boldsymbol{\omega}_i, t | f_i, f_j]$$

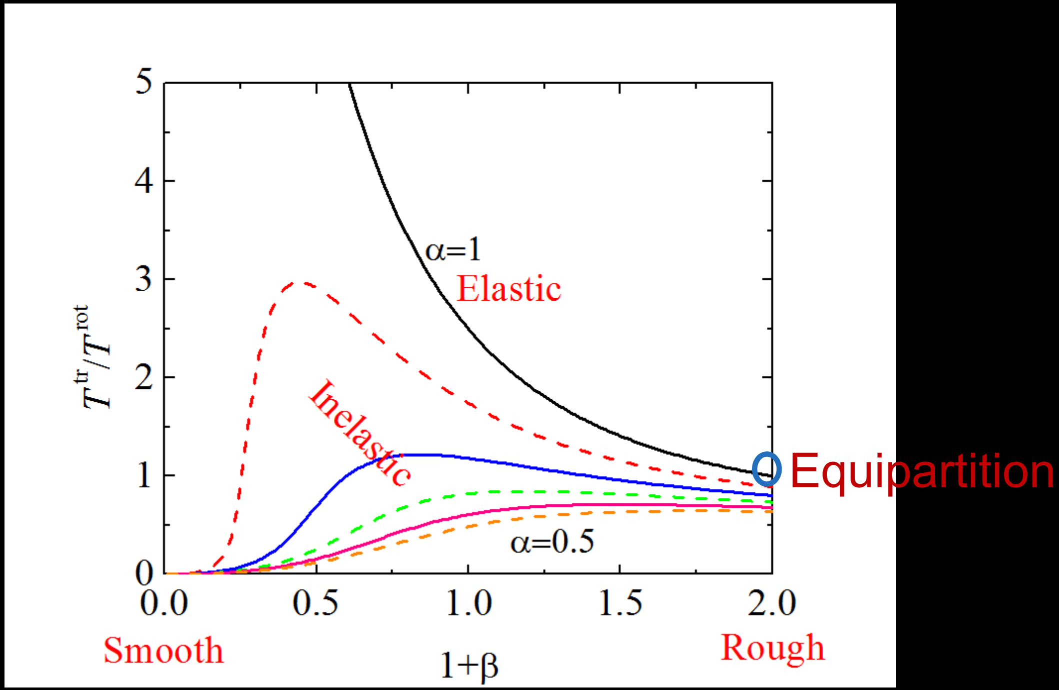
The HCS is
- Spatially homogeneous
- Isotropic
- Undriven
- Freely cooling

$$\frac{\partial T}{\partial t} = -\zeta T$$

$$\frac{\partial}{\partial t} \frac{T_i^{\mathrm{tr}}}{T} = -\left(\xi_i^{\mathrm{tr}} - \zeta\right) \frac{T_i^{\mathrm{tr}}}{T}, \quad \frac{\partial}{\partial t} \frac{T_i^{\mathrm{rot}}}{T} = -\left(\xi_i^{\mathrm{rot}} - \zeta\right) \frac{T_i^{\mathrm{rot}}}{T}$$

$$t \to \infty \Rightarrow \xi_1^{\mathrm{tr}} = \xi_2^{\mathrm{tr}} = \cdots = \xi_1^{\mathrm{rot}} = \xi_2^{\mathrm{rot}} = \cdots$$

Single-component case (κ=2/5)



$$\left.\begin{array}{l}\alpha < 1\\ \beta \to -1\end{array}\right\} \Rightarrow \left\{\begin{array}{l}\xi^{\rm tr} \sim (1-\alpha^2) \Rightarrow \partial_t T^{\rm tr} < 0\\ \xi^{\rm rot} \to 0 \Rightarrow T^{\rm rot} \to {\rm const}\end{array}\right\} \Rightarrow \boxed{\frac{T^{\rm tr}}{T^{\rm rot}} \to 0}$$

$$\left.\begin{array}{l}\alpha = 1\\ \beta \to -1\end{array}\right\} \Rightarrow \left\{\begin{array}{l}\xi^{\rm tr} \sim \kappa(1+\beta) \Rightarrow \partial_t T^{\rm tr} < 0\\ \xi^{\rm rot} \sim (1+\beta) \Rightarrow \partial_t T^{\rm rot} < 0\end{array}\right\} \Rightarrow \xi^{\rm tr} < \xi^{\rm rot} \Rightarrow \boxed{\frac{T^{\rm tr}}{T^{\rm rot}} \to \infty}$$

Binary mixture

Three independent temperature ratios: $\frac{T_1^{\rm tr}}{T_1^{\rm rot}}, \frac{T_2^{\rm tr}}{T_1^{\rm tr}}, \frac{T_2^{\rm rot}}{T_1^{\rm rot}}$

Eleven parameters:

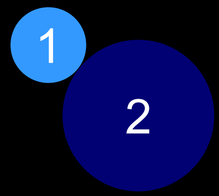
- Coefficients of normal restitution α_{11}, α_{12}, α_{22}
- Coefficients of tangential restitution β_{11}, β_{12}, β_{22}
- Inertia-moment parameters κ_1, κ_2
- Size ratio σ_2/σ_1
- Mass ratio m_2/m_1
- Mole fraction $n_1/(n_1+n_2)$

Binary mixture

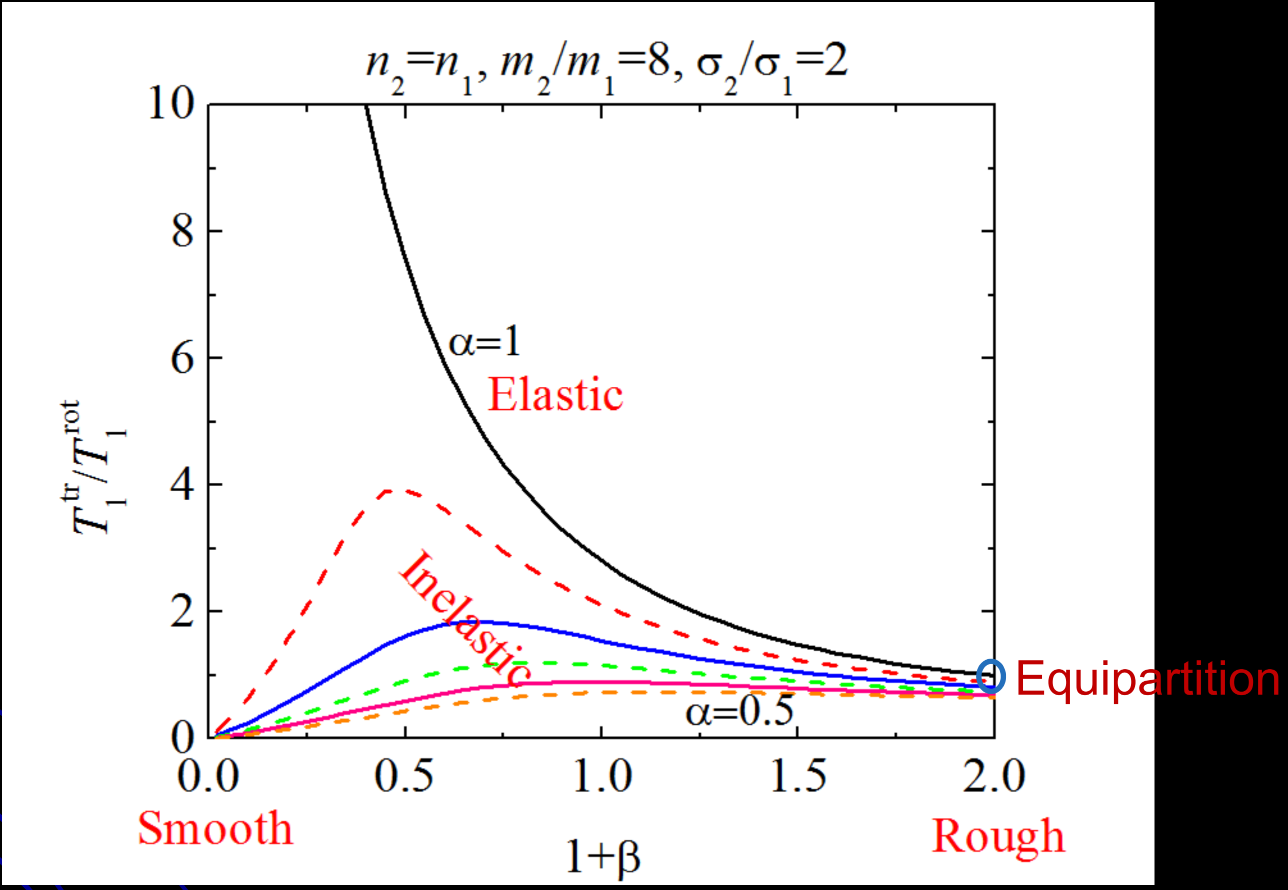
Three independent temperature ratios: $\frac{T_1^{\mathrm{tr}}}{T_1^{\mathrm{rot}}}, \frac{T_2^{\mathrm{tr}}}{T_1^{\mathrm{tr}}}, \frac{T_2^{\mathrm{rot}}}{T_1^{\mathrm{rot}}}$

