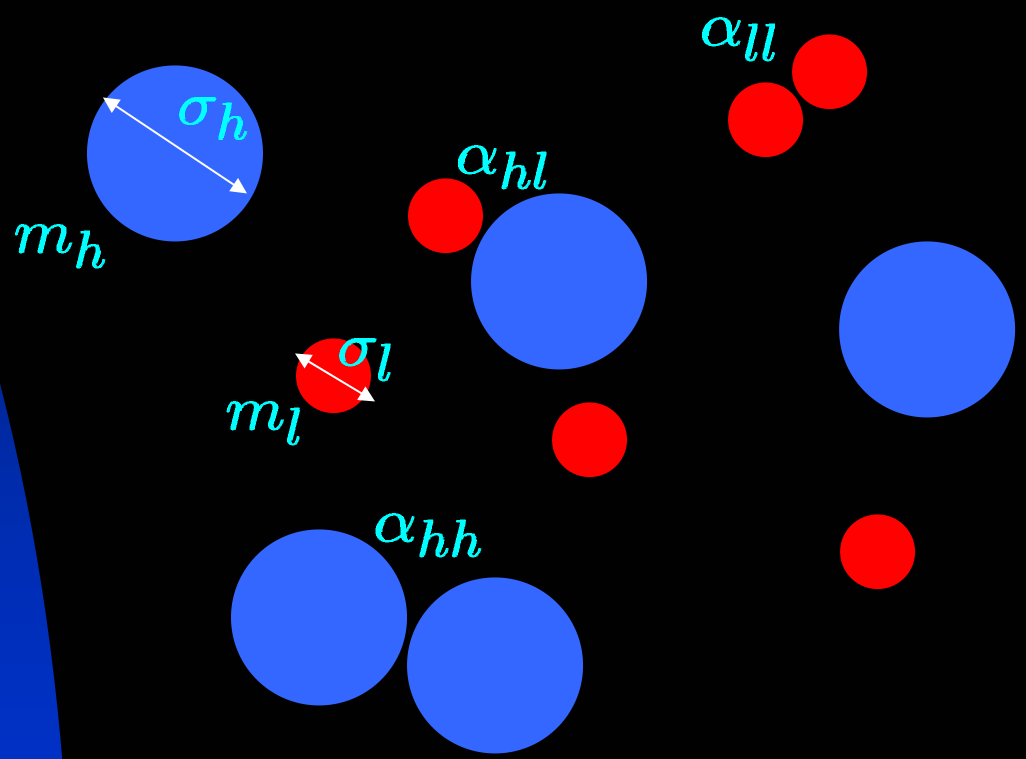
Nonequipartition in uniform granular mixtures

Andrés Santos*

Departmento de Física, Universidad de Extremadura, Badajoz (Spain)

* In collaboration with J.W. Dufty

Binary granular mixture



VOLUME 88, NUMBER 19 PHYSICAL REVIEW LETTERS 13 MAY 2002

Breakdown of Energy Equipartition in a 2D Binary Vibrated Granular Gas

Klebert Feitosa* and Narayanan Menon†
Department of Physics, University of Massachusetts, Amherst, Massachusetts 01003-3720

TABLE I. Some material properties of the spheres used in the experiment.

Particle	Mass [mg]	Effective inelasticity[a]	Mass ratio w/glass
Glass	5.24	0.17	–
Aluminum	5.80	0.31	0.92
Steel	15.80	0.21	0.33
Brass	18.00	0.39	0.28

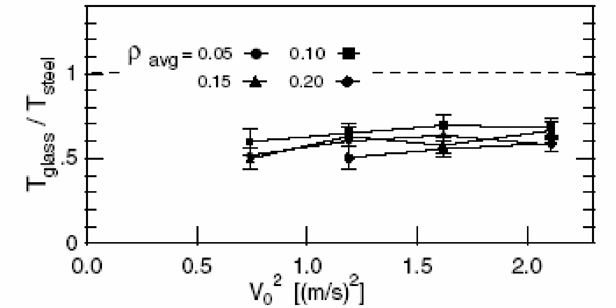


FIG. 4. Temperature ratio, $\gamma = T_{\text{glass}}/T_{\text{steel}}$, in a steel-glass mixture plotted against squared vibration velocity, v_0^2. Different markers represent different number densities of the mixture. The number fraction is fixed at $x = 1/2$. The horizontal dashed line represents equipartition ($\gamma = 1$).

