Energy production rates in two-dimensional granular gases: The interplay between polydispersity, inelasticity, and roughness

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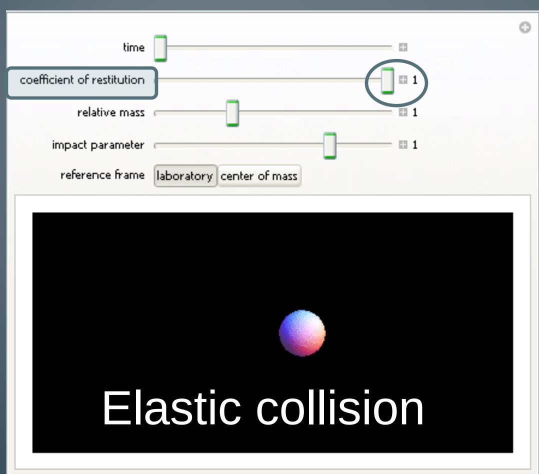
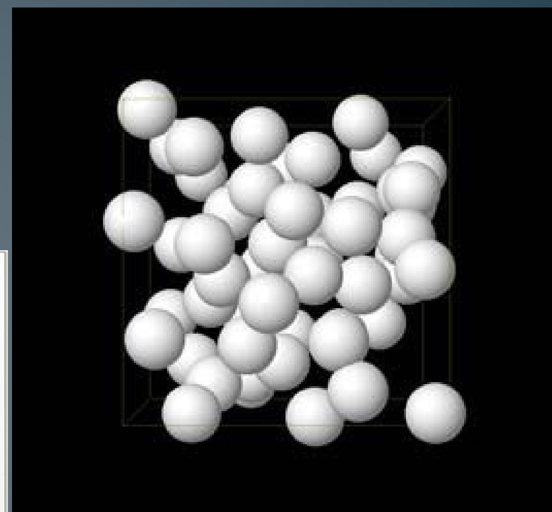
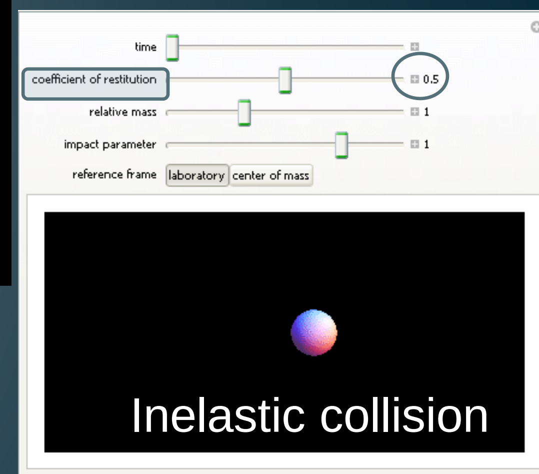
What is a granular gas?

- It is a conglomeration of discrete solid, macroscopic particles characterized by a loss of energy whenever the grains collide.
- If the granular material is driven or shaken, such that contacts between the grains become highly infrequent, the material enters a gaseous state.

Minimal model of a granular gas:
A gas of *identical smooth inelastic* hard spheres