Eleven parameters:

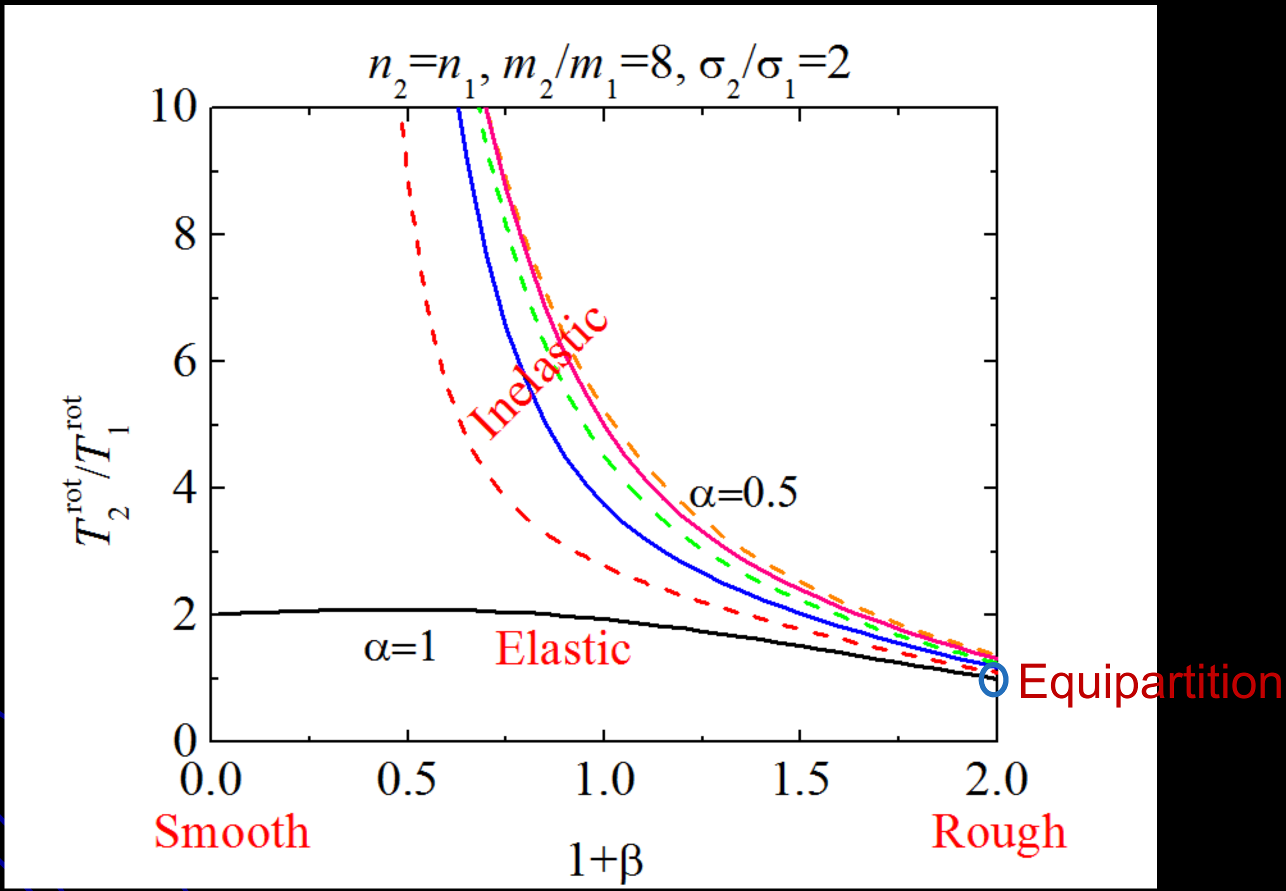
- Coefficients of normal restitution $\alpha_{11} = \alpha_{12} = \alpha_{22} = \alpha$
- Coefficients of tangential restitution $\beta_{11} = \beta_{12} = \beta_{22} = \beta$
- Inertia-moment parameters $\kappa_1 = \kappa_2 = \frac{2}{5}$
- Size ratio $\sigma_2/\sigma_1 = 2$
- Mass ratio $m_2/m_1 = 8$
- Mole fraction $n_1/(n_1 + n_2) = \frac{1}{2}$