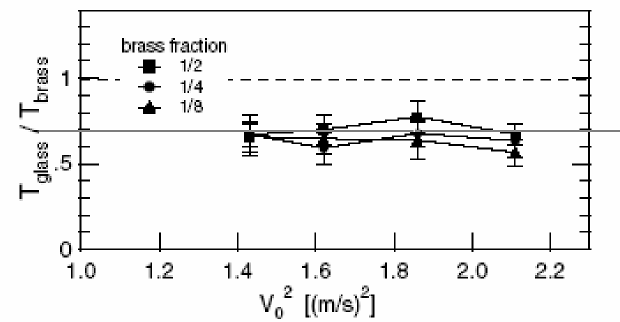


FIG. 5. Temperature ratio, $\gamma = T_{\text{glass}}/T_{\text{brass}}$, in a brass-glass mixture versus the squared vibration velocity of the cell, v_0^2. Different markers represent different number fractions of brass for the same total number of particles ($\rho_{\text{avg}} = 0.049$). The horizontal dashed line represents equipartition ($\gamma = 1$).

In general, the lighter species has a smaller temperature

Formulation of the problem

- Binary mixture of smooth inelastic hard spheres
  - ✓ Heavy species (H):

    $m_h, \sigma_h, x_h = n_h/n, \alpha_{hh}, \alpha_{hl}$
  - ✓ Light species (L):

    $m_l, \sigma_l, x_l = n_l/n = 1 - x_l, \alpha_{ll}, \alpha_{lh} = \alpha_{hl}$
- In the homogeneous cooling state,

  $m_h \gg m_l \Rightarrow \langle v_h^2 \rangle / \langle v_l^2 \rangle = ?$

Enskog-Boltzmann equation

$$\partial_t f_h(v) = J_{hh}[f_h, f_h] + J_{hl}[f_h, f_l]$$

$$\partial_t f_l(v) = J_{lh}[f_l, f_h] + J_{ll}[f_l, f_l]$$

$$\partial_t \langle v_h^2 \rangle = -\xi_h \langle v_h^2 \rangle, \quad \partial_t \langle v_l^2 \rangle = -\xi_l \langle v_l^2 \rangle$$

Rates of change

$$\xi_h = \xi_{hh} + \xi_{hl}, \quad \xi_l = \xi_{lh} + \xi_{ll}$$

Cooling rates

Thermalization rates

“Order” parameter

$$\phi \equiv \frac{\langle v_h^2 \rangle}{\langle v_l^2 \rangle}$$

$$\partial_t \phi = -(\xi_h - \xi_l)\phi$$

Scaling solution for long times (HCS):

$$\xi_h = \xi_l \Rightarrow \xi_{hh} + \xi_{hl} = \xi_{ll} + \xi_{lh}$$

$$\lim_{m_l/m_h \to 0} \phi \begin{cases} = 0 : & \text{“Normal” state} \\ \neq 0 : & \text{“Ordered” state} \end{cases}$$

Maxwellian approximation

PHYSICAL REVIEW E VOLUME 60, NUMBER 5 NOVEMBER 1999

Homogeneous cooling state for a granular mixture

Vicente Garzó* and James Dufty

$$f_i(v) = n_i \left(\frac{m_i}{2\pi T_i}\right)^{3/2} \exp\left(-\frac{m_i v^2}{2T_i}\right), \quad T_i \equiv \frac{m_i}{3}\langle v^2\rangle \quad (i = h, l)$$

$$\xi_{hh} \to x_h \phi^{1/2} \beta_h, \qquad \xi_{hl} \to x_l (1+\phi)^{1/2} \left(1 + \phi_0 - \frac{\mu}{\phi}\right)$$

$$\mu \equiv \frac{m_l}{m_h} \ll 1$$

cooling rates — thermalization rates

$$\xi_{ll} \to x_l \beta_l, \qquad \xi_{lh} \to x_h (1+\phi)^{1/2} \left(1 - \phi_0 + \frac{\phi_0 - \phi}{\mu}\right)$$

$$\beta_h \sim \frac{1-\alpha_{hh}^2}{\mu}, \qquad \beta_l \sim \frac{1-\alpha_{ll}^2}{\mu}\frac{\sigma_l^2}{\sigma_h^2}, \qquad \phi_0 \equiv \frac{1-\alpha_{hl}}{1+\alpha_{hl}}$$