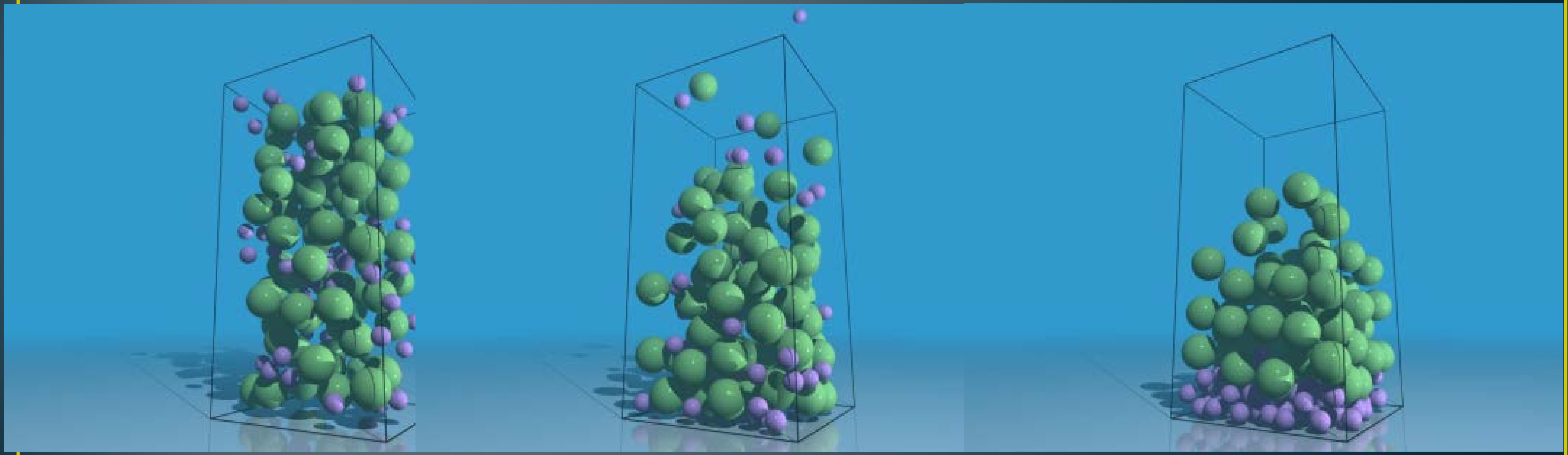




http://demonstrations.wolfram.com/InelasticCollisionsOfTwoSpheres/

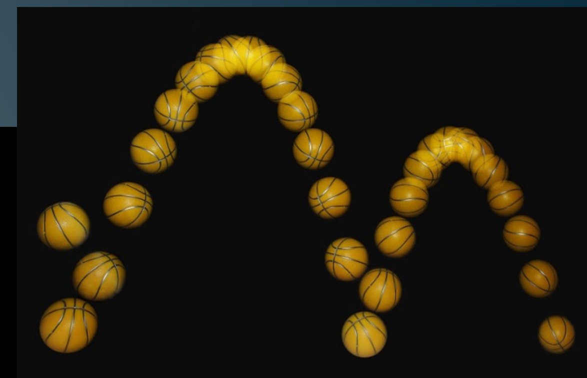
This minimal model ignores …

Polydispersity



http://www.cmt.york.ac.uk/~ajm143/nuts.html

and roughness



Simple model of a granular gas: A *collection* of *inelastic* **rough** hard particles

This model unveils an inherent ***breakdown*** of energy equipartition in granular fluids, even in homogeneous and isotropic states

Several circles
(Kandinsky, 1926)

Galatea of the Spheres
(Dalí, 1952)

3Tr+3Rot

2Tr+3Rot

https://www.shutterstock.com/video/

2Tr+1Rot

A motivation

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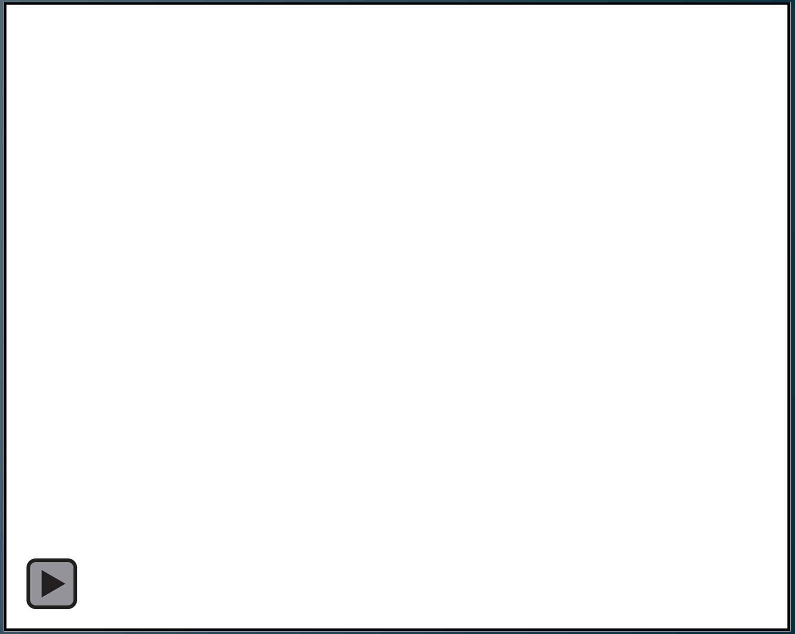
Velocity Distribution of a Homogeneously Driven Two-Dimensional Granular Gas

Christian Scholz and Thorsten Pöschel

Vibrot



Material parameters:

- Masses m_i
- Diameters σ_i
- Moments of inertia I_i
- Coefficients of normal restitution α_{ij}
- Coefficients of tangential restitution β_{ij}
- $\alpha_{ij}=1$ for perfectly elastic particles
- $\beta_{ij}=-1$ for perfectly smooth particles
- $\beta_{ij}=+1$ for perfectly rough particles