Translational/Rotational

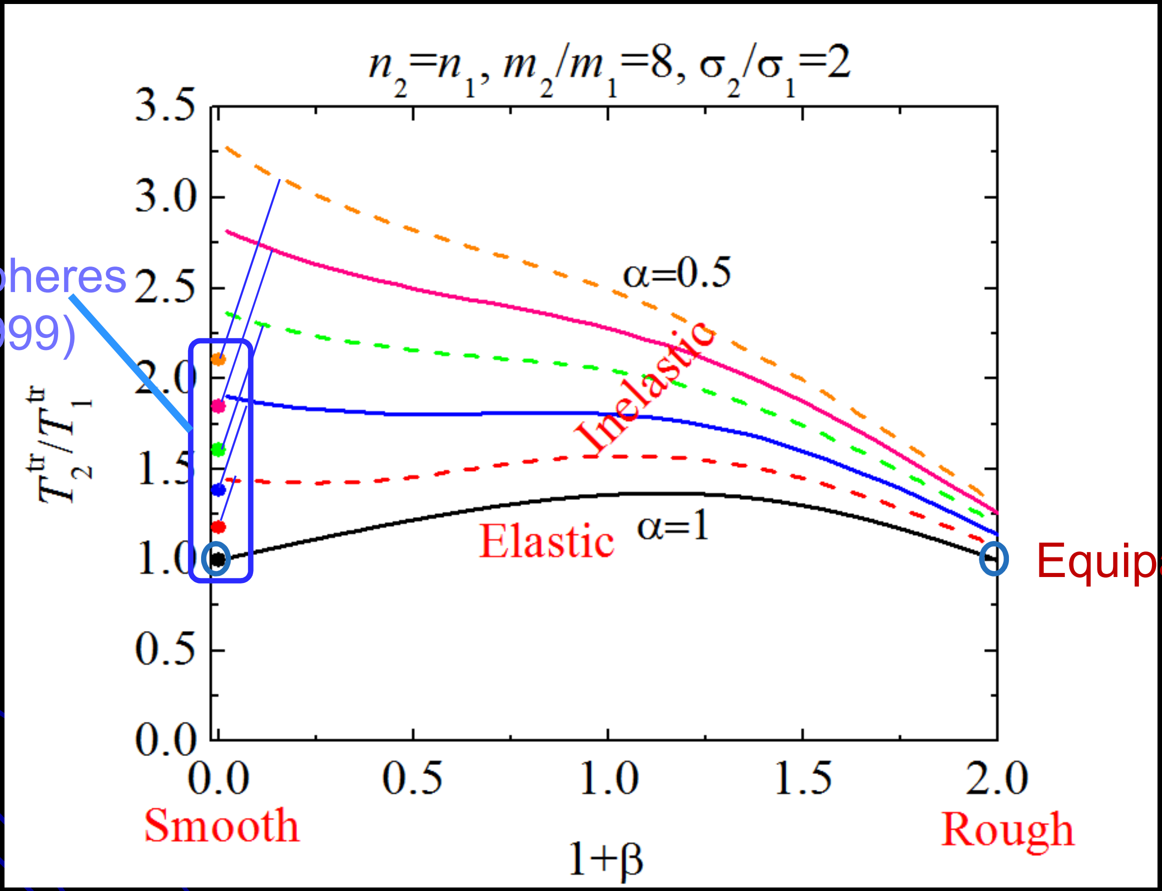


Rotational/Rotational



Translational/Translational

“Pure” smooth spheres
(Garzó&Dufty, 1999)



Equipartition

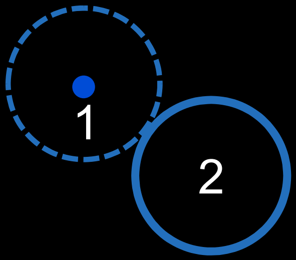
“Ghost” effect: A tiny amount of roughness has dramatic effects on the temperature ratio (enhancement of non-equipartition)

Binary mixture

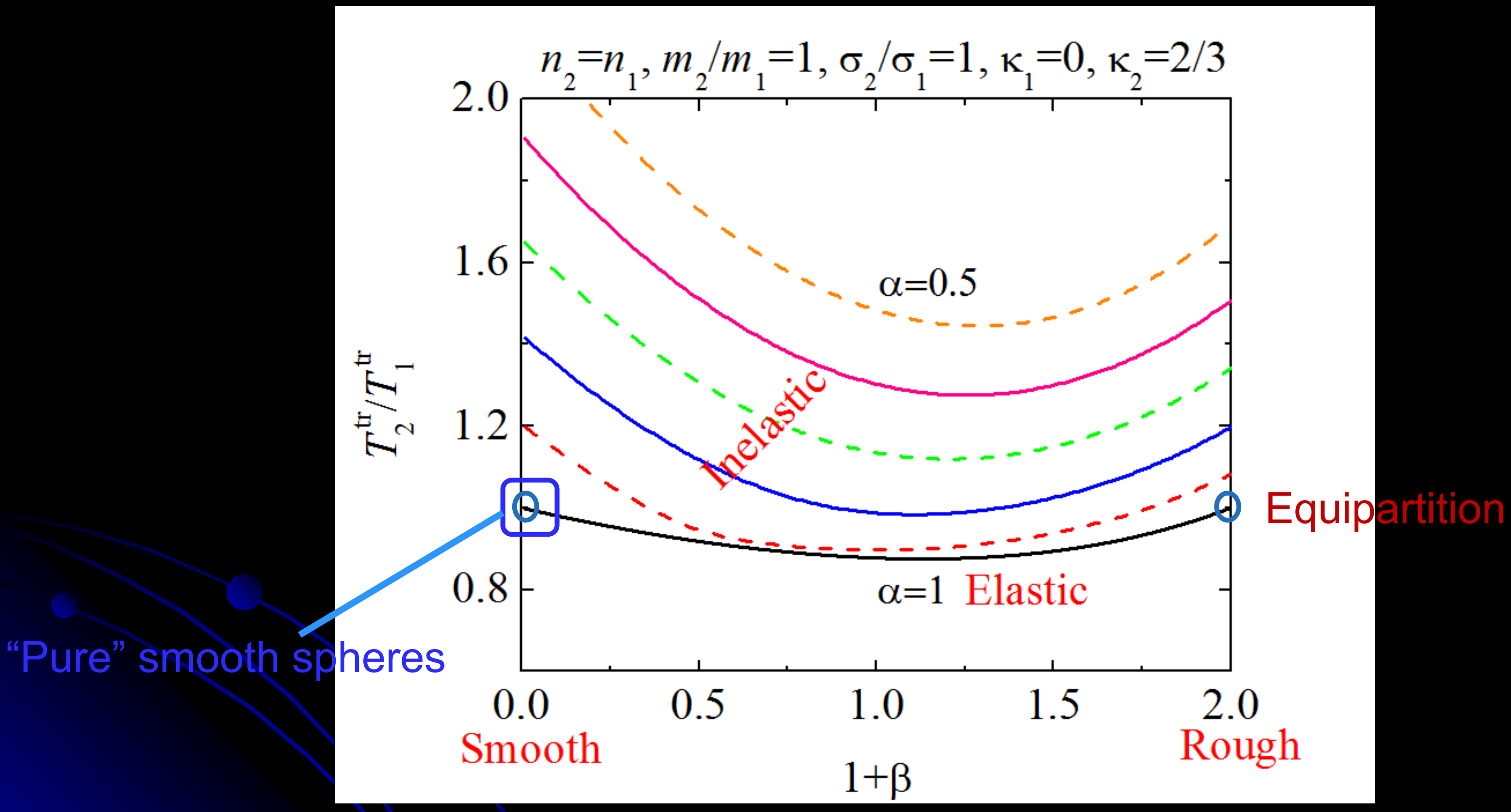
Three independent temperature ratios: $\frac{T_1^{\rm tr}}{T_1^{\rm rot}}, \frac{T_2^{\rm tr}}{T_1^{\rm tr}}, \frac{T_2^{\rm rot}}{T_1^{\rm rot}}$

Eleven parameters:

- Coefficients of normal restitution $\alpha_{11} = \alpha_{12} = \alpha_{22} = \alpha$
- Coefficients of tangential restitution $\beta_{11} = \beta_{12} = \beta_{22} = \beta$
- Inertia-moment parameters $\kappa_1 = 0$, $\kappa_2 = \frac{2}{3}$
- Size ratio $\sigma_2/\sigma_1 = 1$
- Mass ratio $m_2/m_1 = 1$
- Mole fraction $n_1/(n_1 + n_2) = \frac{1}{2}$



Translational/Translational



"Ghost" effect: A tiny amount of roughness has dramatic effects on the temperature ratio