Elastic collisions: $\phi = \mu \Rightarrow T_h = T_l$ **Energy equipartition!**

A few representative cases

- 1. Quasi-elastic cross collisions:

$$\alpha_{hh} = \alpha_{ll} = 1, \quad 1 - \alpha_{hl} \sim \mu$$

$$\xi_{hh} = 0, \quad \xi_{hl} \to x_l \left(1 - \frac{\mu}{\phi}\right)$$

$$\beta_h = \beta_l = 0, \quad \phi_0 \sim \mu$$

$$\xi_{ll} = 0, \quad \xi_{lh} \to x_h \left(1 + \frac{\phi_0 - \phi}{\mu}\right)$$

$$\mu < \phi < \mu + \phi_0$$

Weak breakdown of energy equipartition

$$\phi \sim \mu \Rightarrow T_h \sim T_l$$

“Partitioned” state

2. Inelastic cross collisions:

$$\alpha_{hh} = \alpha_{ll} = 1, \quad 1 - \alpha_{hl} = \text{finite}$$

$$\beta_h = \beta_l = 0, \quad \phi_0 \lesssim 1$$

$$\xi_{hh} = 0, \quad \xi_{hl} \to x_l$$

$$\xi_{ll} = 0, \quad \xi_{lh} \to x_h (1+\phi)^{1/2} \frac{\phi_0 - \phi}{\mu}$$

$$\phi = \phi_0 = \frac{1 - \alpha_{hl}}{1 + \alpha_{hl}}$$

Regardless of the concentrations î
- No Brownian dynamics (x_h ö 0)
- No Lorentz gas (x_l ö 0)

Strong breakdown of energy equipartition

$$\phi \sim 1 \Rightarrow T_h / T_l \to \infty$$

“Ordered” state

- 3. Inelastic light-light collisions + disparate sizes:

$$\alpha_{hh} = \alpha_{hl} = 1, \quad 1 - \alpha_{ll} = \text{finite}, \quad \sigma_i \sim m_i^{1/3}$$

$$\xi_{hh} = 0, \quad \xi_{hl} \to x_l$$

$$\beta_h = \phi_0 = 0, \quad \beta_l \sim \mu^{-1/3}$$

$$\xi_{ll} \to x_l \beta_l, \quad \xi_{lh} \to -x_h \frac{\phi}{\mu}$$

$$\phi \sim \mu^{2/3} \to 0, \quad T_h/T_l \to \mu^{-1/3} \to \infty$$

It is a **normal** state, but the energy is not partitioned

"Mono-energetic H" state

- 4. Inelastic heavy-heavy collisions:

$$\alpha_{hl} = \alpha_{ll} = 1, \quad 1 - \alpha_{hh} = \text{finite}$$

$$\xi_{hh} \to x_l \phi^{1/2} \beta_h, \quad \xi_{hl} \to -x_l \frac{\mu}{\phi}$$

$$\beta_l = \phi_0 = 0, \quad \beta_h \sim \mu^{-1}$$

$$\xi_{ll} = 0, \quad \xi_{lh} \to x_h$$

$$\phi \sim \mu^{4/3} \to 0, \quad T_h/T_l \to \mu^{1/3} \to 0$$

Normal state, but again **strong** breakdown of energy equipartition

"Mono-energetic L" state

- 5. Inelastic light-light collisions + Brownian limit:

$$\alpha_{hh} = \alpha_{hl} = 1, \quad 1 - \alpha_{ll} = \text{finite}, \quad x_h \sim \mu$$

