Singular Properties of a Heavy Impurity Particle Immersed in a Granular Fluid

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- **Homogeneous cooling state (HCS)**
- **Velocity fluctuations as order parameter**
- **Order parameter dynamics, normal and ordered phases**
- **Thermodynamic analogy, second order transition**
- **Critical dynamics**
- **Stationary state velocity distribution**

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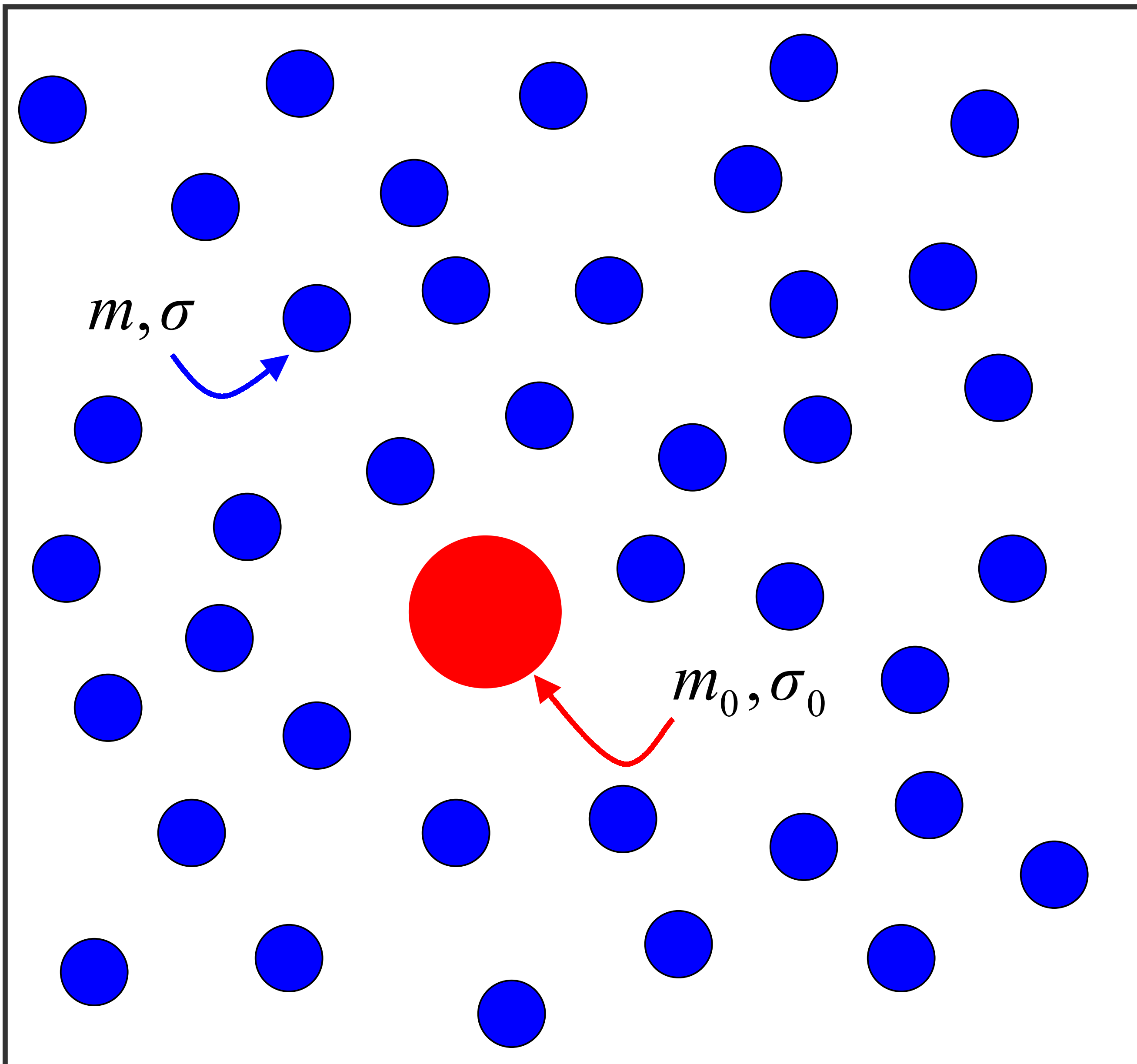
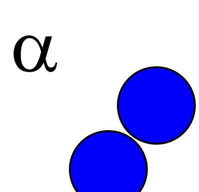
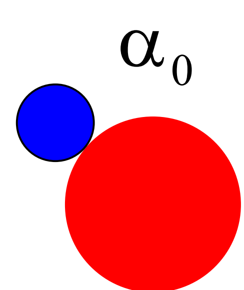
(thanks to NSF and MCyT for support)