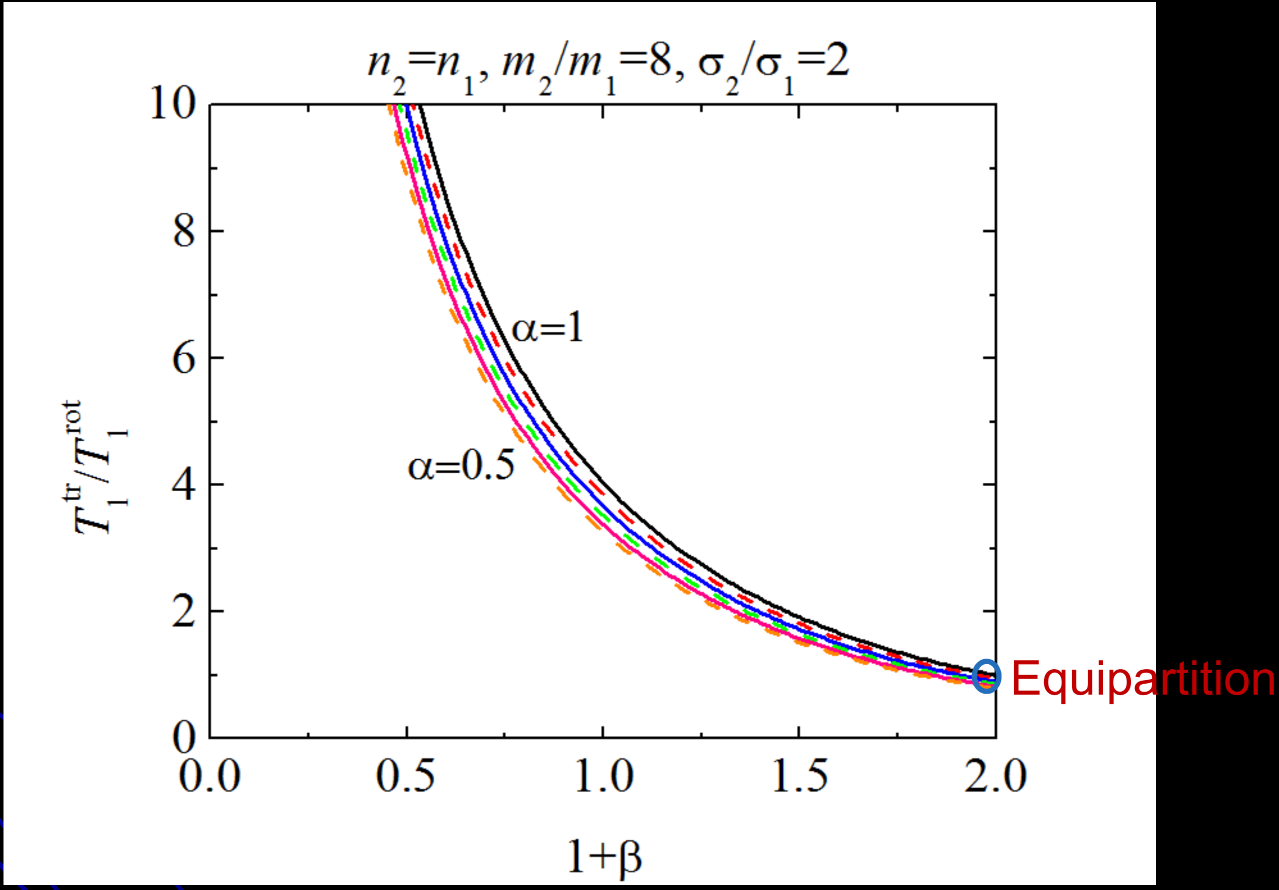
Simple application:
White-noise heating (steady state)

$$-\frac{\chi_0^2}{2}\left(\frac{\partial}{\partial \mathbf{v}_i}\right)^2 f_i(\mathbf{v}_i,\boldsymbol{\omega}_i) = \sum_j J_{ij}[\mathbf{v}_i,\boldsymbol{\omega}_i,t|f_i,f_j]$$

$$T_1^{\mathrm{tr}}\xi_1^{\mathrm{tr}} = T_2^{\mathrm{tr}}\xi_2^{\mathrm{tr}} = \cdots$$

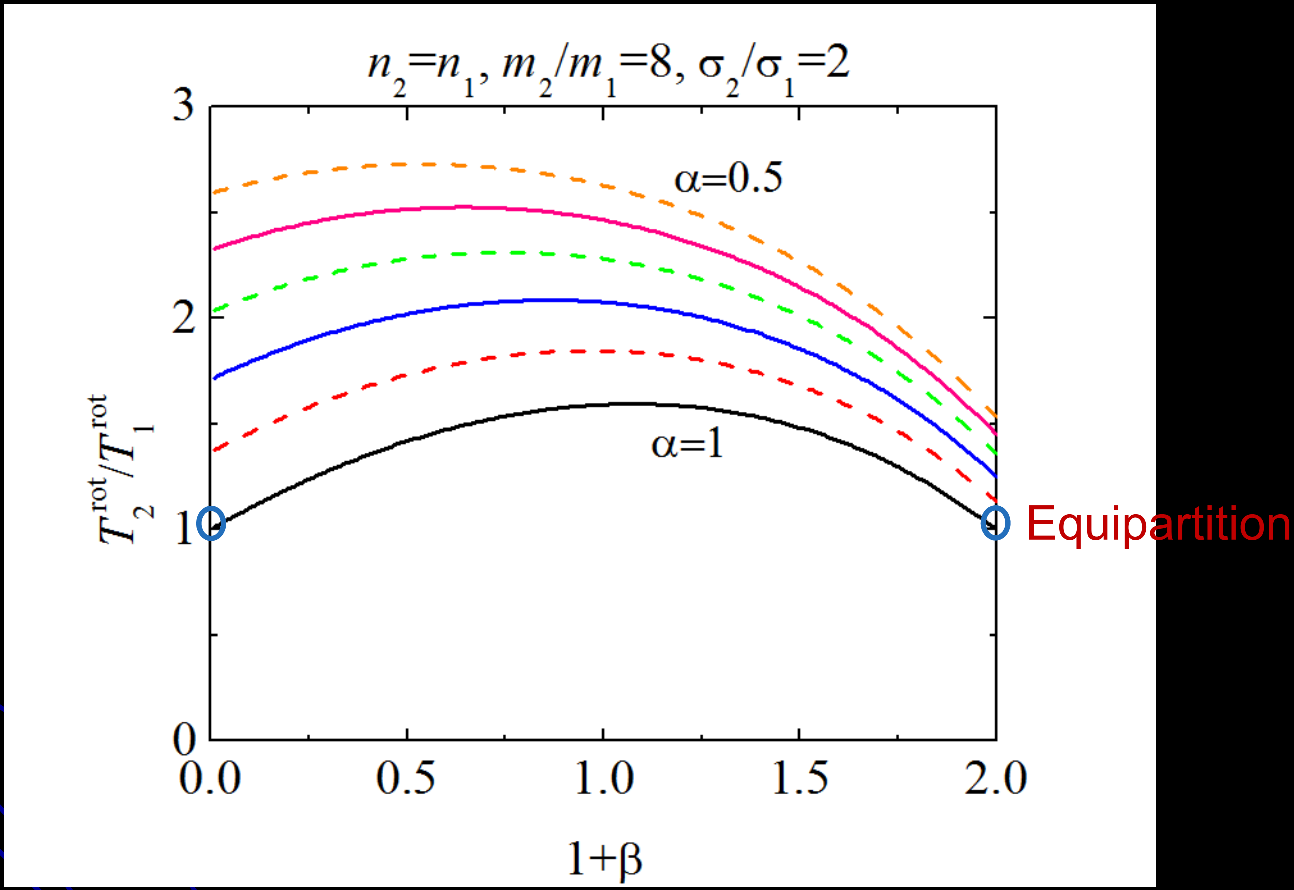
$$\xi_1^{\mathrm{rot}} = \xi_2^{\mathrm{rot}} = \cdots = 0$$