$$\xi_{hh} = 0, \quad \xi_{hl} \to \phi^{1/2}$$

$$\beta_h = \phi_0 = 0, \quad \beta_l \sim \mu^{-1}$$

$$\xi_{ll} \to \beta_l, \quad \xi_{lh} \to -x_h \frac{\phi^{3/2}}{\mu}$$

$$\phi \sim \mu^{-2/3} \to \infty, \quad T_h/T_l \to \mu^{-5/3} \to \infty$$

Ordered state, but with a very strong breakdown of energy equipartition

“Super-ordered” state

Classification of states

$$\mu \equiv \frac{m_l}{m_h}, \qquad \phi \equiv \frac{\langle v_h^2 \rangle}{\langle v_l^2 \rangle} \sim \mu^\eta, \qquad \frac{T_h}{T_l} \sim \mu^{\eta - 1}$$

Class	Subclass	η	$\langle v_h^2\rangle/\langle v_l^2\rangle$	T_h/T_l	Example
Normal	Mono-energetic L	$\eta > 1$	0	0	$\alpha_{hh} < 1$
	Partitioned	$\eta = 1$	0	finite	$1 - \alpha_{hl} \sim m_l/m_h$
	Mono-energetic H	$0 < \eta < 1$	0	∞	$\alpha_{hl} < 1$
Ordered	Ordered	$\eta = 0$	finite	∞	$\alpha_{ll} < 1, \sigma_i \sim m_i^{1/3}$
	Super-ordered	$\eta < 0$	∞	∞	$\alpha_{ll} < 1, x_h \sim m_l/m_h$

Scaling laws

$$1-\alpha_{hh}\sim\mu^{a_1},\quad (1-\alpha_{ll})\sigma_l^2/\sigma_h^2\sim\mu^{a_2},\quad 1-\alpha_{hl}\sim\mu^{b}$$

a_1=0 ï Inelastic heavy-heavy collisions
a_1= ¶ ï Elastic heavy-heavy collisions

b=0 ï Inelastic cross collisions
b= ¶ ï Elastic cross collisions

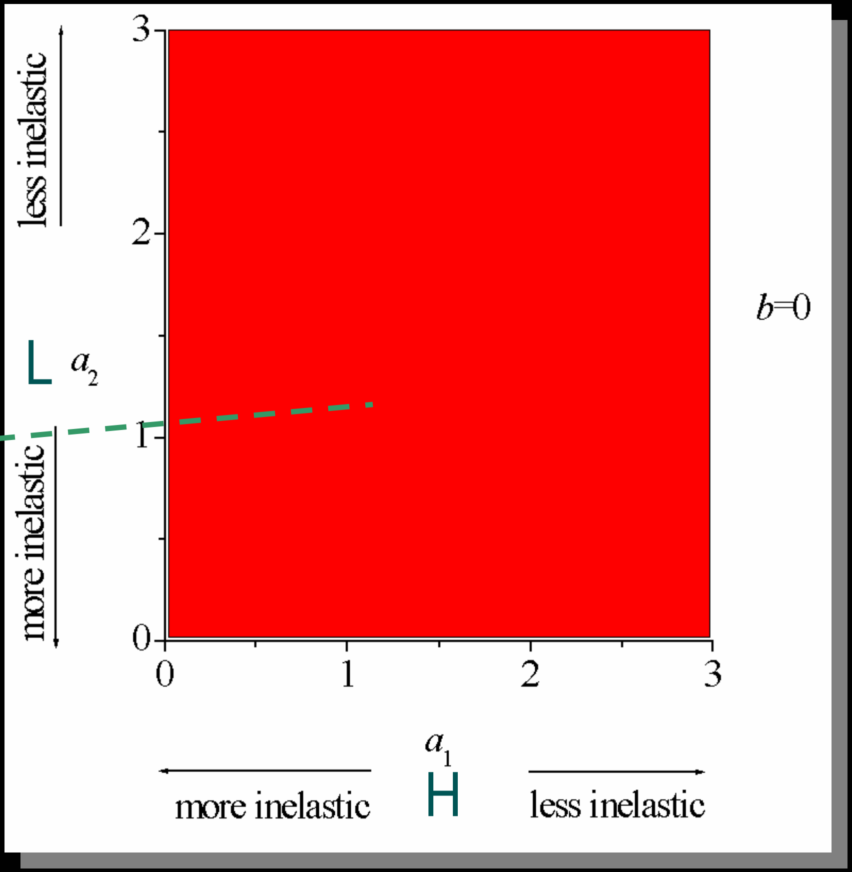
a_2=0 ï Inelastic light-light collisions + comparable sizes
a_2= ¶ ï Elastic light-light collisions