Collision rules

Cons. linear momentum:

$$m_i \mathbf{v}_i' + m_j \mathbf{v}_j' = m_i \mathbf{v}_i + m_j \mathbf{v}_j$$

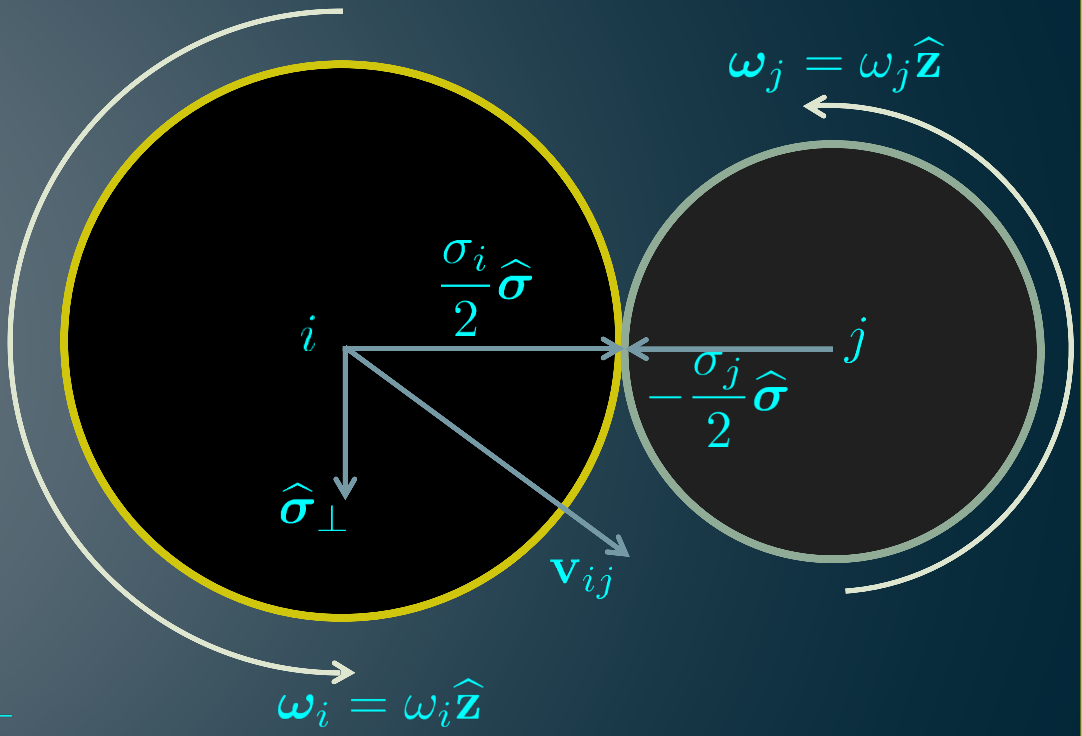
Cons. angular momentum:

$$I_{\substack{i\\j}} \omega_{\substack{i\\j}}' \pm \frac{1}{2} m_{\substack{i\\j}} \sigma_{\substack{i\\j}} \left(\mathbf{v}_{\substack{i\\j}}' \cdot \widehat{\boldsymbol{\sigma}}_\perp \right)$$

$$= I_{\substack{i\\j}} \omega_{\substack{i\\j}} \pm \frac{1}{2} m_{\substack{i\\j}} \sigma_{\substack{i\\j}} \left(\mathbf{v}_{\substack{i\\j}} \cdot \widehat{\boldsymbol{\sigma}}_\perp \right)$$

Relative velocity of the points of the spheres at contact:

$$\mathbf{w}_{ij} = \mathbf{v}_{ij} - \left(\frac{\sigma_i}{2} \omega_i + \frac{\sigma_j}{2} \omega_j \right) \widehat{\boldsymbol{\sigma}}_\perp$$



$$\mathbf{w}_{ij}' \cdot \widehat{\boldsymbol{\sigma}} = -\alpha_{ij} \mathbf{w}_{ij} \cdot \widehat{\boldsymbol{\sigma}}, \quad \mathbf{w}_{ij}' \cdot \widehat{\boldsymbol{\sigma}}_\perp = -\beta_{ij} \mathbf{w}_{ij} \cdot \widehat{\boldsymbol{\sigma}}_\perp$$

Energy collisional loss

$$E_{ij} = \frac{1}{2}m_i v_i^2 + \frac{1}{2}m_j v_j^2 + \frac{1}{2}I_i\omega_i^2 + \frac{1}{2}I_j\omega_j^2$$

$$\begin{aligned} E'_{ij} - E_{ij} &= -(1-\alpha_{ij}^2)\times\cdots \\ &\quad -(1-\beta_{ij}^2)\times\cdots \end{aligned}$$

Energy is conserved *only* if the spheres are

- elastic (α_{ij}=1) and
- either
  - perfectly smooth (β_{ij}=-1) or
  - perfectly rough (β_{ij}=+1)

Partial (granular) temperatures

Translational temperatures: $T_i^{\mathrm{tr}} = \frac{m_i}{2}\langle(\mathbf{v}_i - \mathbf{u})^2\rangle$

Rotational temperatures: $T_i^{\mathrm{rot}} = I_i\langle\omega_i^2\rangle$

Total temperature: $T = \sum_i \frac{n_i}{3n}\left(2T_i^{\mathrm{tr}} + T_i^{\mathrm{rot}}\right)$