HEAVY PARTICLE
IN A GRANULAR FLUID



Inelastic collisions, both 

and 

COLLISIONAL COOLING

Time dependence through mean speeds

$$v_T^2(t) = \left\langle v^2(t) \right\rangle \qquad v_{T0}^2(t) = \left\langle v_0^2(t) \right\rangle$$

Cooling rates for fluid, impurity particles

$$\partial_t \ln v_T^2(t) = -\xi(t) \qquad \partial_t \ln v_{T0}^2(t) = -\xi_0(t)$$

***Order parameter*:**

$$\bar{\phi}(t) = \frac{v_{T0}^2(t)}{v_T^2(t)}$$

(<u>Elastic</u> collisions, equilibrium $\Rightarrow \bar{\phi}_{\text{Gibbs}} = \dfrac{m}{m_0}$ **)**

HOMOGENEOUS COOLING STATE (HCS)

$$h \equiv \frac{1+\alpha_0}{2}\frac{m/m_0}{1+m/m_0} \approx m/m_0 << 1$$

Order parameter dynamics

$$\left[\partial_s + \gamma^*(\overline{\phi}) - \xi^*\right]\overline{\phi} = hn(\overline{\phi})$$

Friction coefficient

$$ds = \nu(t)dt$$

Impurity collision frequency

Fluid cooling rate *relative to impurity collision rate*

$$\xi^* = \xi(t)/\nu(t)$$

HCS:

$$\left[\gamma^*(\overline{\phi}_s) - \xi^*\right]\overline{\phi}_s = hn(\overline{\phi}_s)$$

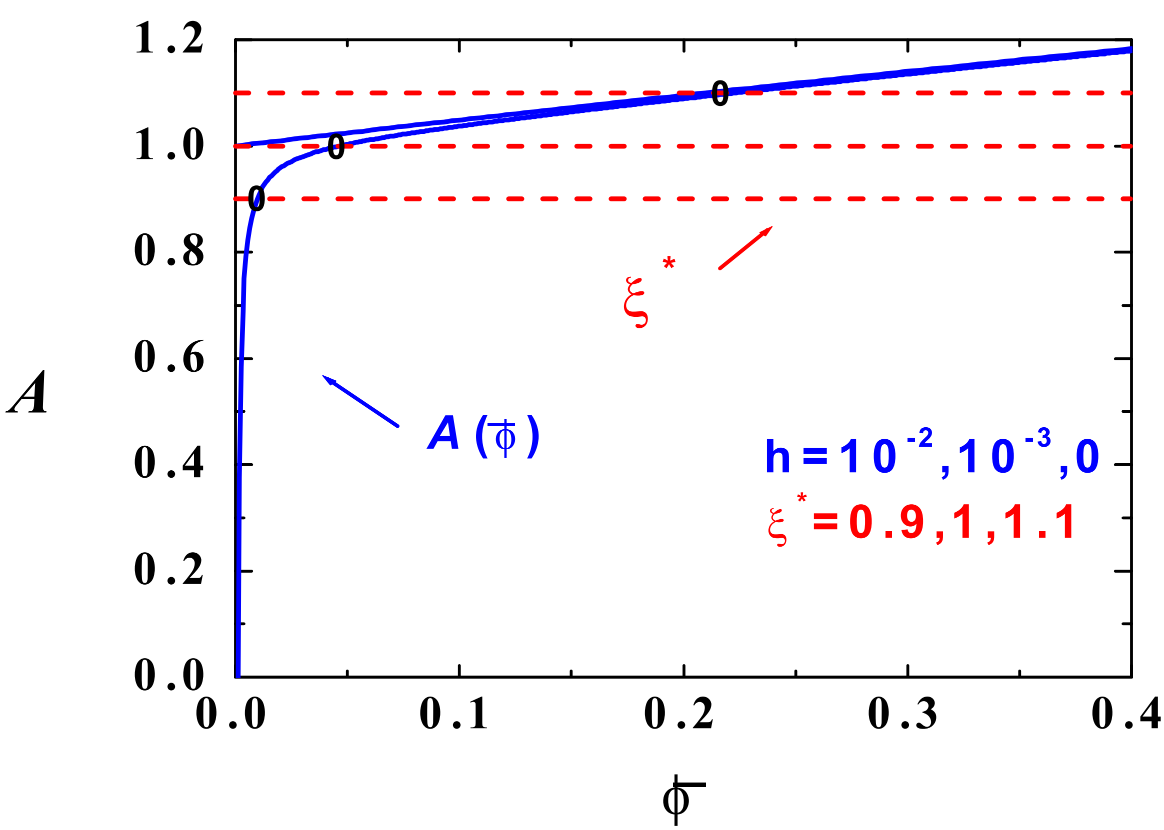
Comment: $\overline{\phi}_s \neq \frac{h}{1-h} = \frac{m}{m_0} \Rightarrow$ two temperatures!