$$\phi\sim\mu^{\eta},\quad \eta\overset{?}{=}\eta(a_1,a_2,b)$$

Phase diagram (finite concentrations)

Inelastic cross collisions $1 - \alpha_{hl} = \text{finite}$

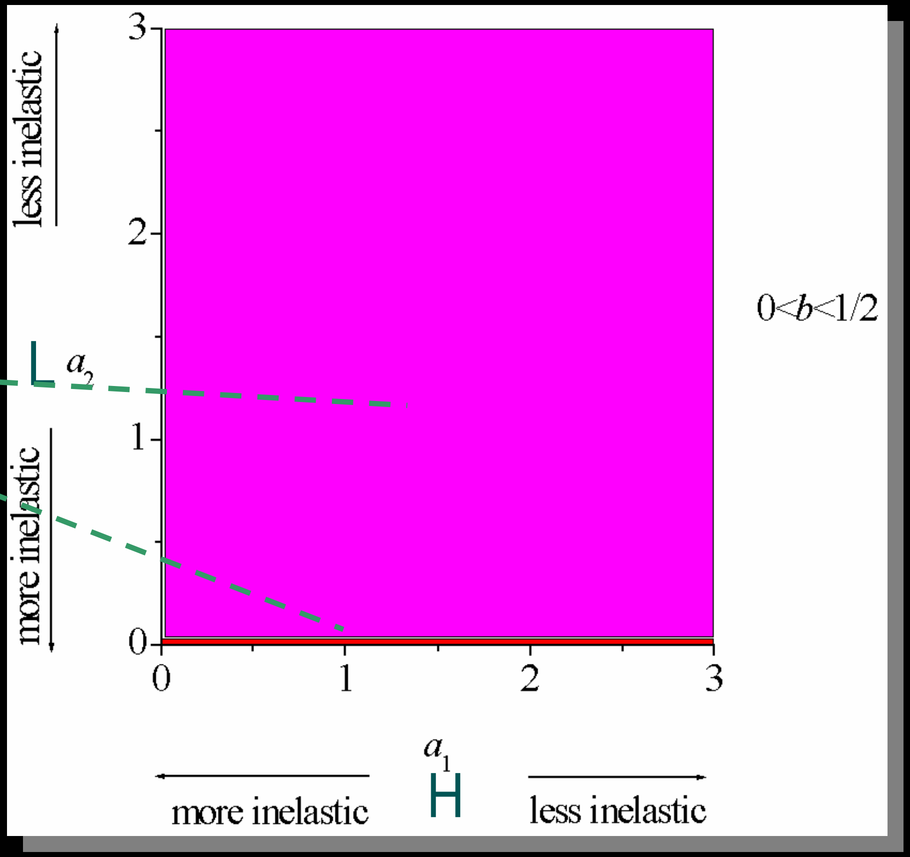
Mono-energetic L	
Partitioned	
Mono-energetic H	
Ordered	
Super-ordered	



Phase diagram (finite concentrations)