Collisional rates of change for temperatures

Energy production rates:

$$\xi_i^{\rm tr} = -\frac{1}{T_i^{\rm tr}}\left(\frac{\partial T_i^{\rm tr}}{\partial t}\right)_{\rm coll}, \quad \xi_i^{\rm tr} = \sum_j \xi_{ij}^{\rm tr}$$

Binary collisions

$$\xi_i^{\rm rot} = -\frac{1}{T_i^{\rm rot}}\left(\frac{\partial T_i^{\rm rot}}{\partial t}\right)_{\rm coll}, \quad \xi_i^{\rm rot} = \sum_j \xi_{ij}^{\rm rot}$$

Net *cooling* rate:

$$\zeta = -\frac{1}{T}\left(\frac{\partial T}{\partial t}\right)_{\rm coll}, \quad \zeta = \sum_i \frac{n_i}{3nT}\left(2\xi_i^{\rm tr}T_i^{\rm tr} + \xi_i^{\rm rot}T_i^{\rm rot}\right)$$

Methodology: Kinetic Theory

Ansatz on the precollisional two-body velocity distribution function:

$$f_{ij}(\mathbf{v}_i,\omega_i;\mathbf{v}_j,\omega_j) \;\to\; n_i n_j \frac{m_i m_j}{4\pi^2 T_i^{\rm tr} T_j^{\rm tr}} e^{-m_i \frac{(\mathbf{v}_i-\mathbf{u})^2}{2T_i^{\rm tr}} - m_j \frac{(\mathbf{v}_j-\mathbf{u})^2}{2T_j^{\rm tr}}} \times f_i^{\rm rot}(\omega_i) f_j^{\rm rot}(\omega_j)$$

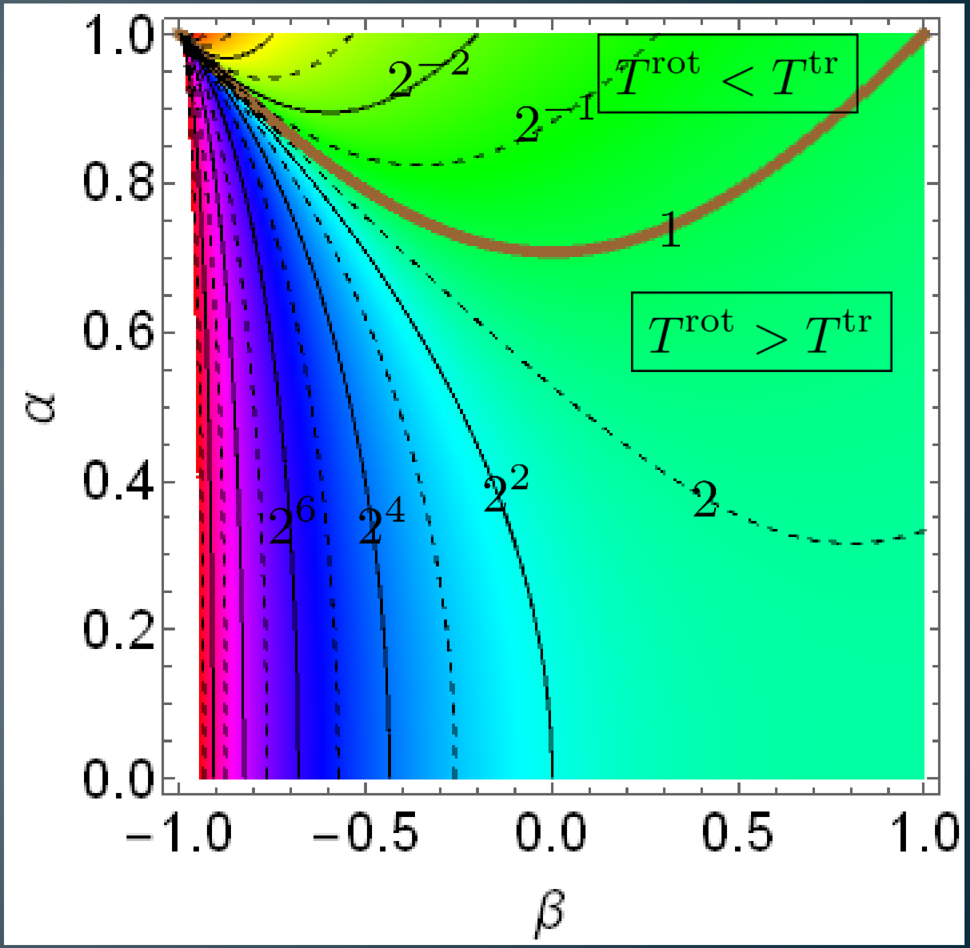
Molecular chaos+Maxwellian approx. for translational distribution