NORMAL AND ORDERED PHASES

Estimate $\gamma^*(\bar{\phi}), n(\bar{\phi})$ **(maximum entropy ensemble)**

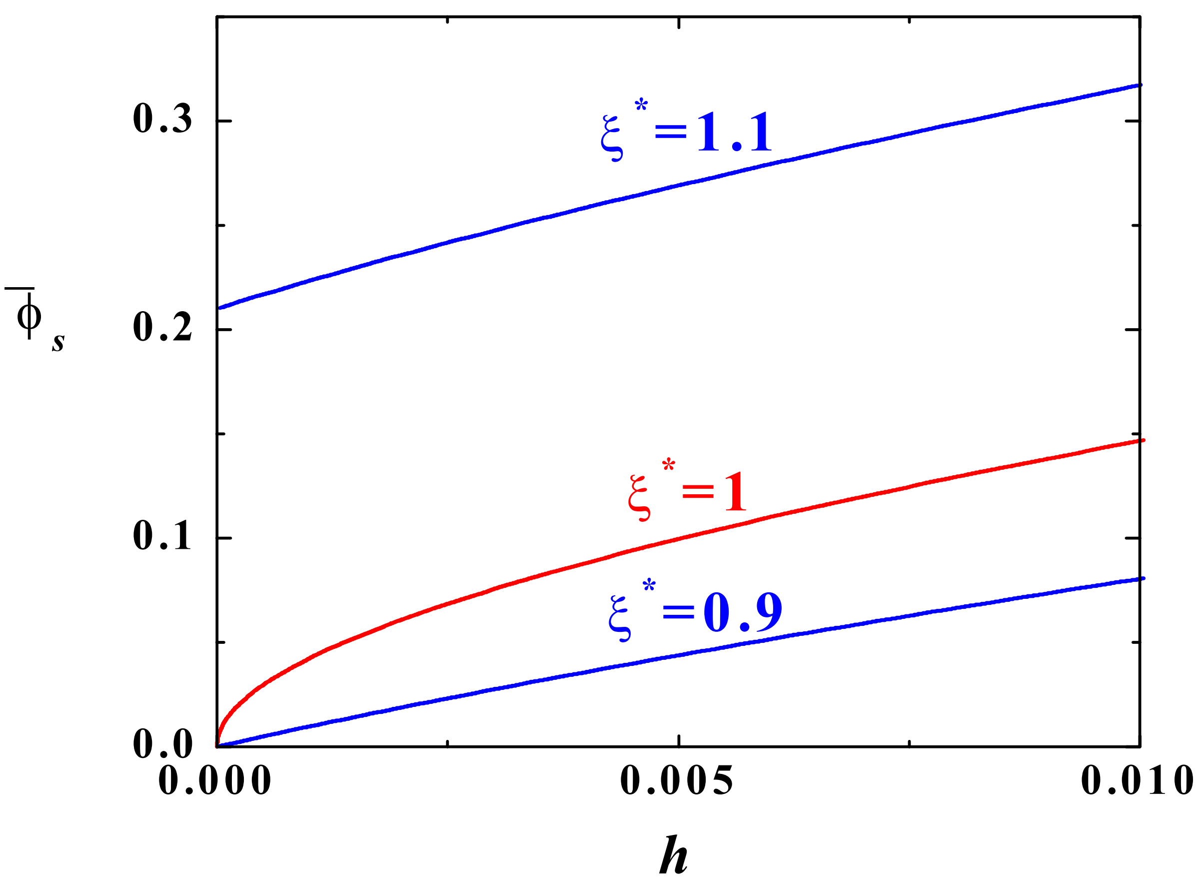
$$\gamma^*(\bar{\phi}) = \left(1+\bar{\phi}\right)^{1/2} \qquad n(\bar{\phi}) = \left(1+\bar{\phi}\right)^{3/2}$$