Weakly quasi-elastic cross collisions $1 - \alpha_{hl} \sim \mu^b$

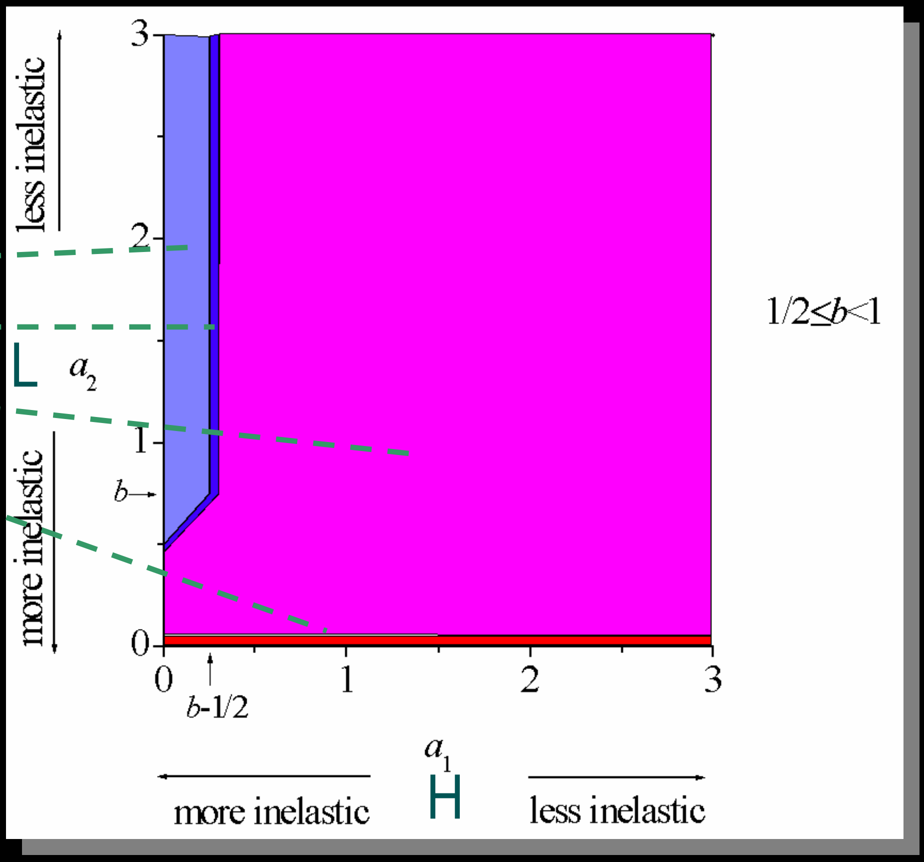
Mono-energetic L	
Partitioned	
Mono-energetic H	
Ordered	
Super-ordered	



Phase diagram (finite concentrations)