Results

$$\xi_{ij}^{\mathrm{tr}} \;\&\; \xi_{ij}^{\mathrm{rot}} = \text{functions of} \begin{cases} m_i, m_j, I_i, I_j, \sigma_i, \sigma_j, \\ \alpha_{ij}, \beta_{ij}, \\ n_i, \quad n_j, \\ \boxed{T_i^{\mathrm{tr}}, T_j^{\mathrm{tr}}, T_i^{\mathrm{rot}}, T_j^{\mathrm{rot}}} \end{cases}$$

Application to the Homogeneous Cooling State. *Monodisperse* system

$$\frac{T^{\mathrm{rot}}}{T^{\mathrm{tr}}}$$



Application to the Homogeneous Cooling State. *Bidisperse* system

$$\alpha_{ij} = \alpha$$

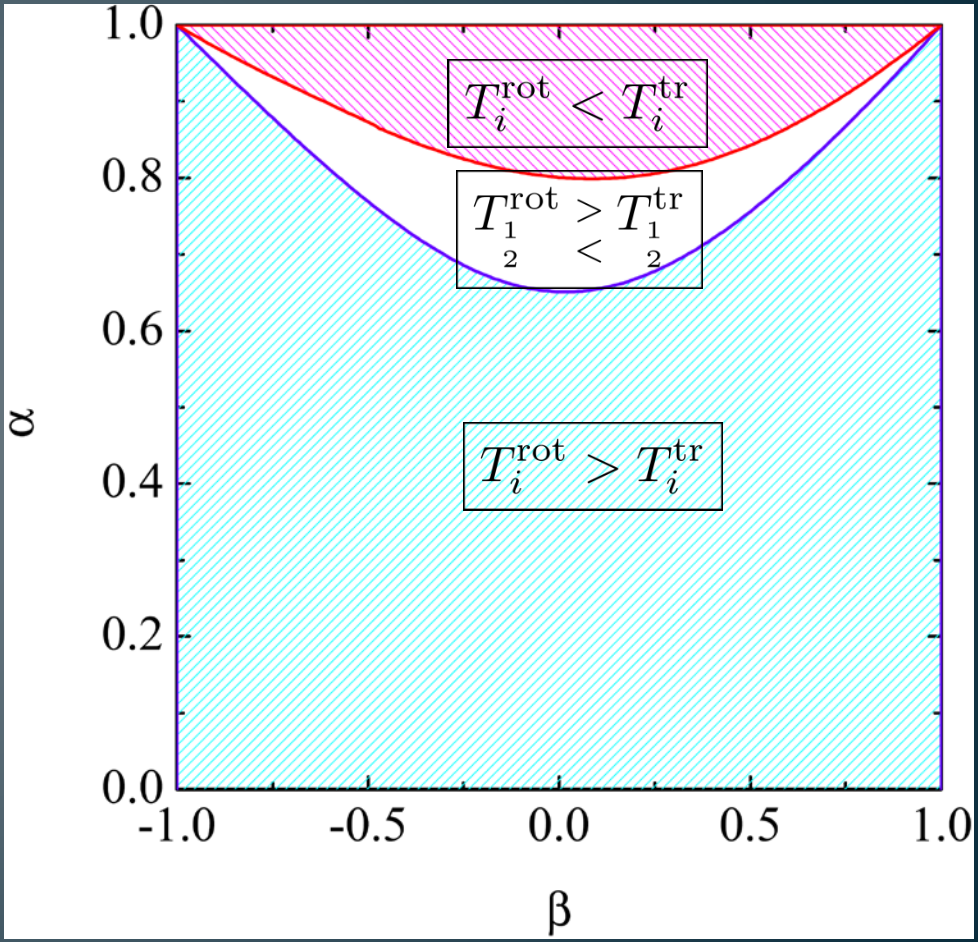
$$\beta_{ij} = \beta$$

$$\frac{n_1}{n_2} = 1$$

$$\frac{\sigma_1}{\sigma_2} = 2$$

$$\boxed{\frac{m_1}{m_2} = 4}$$

$$T_1^{\text{tr}} > T_2^{\text{tr}}$$



Application to the Homogeneous Cooling State. *Bidisperse* system

$\alpha_{ij} = \alpha$

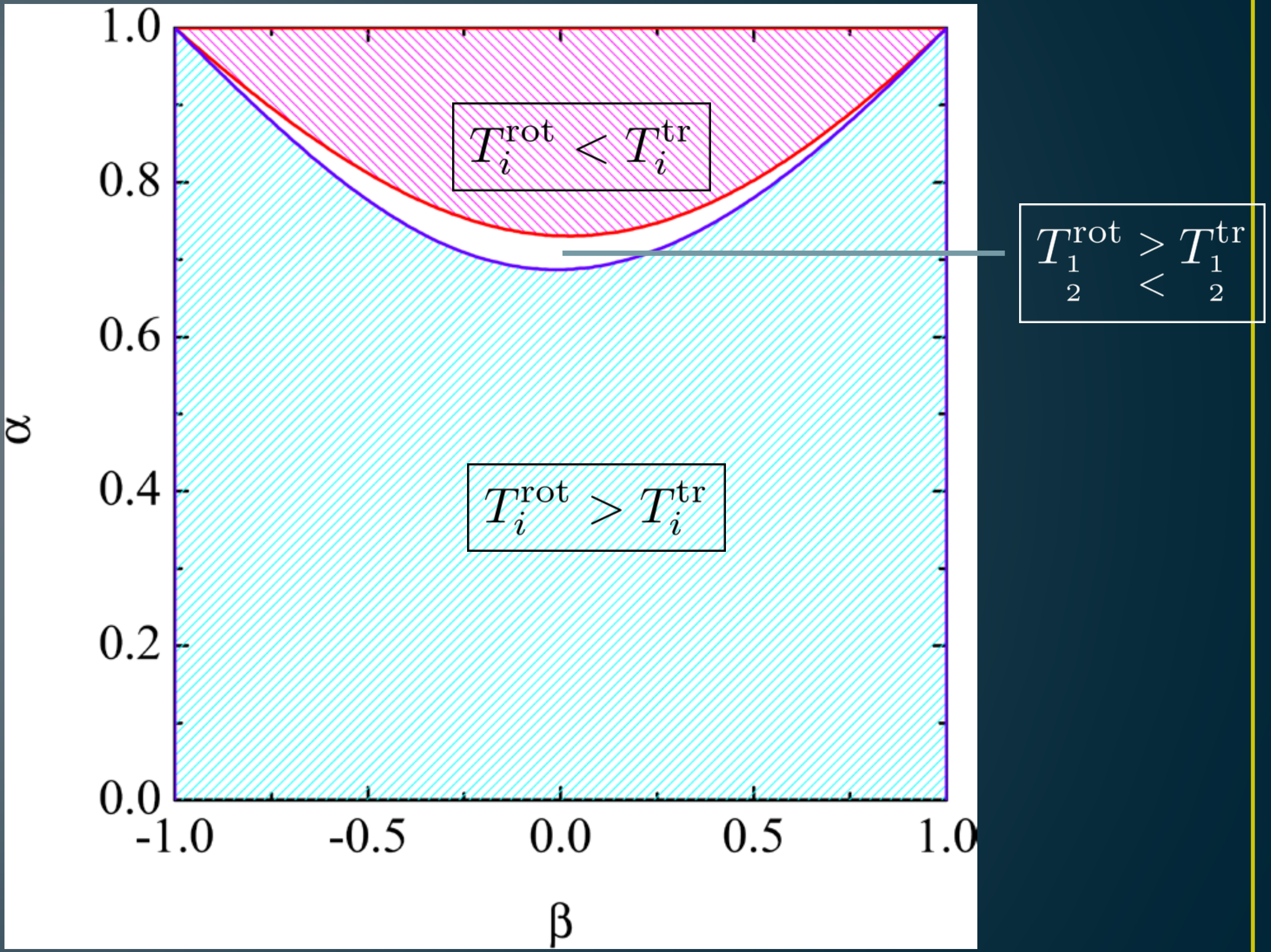