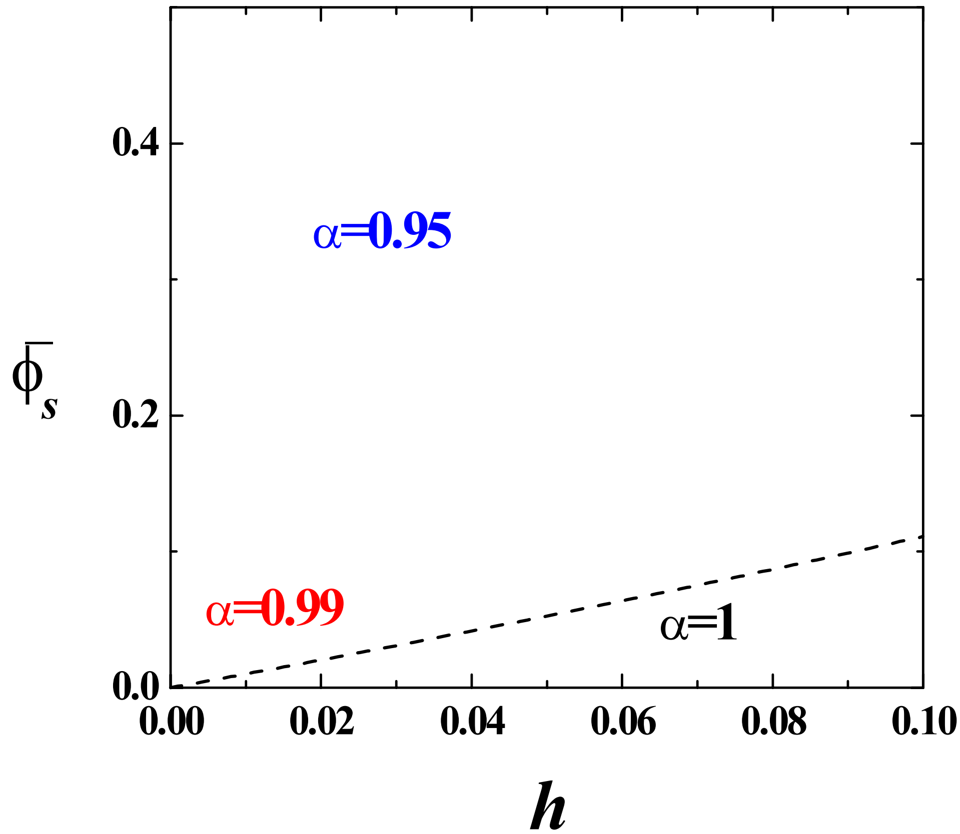
HCS : $$A(\bar{\phi}_s) \equiv \gamma^*(\bar{\phi}_s) - h\frac{n(\bar{\phi}_s)}{\bar{\phi}_s} = \xi^*$$



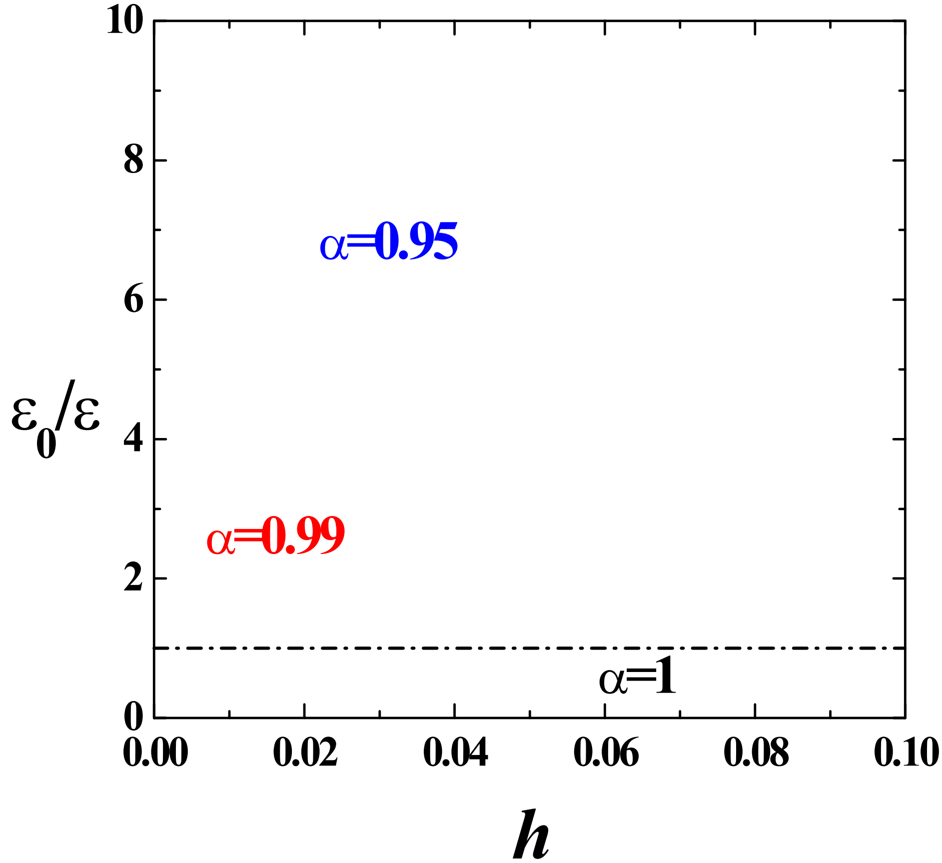
- **Unique positive solution for finite** ξ^*, h
- $\overline{\phi}_s = 0$ **for** $\xi^* < 1, h = 0$ **(normal)**
- $\overline{\phi}_s > 0$ **for** $\xi^* > 1, h = 0$ **(ordered)**