Quasi-elastic cross collisions $1 - \alpha_{hl} \sim \mu^b$

Mono-energetic L
Partitioned
Mono-energetic H
Ordered
Super-ordered

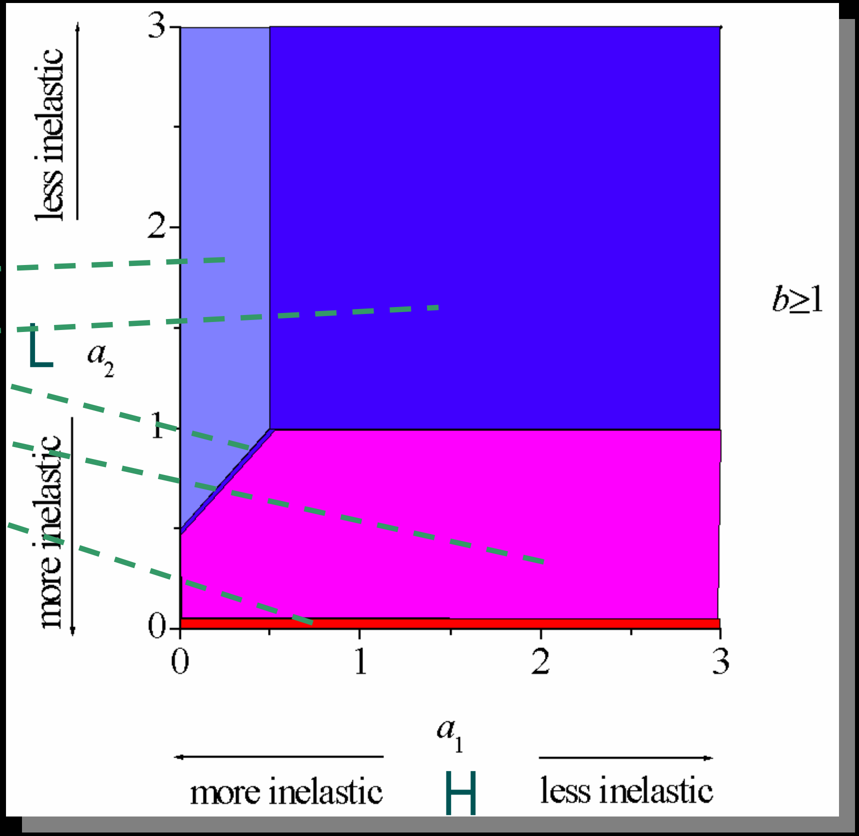


Phase diagram (finite concentrations)

Strongly quasi-elastic cross collisions

$1 - \alpha_{hl} \sim \mu^b$

- Mono-energetic L
- Partitioned
- Mono-energetic H
- Ordered
- Super-ordered



Phase diagram (Brownian limit)