Translational/Rotational



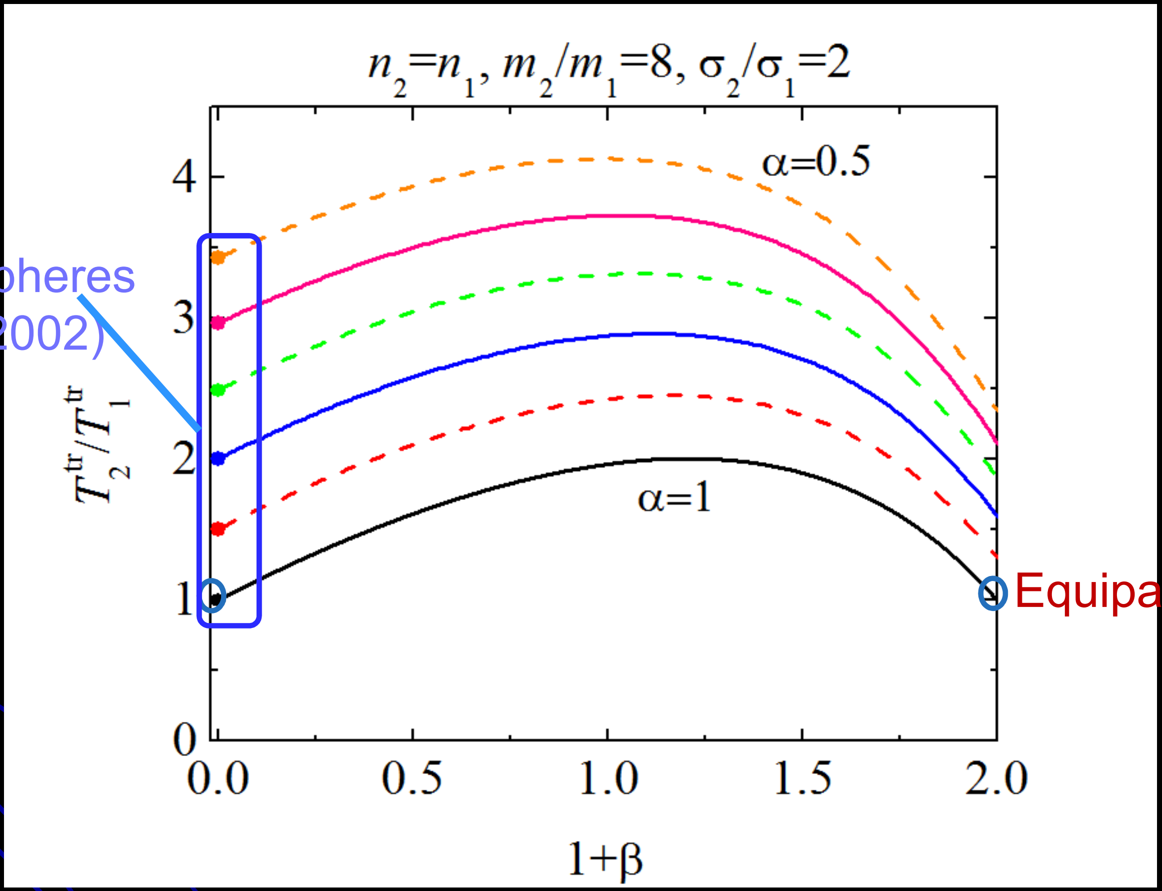
Weak influence of inelasticity

Rotational/Rotational



Same qualitative behavior for different inelasticities

Translational/Translational



"Pure" smooth spheres (Barrat&Trizac, 2002)

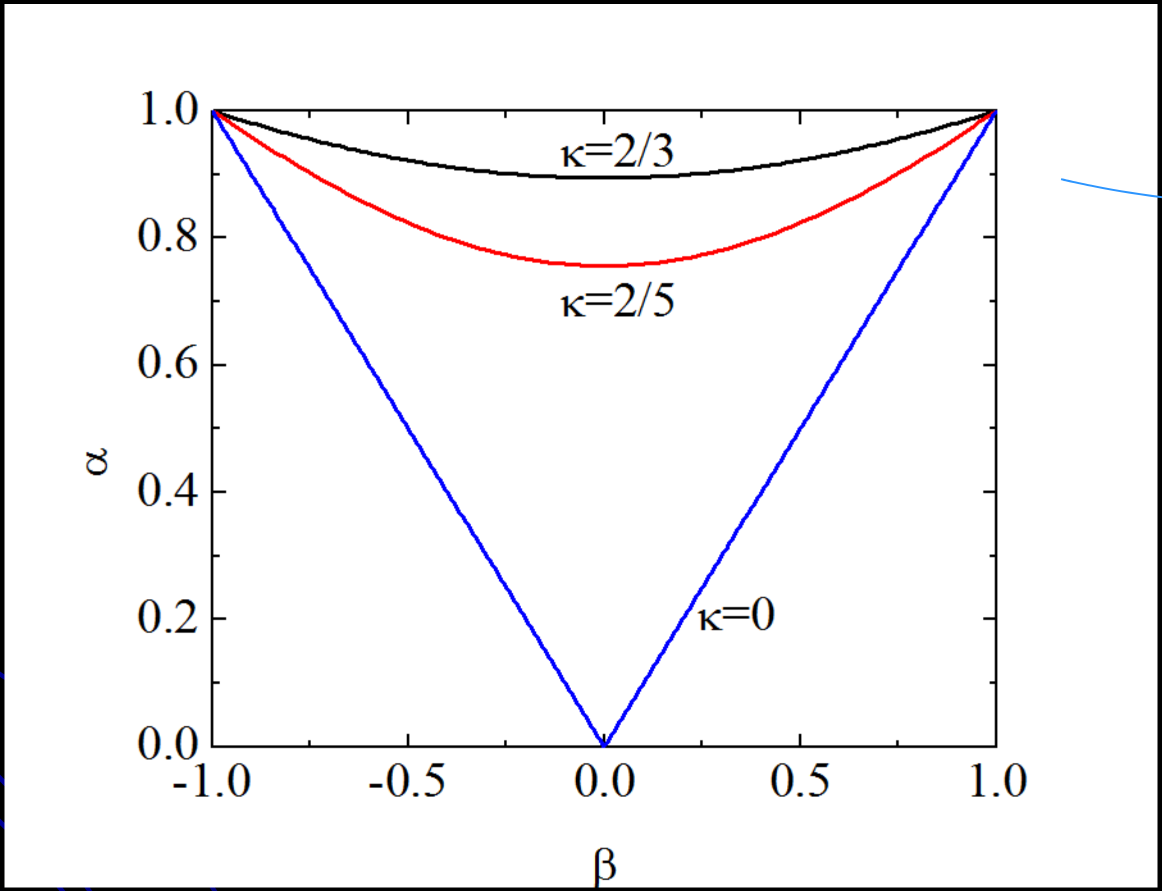
Equipartition

No "ghost" effect! (steady state)