$$\overline{\phi}_s \to \begin{cases} h/(1-\xi^*), & \xi^* < 1 \\ \sqrt{2h}, & \xi^* = 1 \\ \xi^{*2} - 1, & \xi^* > 1 \end{cases}$$





0.4
0.2
0.0
$\bar{\phi}_s$
α=0.95
α=0.99
α=1
0.00
0.02
0.04
0.06
0.08
0.10
h



10
8
6
4
2
0
$\varepsilon_0/\varepsilon$
α=0.95
α=0.99
α=1
0.00
0.02
0.04
0.06
0.08
0.10
h

THERMODYNAMIC ANALOGY

$$h = h(\xi^*, \bar{\phi}_s)$$ **(equation of state)**

$$F(\xi^*, \bar{\phi}_s) = \int_0^{\bar{\phi}_s} h(\xi^*, x)\,dx$$ **(Helmholtz free energy)**

$$G(\xi^*, h) = F - h\bar{\phi}_s$$ **(Gibbs free energy)**

Landau-like free energy near critical point

$$h \to \left(1 - \xi^*\right)\bar{\phi}_s + \frac{1}{2}\bar{\phi}_s^{\,2}$$

$$G \to \frac{1}{2}\left(1 - \xi^*\right)\bar{\phi}_s^{\,2} + \frac{1}{6}\bar{\phi}_s^{\,3} - h\bar{\phi}_s$$

SUSCEPTIBILITIES

$$\chi = -\frac{\partial^2 G}{\partial h^2} = \frac{\partial \bar{\phi}}{\partial h} \to \frac{1}{\left|\xi^* - 1\right|} \begin{cases} 1, & \xi^* < 1 \\ \dfrac{2\xi^{*4}}{\xi^* + 1}, & \xi^* > 1 \end{cases}$$