Inelastic cross collisions

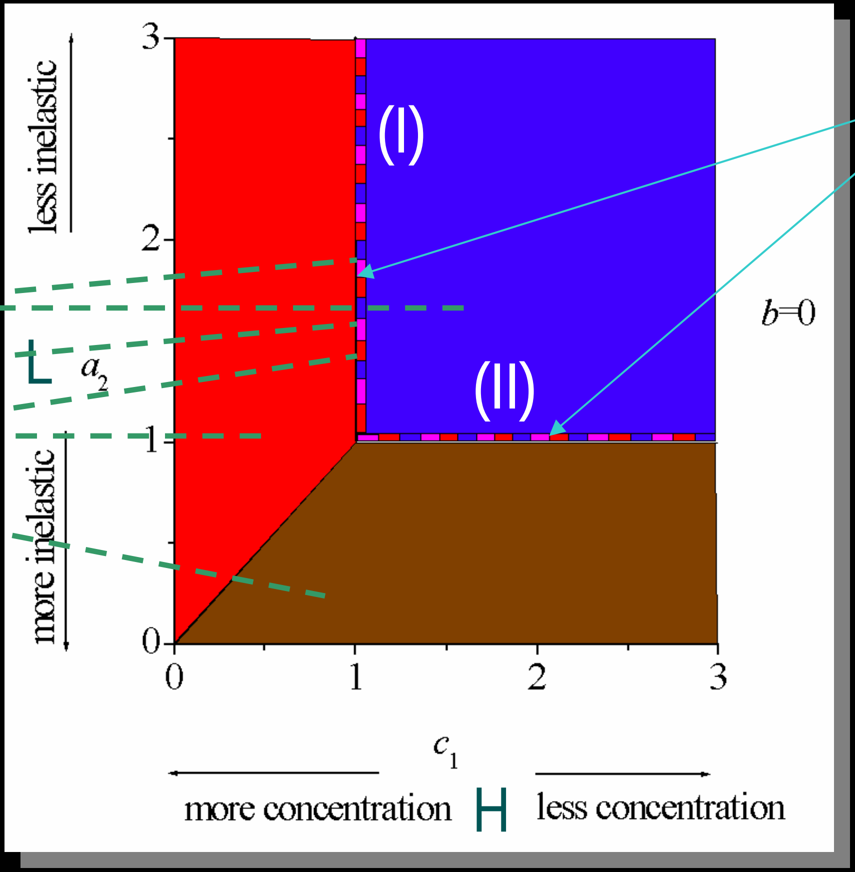
$$1 - \alpha_{hl} = \text{finite}$$

$$1 - \alpha_{hh} \sim \mu^{a_1}$$

$$(1 - \alpha_{ll})\sigma_l^2/\sigma_h^2 \sim \mu^{a_2}$$

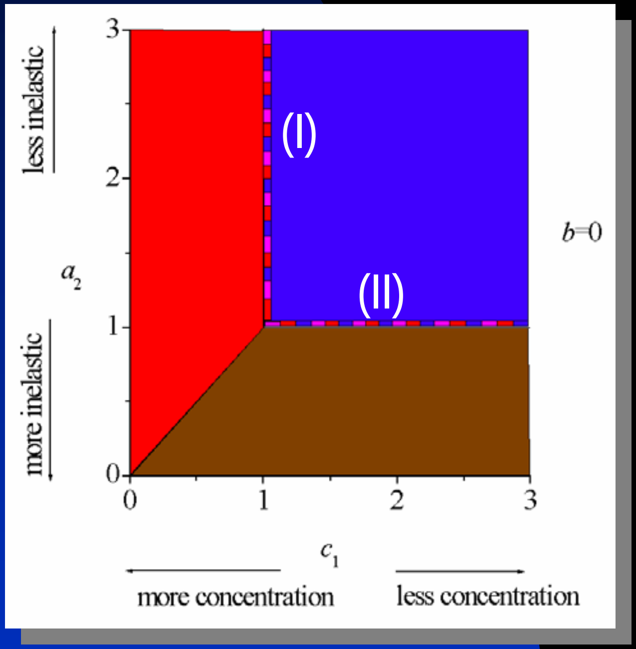
$$\boxed{x_h \sim \mu^{c_1}}$$

Mono-energetic L
Partitioned
Mono-energetic H
Ordered
Super-ordered



Critical lines

Critical lines (Brownian limit)



(I) $x_h \sim \mu, \quad \mu^{-1}(1-\alpha_{ll}^2)\sigma_l^2/\sigma_h^2 \to 0$

$$\frac{x_h}{\mu}\frac{1-\alpha_{hl}^2}{4}\begin{cases} <1 \Rightarrow & \phi \sim \mu \\ =1 \Rightarrow & \phi \sim \mu^{1/2} \\ >1 \Rightarrow & \phi \sim 1 \end{cases}$$

Partitioned

Mono-energetic H

ordered

(II) $x_h/\mu \to 0, \quad 1-\alpha_{ll} \sim \mu$

$$\frac{1-\alpha_{ll}^2}{4\sqrt{2}\mu}\frac{\sigma_l^2}{\sigma_h^2}\begin{cases} <1 \Rightarrow & \phi \sim \mu \\ =1 \Rightarrow & \phi \sim \mu^{1/2} \\ >1 \Rightarrow & \phi \sim 1 \end{cases}$$

Partitioned

Mono-energetic H

ordered

And if the system is heated?

- Gaussian thermostat

$$\partial_t f_i(v) \to \partial_t f_i(v) + \gamma \frac{\partial}{\partial \mathbf{v}} \cdot \mathbf{v} f_i(v)$$

NESS: $\xi_h = \xi_l$ NO CHANGES

- White noise

$$\partial_t f_i(v) \to \partial_t f_i(v) - D \frac{\partial^2}{\partial v^2} f_i(v)$$

NESS: ξ_h f $= \xi_l$ MINOR CHANGES (e.g., No Mono-energetic L state)

Conclusions

❑ Depending on the control parameters (coefficients of restitution, size ratio, and concentrations), the mean square velocity ratio ‚v_h^2Ú/‚v_l^2Ú and the temperature ratio T_h/T_l in a free cooling granular mixture exhibit a rich diversity of scaling behaviors in the disparate-mass limit m_l/m_h ö 0, ranging from the “mono-energetic L” state (T_h/T_l ö 0) to the “super-ordered” state (‚v_h^2Ú/‚v_l^2Ú ö ¶).

❑ If the cross collisions are **inelastic** ($a_{hl}<1$), the state is always “ordered” (‚v_h^2Ú/‚v_l^2Ú~1). As a consequence, in this case there is neither Brownian dynamics (when x_h ö 0) nor Lorentz gas (when x_l ö 0).

Conclusions

- A "partitioned" state (T_h/T_l~1) is only possible if the three types of collisions are sufficiently quasi-elastic.

- A "super-ordered" state ($\langle v_h^2\rangle/\langle v_l^2\rangle \to \infty$) is only possible in the Brownian limit (when $x_h \to 0$). There is no "mono-energetic L" state in that case.

- In the Brownian limit, there exist critical lines in the phase diagram where the state can be partitioned, ordered, or mono-energetic H.

- The same scenario as for free cooling mixtures holds essentially in the heated case, except that the mono-energetic L state disappears.

THANKS!