$\beta_{ij} = \beta$

$\frac{n_1}{n_2} = 1$

$\frac{\sigma_1}{\sigma_2} = 2$

$\boxed{\frac{m_1}{m_2} = 2}$

$T_1^{\text{tr}} \gtrsim T_2^{\text{tr}}$



Application to the Homogeneous Cooling State. *Bidisperse* system

$$\alpha_{ij} = \alpha$$

$$\beta_{ij} = \beta$$

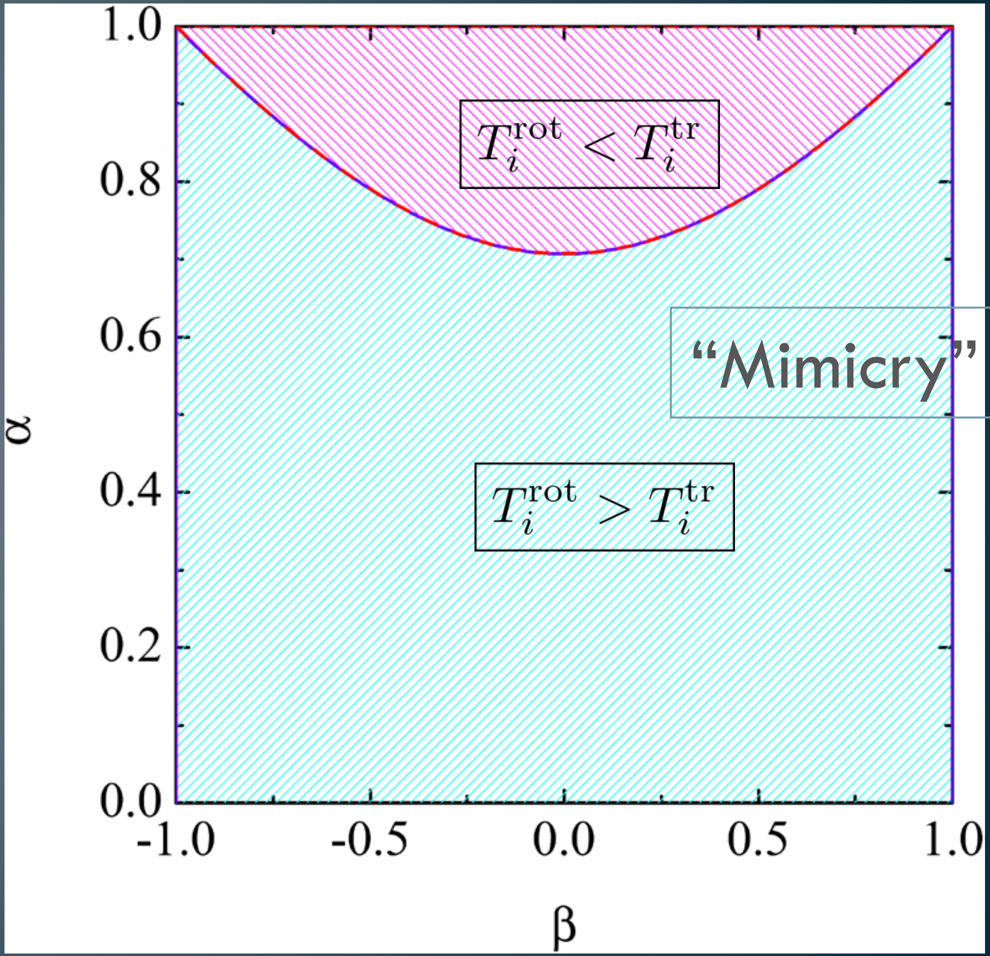
$$\frac{n_1}{n_2} = 1$$

$$\frac{\sigma_1}{\sigma_2} = 2$$

$$\boxed{\frac{m_1}{m_2} = 1.565}$$