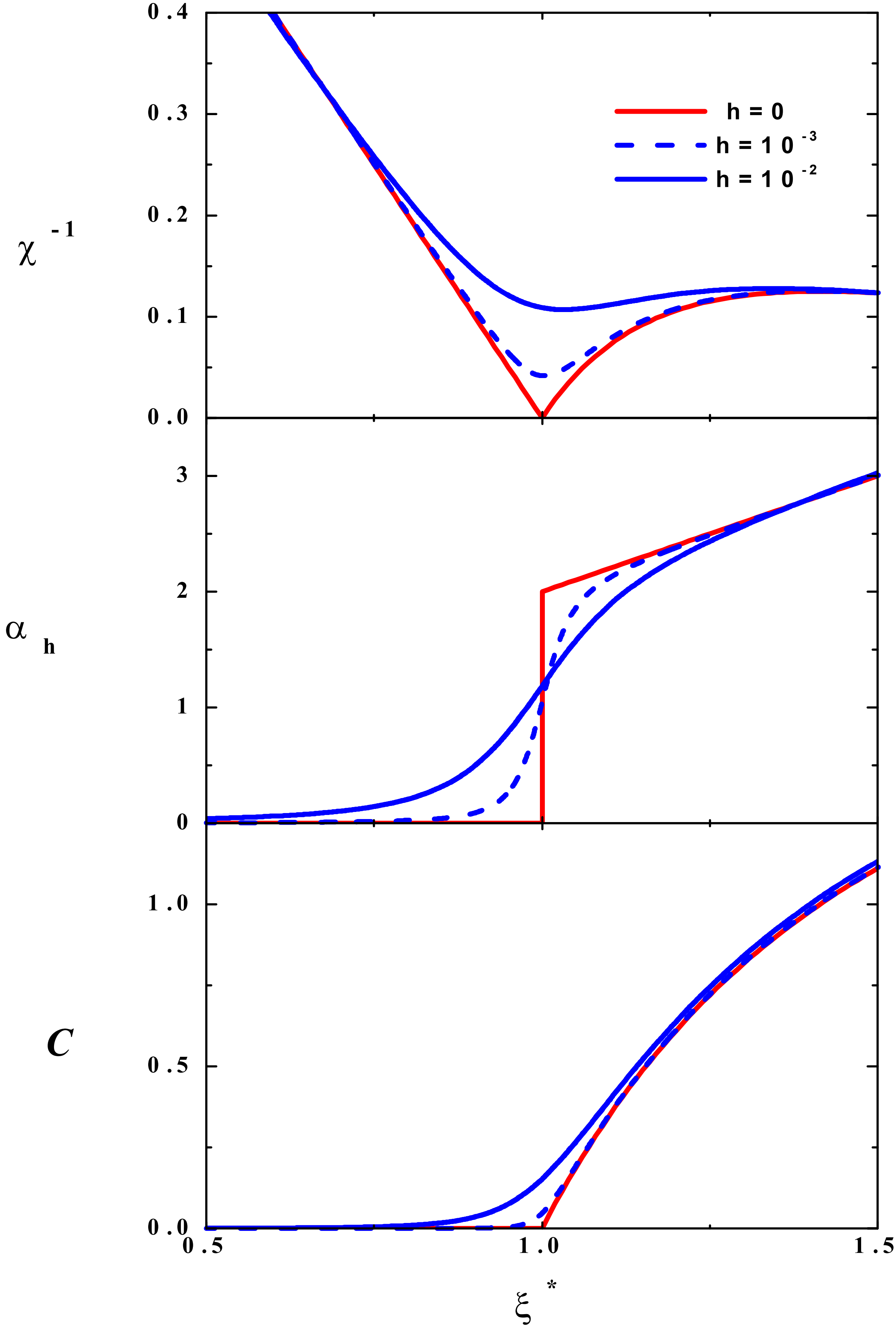
divergent **at** $\xi^* = 1$

$$\alpha_h = -\frac{\partial^2 G}{\partial h \partial \xi^*} = \frac{\partial \bar{\phi}}{\partial \xi^*} \to \begin{cases} 0, & \xi^* < 1 \\ 2\xi^*, & \xi^* > 1 \end{cases}$$

discontinuous **at** $\xi^* = 1$

$$C = -\frac{\partial^2 G}{\partial \xi^{*2}} \to \frac{2}{\xi^{*2}} \begin{cases} 0, & \xi^* < 1 \\ \xi^{*2} - 1, & \xi^* > 1 \end{cases}$$

Second order phase transition!