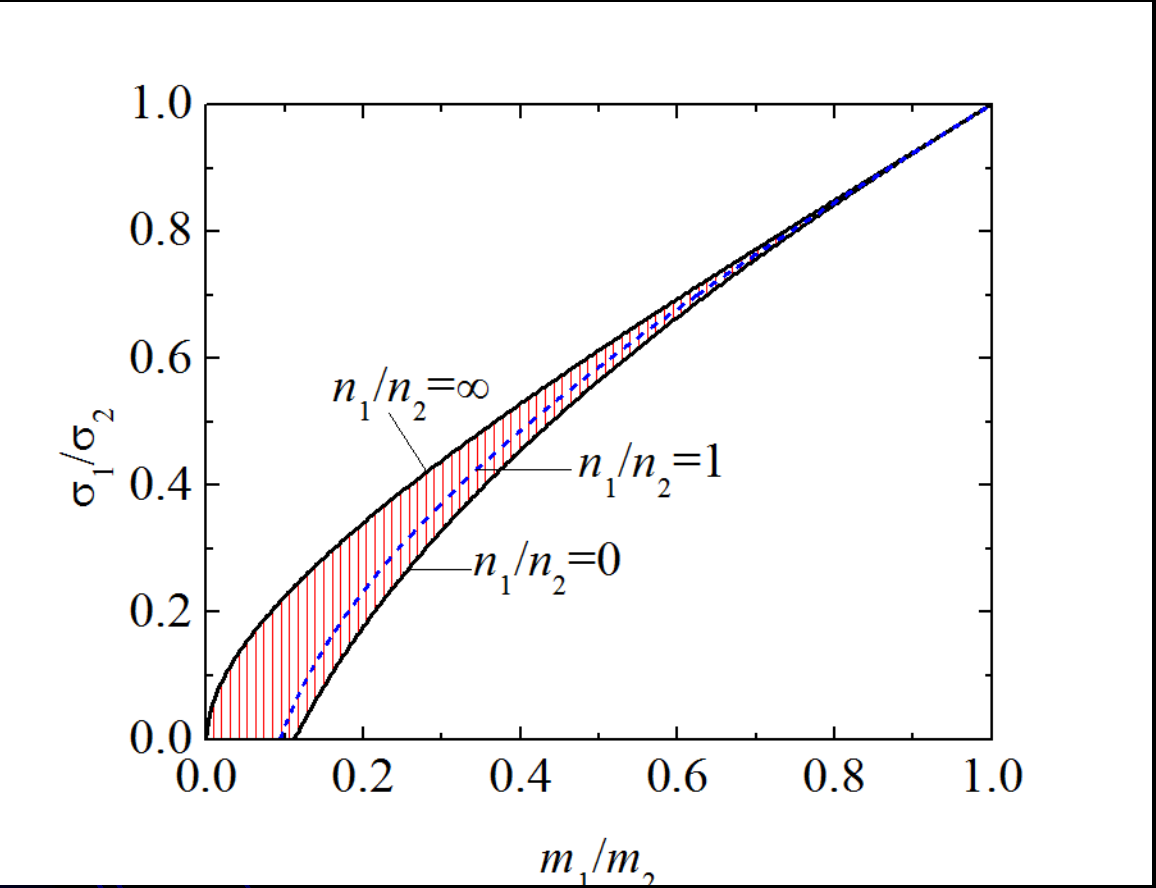
Locus of equipartition: Under which conditions does equipartition hold?

- Coefficients of normal restitution $\alpha_{11} = \alpha_{12} = \alpha_{22} = \alpha$
- Coefficients of tangential restitution $\beta_{11} = \beta_{12} = \beta_{22} = \beta$
- Inertia-moment parameters $\kappa_1 = \kappa_2 = \kappa$
- Size ratio $\sigma_1/\sigma_2 = \text{free}$
- Mass ratio $m_1/m_2 = \text{free}$
- Mole fraction $n_1/(n_1 + n_2) = \text{free}$

First condition: $\begin{cases} 1-\alpha^2 = \frac{1-\kappa}{1+\kappa}(1-\beta^2) & \text{HCS} \\ \beta = \pm 1 & \text{White noise} \end{cases}$



Second condition: $$\frac{n_1}{n_2}=\frac{\sigma_{12}^2\sqrt{\frac{m_2}{m_1}}-\sigma_2^2\sqrt{\frac{m_1+m_2}{2m_2}}}{\sigma_{12}^2\sqrt{\frac{m_1}{m_2}}-\sigma_1^2\sqrt{\frac{m_1+m_2}{2m_1}}}$$



Simple kinetic model for *monodisperse* inelastic rough hard spheres

Three key ingredients we want to keep:

1. $(\partial_t T^{\rm tr})_{\rm coll} = -\xi^{\rm tr} T^{\rm tr}$

2. $(\partial_t T^{\rm rot})_{\rm coll} = -\xi^{\rm rot} T^{\rm rot}$

3. $$\int d\mathbf{v}_i \int d\boldsymbol{\omega}_i\, \mathbf{v}_i J_{ij}[\mathbf{v}_i, \boldsymbol{\omega}_i | f_i, f_j] = \frac{1+\alpha_{ij}+\beta_{ij}\kappa_{ij}/(1+\kappa_{ij})}{2} \times \int d\mathbf{v}_i \int d\boldsymbol{\omega}_i\, \mathbf{v}_i J_{ij}[\mathbf{v}_i, \boldsymbol{\omega}_i | f_i, f_j]\Big|_{\substack{\alpha_{ij}=1 \\ \beta_{ij}=-1}}$$

Elastic smooth spheres

$$\partial_t f(\mathbf{r},\mathbf{v},\boldsymbol{\omega},t) + \mathbf{v}\cdot\nabla f(\mathbf{r},\mathbf{v},\boldsymbol{\omega},t) = J[\mathbf{r},\mathbf{v},\boldsymbol{\omega},t|f,f]$$

$$\begin{aligned} J[f,f] \;&\rightarrow\; -\lambda\nu_0\,(f-f_0) \\ &+\frac{\xi^{\mathrm{tr}}}{2}\frac{\partial}{\partial\mathbf{v}}\cdot[(\mathbf{v}-\mathbf{u})f] + \frac{\xi^{\mathrm{rot}}}{2}\frac{\partial}{\partial\boldsymbol{\omega}}\cdot(\boldsymbol{\omega}f) \end{aligned}$$

$$\lambda \equiv \frac{1+\alpha}{2} + \frac{\kappa}{1+\kappa}\frac{1+\beta}{2}, \quad \nu_0 = \frac{16\sqrt{\pi}}{5} n\sigma^2\sqrt{T^{\mathrm{tr}}/m}$$