h = 0
h = 10 -3
h = 10 -2
χ -1
α h
C
ξ *
0.0
0.1
0.2
0.3
0.4
0
1
2
3
0.5
1.0
1.5

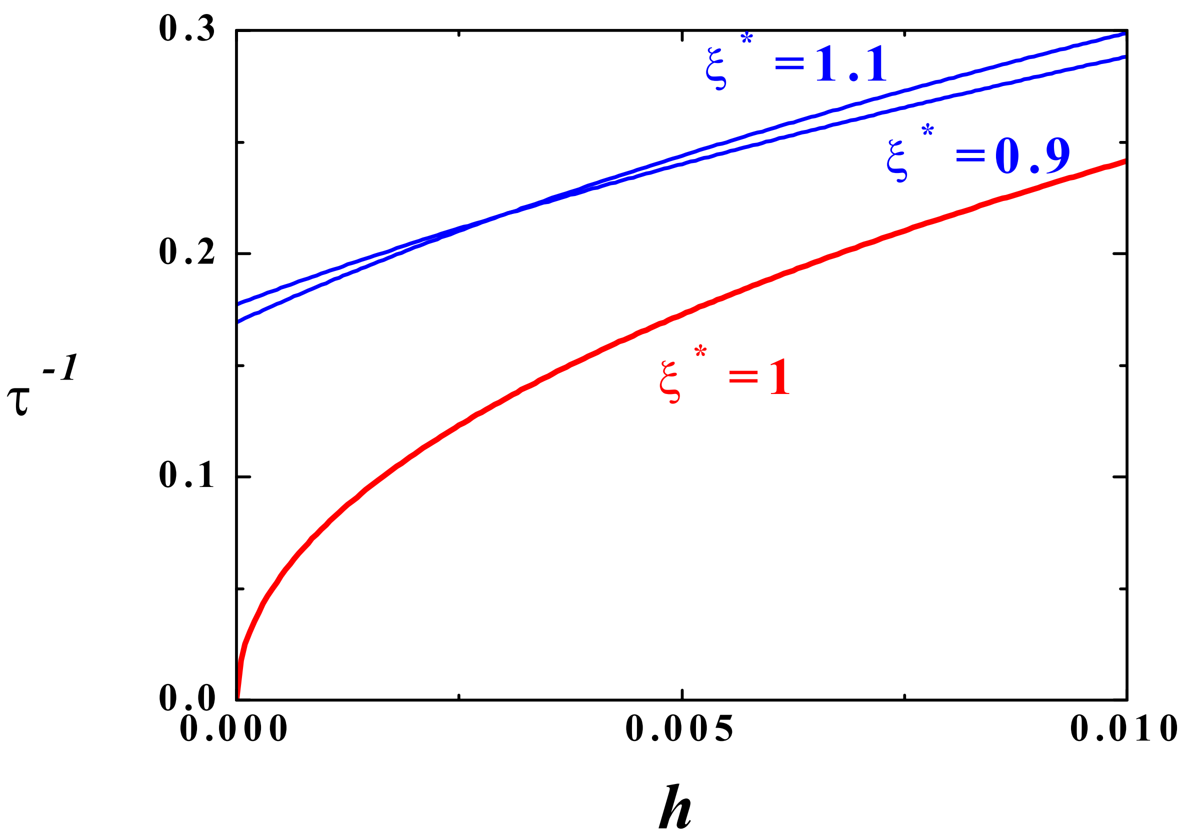
CRITICAL SLOWING

Ginzburg-Landau dynamics

$$\partial_s \overline{\phi} = -n(\overline{\phi})\,\frac{\partial G(\xi^*, h; \overline{\phi})}{\partial \overline{\phi}}$$

linearize $\Rightarrow \left|\overline{\phi} - \overline{\phi}_s\right| \approx e^{-s/\tau}$

divergent response time $\tau = \chi\left(\overline{\phi}_s\right) / n\left(\overline{\phi}_s\right)$



VELOCITY DISTRIBUTION

(asymptotic analysis of the Enskog-Lorentz kinetic equation)

Critical domain $\xi^* - 1 = \sqrt{h}\delta, \qquad \phi = \sqrt{h}\eta,$

$$h \to 0$$

$$P_s(x) \propto x^{1/2} \exp\left[-\frac{9}{40}\left(\bar{\eta}_s x - \frac{10}{3}\delta \right)^2 \right]$$

$$x = \eta / \bar{\eta}_s = \phi / \bar{\phi}_s, \quad \bar{\eta}_s(\delta) \to \begin{cases} |\delta|^{-1}, & \delta << -1 \\ 1.6, & \delta = 0 \\ \frac{10}{3}\delta, & \delta >> 1 \end{cases}$$