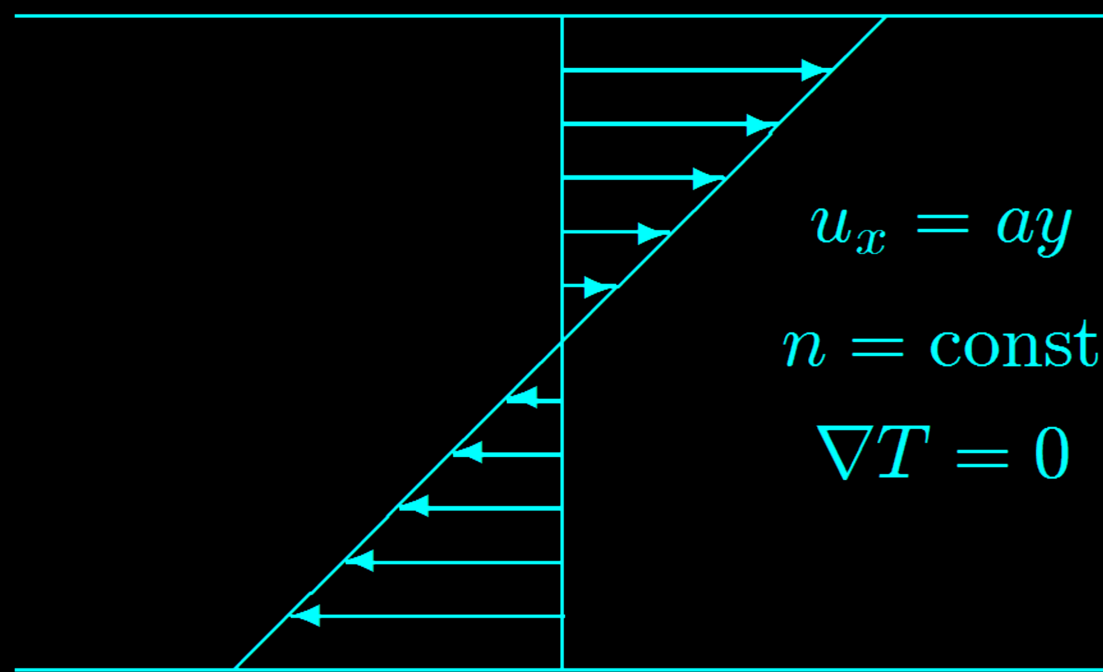
$$f_0 = n\left(\frac{mI}{4\pi^2 T^{\mathrm{tr}} T^{\mathrm{rot}}}\right)^{3/2}\exp\left[-\frac{m(\mathbf{v}-\mathbf{u})^2}{2T^{\mathrm{tr}}} - \frac{I\omega^2}{2T^{\mathrm{rot}}}\right]$$

An even simpler version …

$$\begin{cases} \partial_t f^{\rm tr}(\mathbf{r},\mathbf{v},t) + \mathbf{v}\cdot\nabla f^{\rm tr}(\mathbf{r},\mathbf{v},t) = -\lambda\nu_0\left[f^{\rm tr}(\mathbf{r},\mathbf{v},t) - f_0^{\rm tr}(\mathbf{r},\mathbf{v},t)\right] + \dfrac{\xi^{\rm tr}}{2}\dfrac{\partial}{\partial\mathbf{v}}\cdot\left[(\mathbf{v}-\mathbf{u})f^{\rm tr}(\mathbf{r},\mathbf{v},t)\right] \\ \partial_t T^{\rm rot} + \nabla\cdot\left(\mathbf{u}\,T^{\rm rot}\right) = -\xi^{\rm rot}T^{\rm rot} \end{cases}$$

Application to simple shear flow

$$y = +L/2$$



$$y = -L/2$$

$$u_x = ay$$

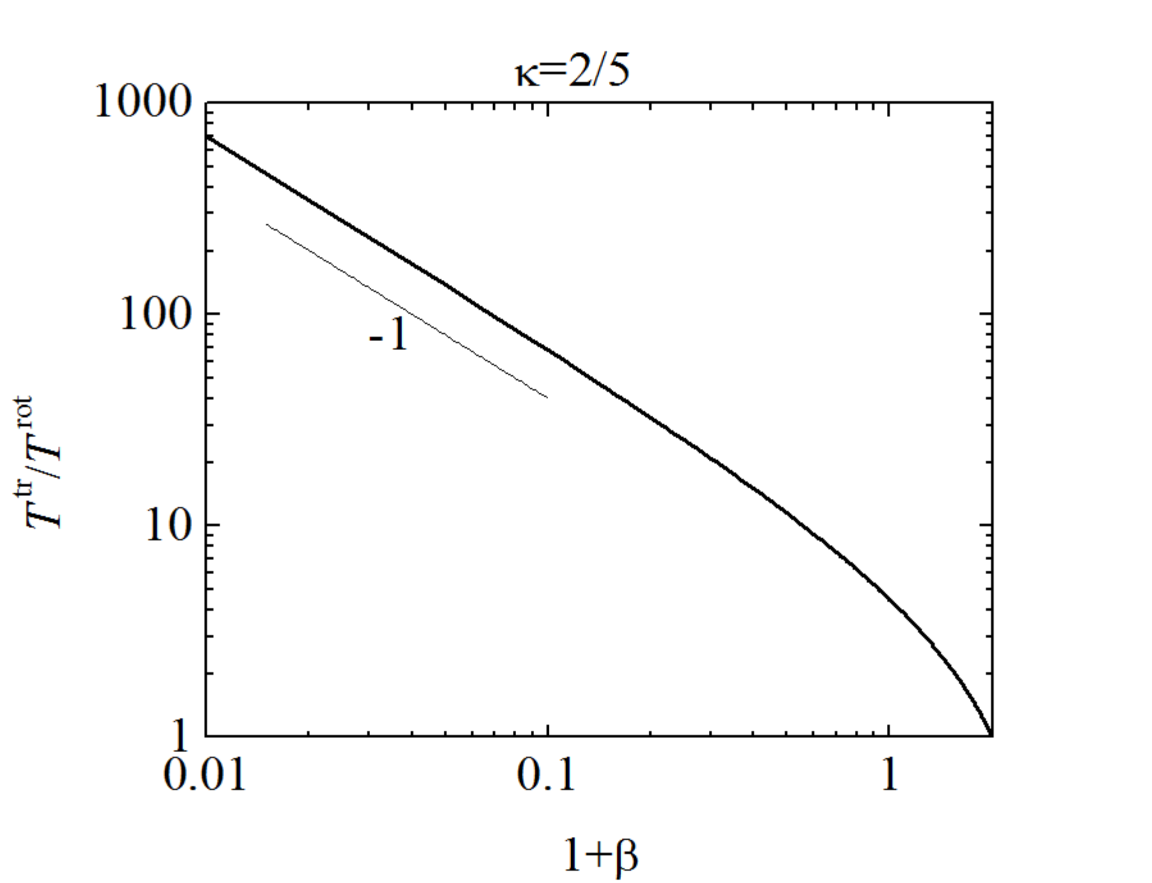
$$n = \text{const}$$

$$\nabla T = 0$$

Application to simple shear flow
Translational/Rotational temperature ratio

$$\xi^{\text{rot}} = 0 \Rightarrow \boxed{\frac{T^{\text{rot}}}{T^{\text{tr}}} = \frac{\kappa(1+\beta)}{2\kappa+1-\beta}}$$