Normal phase: $\delta << -1$

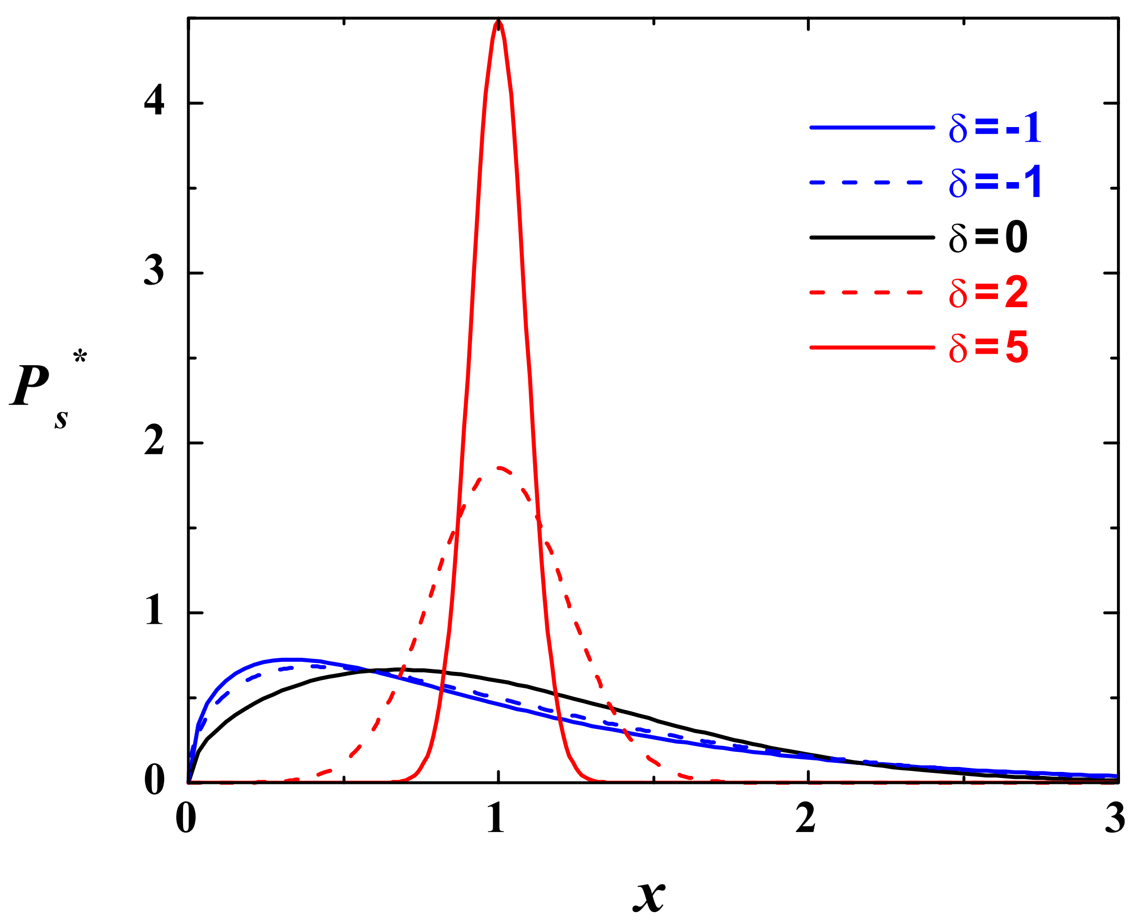
Maxwell-Boltzmann (at $T_0 = T\dfrac{1+\alpha_0}{2\left(1-\xi^*\right)}$ **)**

Critical point: $\delta = 0$

$$P_s(x) \propto x^{1/2} \exp(-0.55\, x^2)$$

Ordered phase: $\delta >> 1$

$$P_s(x) \propto \exp\left[-\frac{5}{2}\delta\left(x-1\right)^2\right]$$



DIFFUSION

$$\partial_s n(\mathbf{r},s) = D^*(s)\nabla^2 n(\mathbf{r},s)$$

$$D^*(s) = \frac{1}{3\bar{\phi}_s}\int_0^s ds'\langle \mathbf{v}(s')\cdot\mathbf{v}\rangle \rightarrow \frac{1}{3}\int_0^s ds' e^{-\omega s'}$$

$$= D\left[1-e^{-\omega s}\right]$$

"Enskog"

Diffusion for $s \gg \omega^{-1}$

$$D = \frac{1}{3\omega} = \frac{2}{3\left(1+\bar{\phi}_s\right)^{3/2}}\frac{\bar{\phi}_s}{h} \rightarrow \begin{cases} \dfrac{2}{3\left(1-\xi^*\right)}, & \xi^* < 1 \\ \infty, & \xi^* \geq 1 \end{cases}$$

Divergent diffusion constant ??

Mean square displacement:

$$\left\langle \left| \mathbf{r}(s) - \mathbf{r}(0) \right|^2 \right\rangle = 2\omega^{-1}\left[s - \omega^{-1}\left(1 - e^{-\omega s}\right)\right]$$

$$\rightarrow \begin{cases} 2\omega^{-1} s = 6Ds, & \xi^* < 1 \\ s^2, & \xi^* \geq 1 \end{cases}$$

ballistic motion

(divergent mean free time)