Independent of α



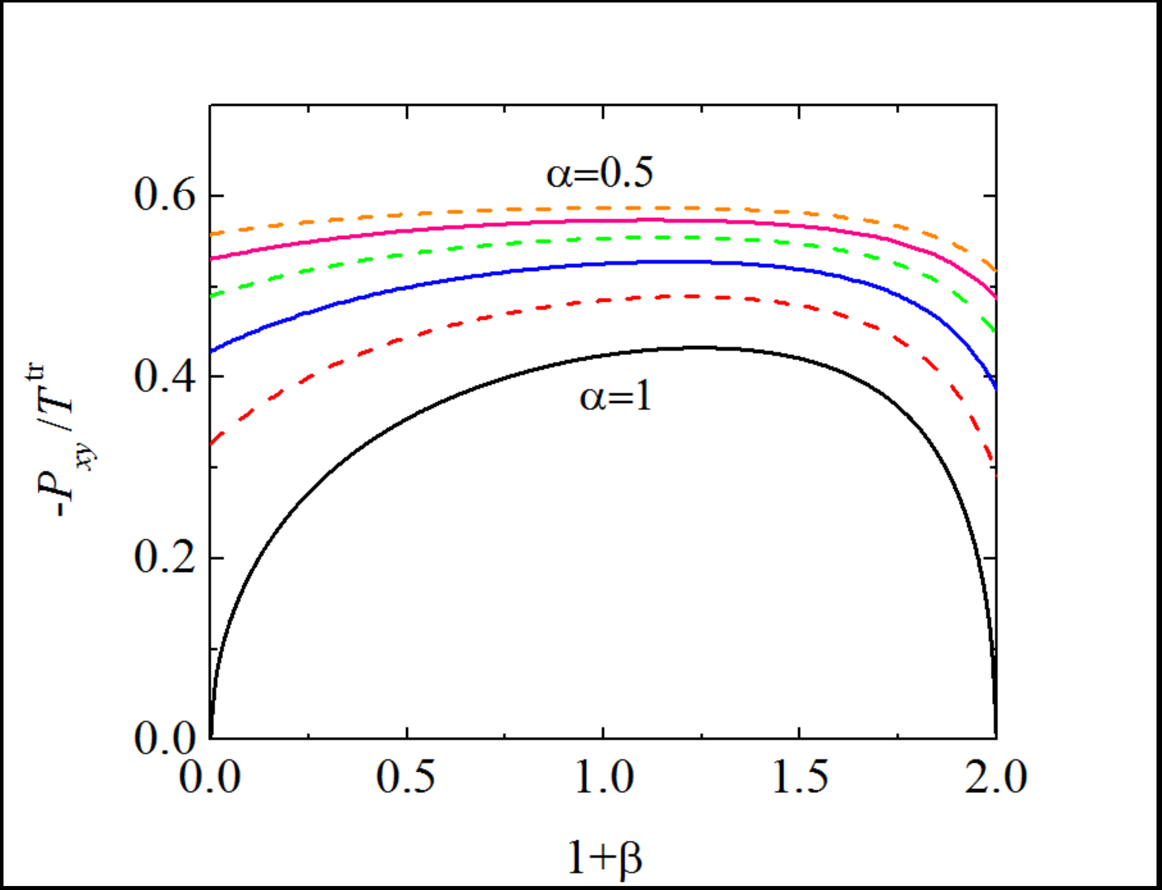
Application to simple shear flow

Shear stress

$$\frac{P_{xy}}{nT^{\text{tr}}}=-\frac{\sqrt{3\widehat{\xi}^{\text{tr}}/2}}{1+\widehat{\zeta}^{\text{tr}}}$$

$$\widehat{\xi}^{\text{tr}}=\frac{5}{6}\frac{1-\alpha^2+2\kappa(1-\beta^2)/(2\kappa+1-\beta)}{1+\alpha+\kappa(1+\beta)/(1+\kappa)}$$

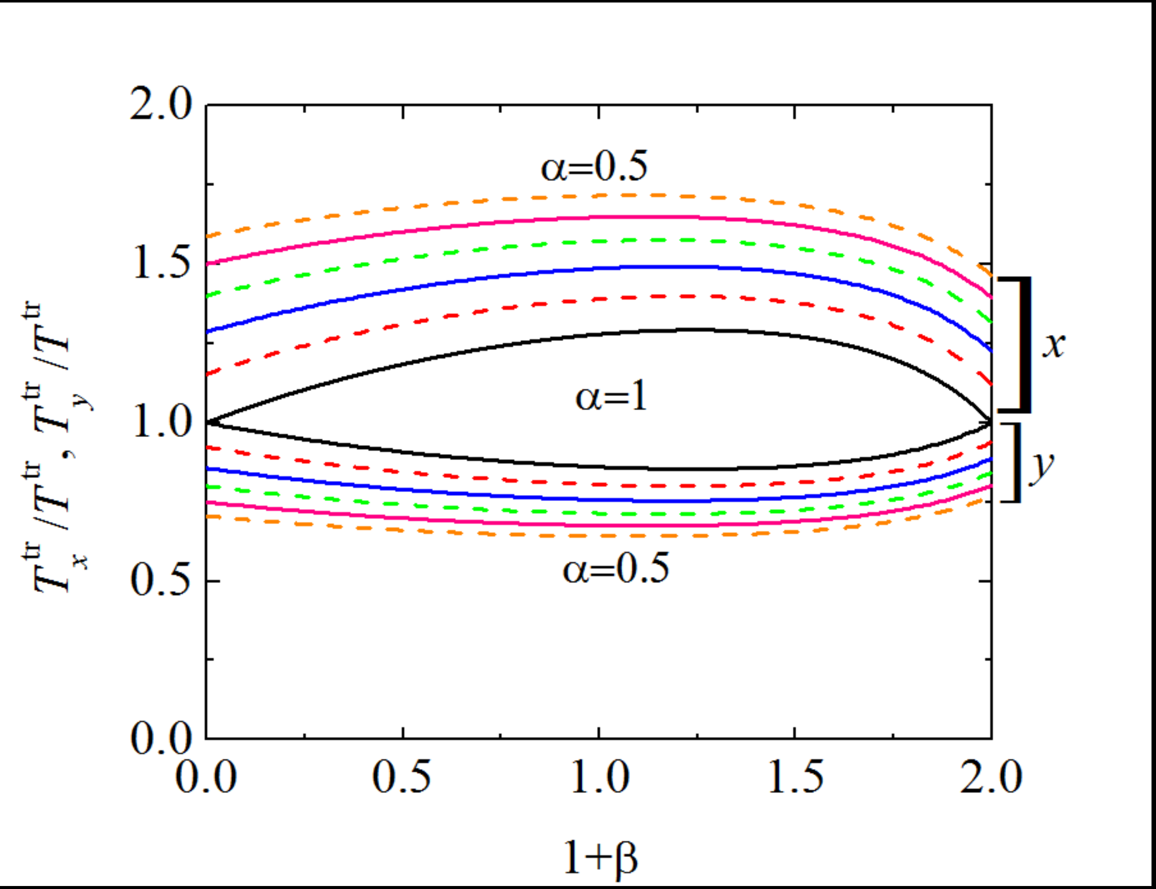
Scaled thermal rate



Application to simple shear flow
Anisotropic translational temperatures

$$\frac{T_x^{\mathrm{tr}}}{T^{\mathrm{tr}}} = \frac{1+3\widehat{\xi}^{\mathrm{tr}}}{1+\widehat{\xi}^{\mathrm{tr}}}$$

$$\frac{T_y^{\mathrm{tr}}}{T^{\mathrm{tr}}} = \frac{T_z^{\mathrm{tr}}}{T^{\mathrm{tr}}} = \frac{1}{1+\widehat{\xi}^{\mathrm{tr}}}$$



Conclusions and outlook

- Collisional thermal rates obtained for mixtures of inelastic rough hard spheres.
- Interesting non-equipartition phenomena in the HCS (“ghost” effect).
- Simulations planned to test the theoretical predictions.
- Proposal of a simple model kinetic equation for the single-component case.
- Solution of the above model in the uniform shear flow. Simulations planned.
- Derivation of the Navier-Stokes constitutive equations.

Thanks for your attention!