$$T_1^{\mathrm{tr}} = T_2^{\mathrm{tr}}$$

$$T_1^{\mathrm{rot}} = T_2^{\mathrm{rot}}$$



"Mimicry" effect

Application to the Homogeneous Cooling State. *Bidisperse* system

$$\alpha_{ij} = \alpha$$

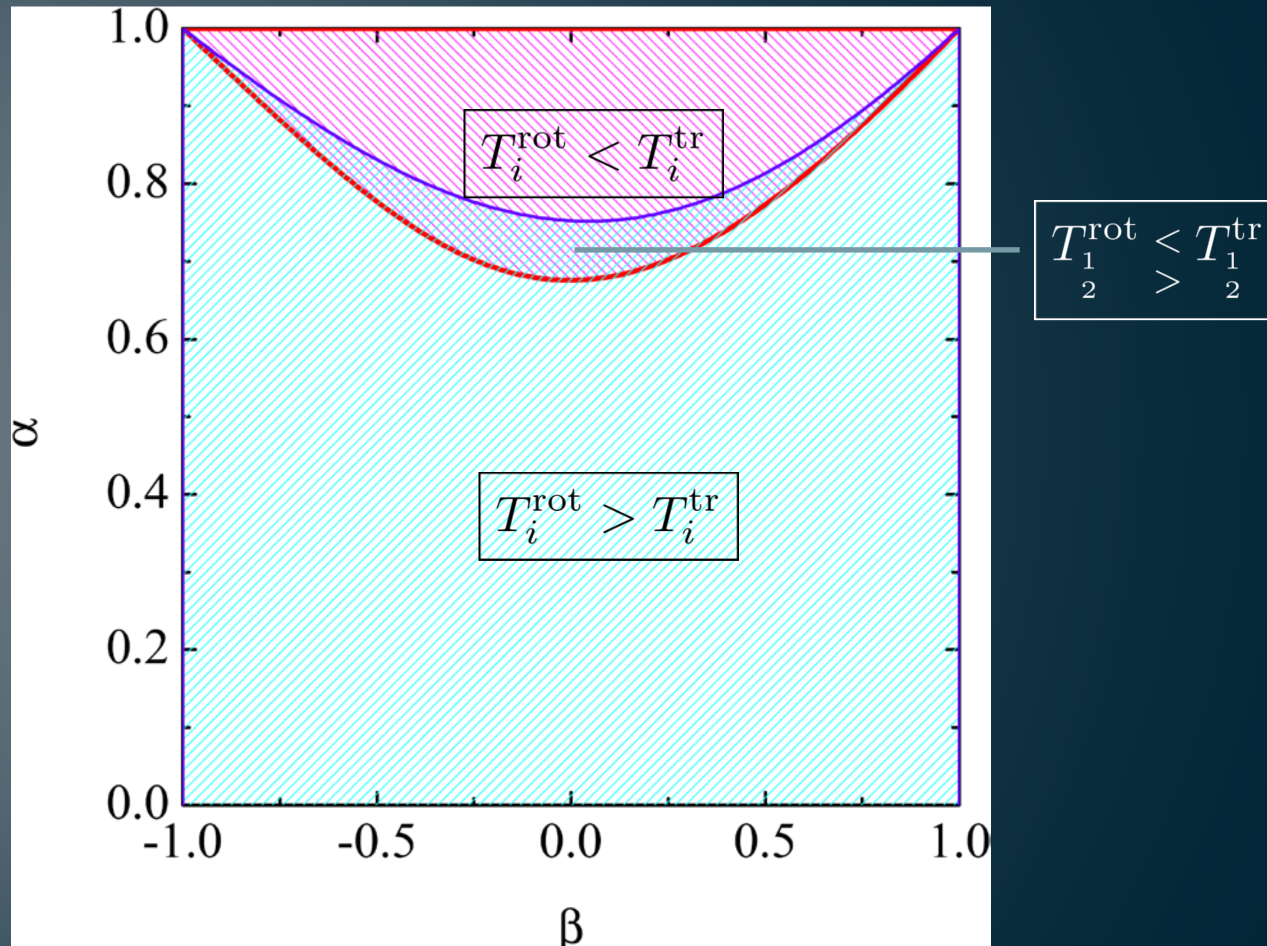
$$\beta_{ij} = \beta$$

$$\frac{n_1}{n_2} = 1$$

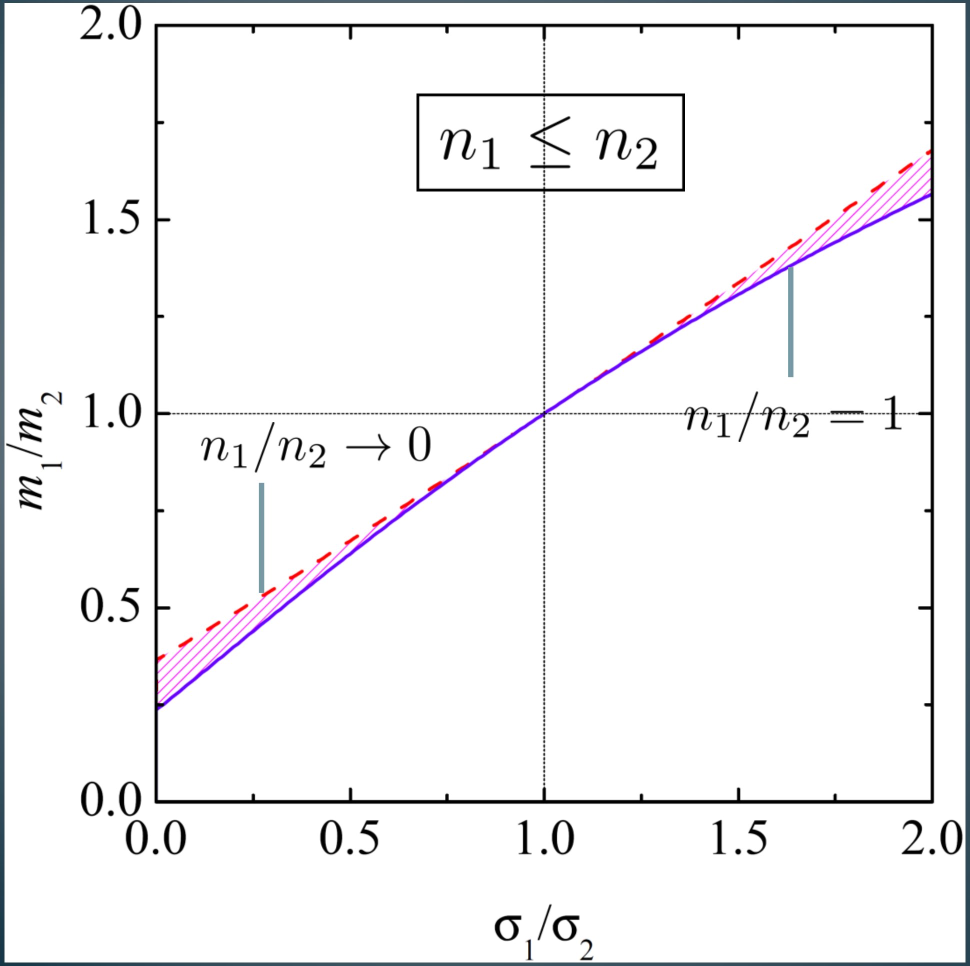
$$\frac{\sigma_1}{\sigma_2} = 2$$

$$\boxed{\frac{m_1}{m_2} = 1}$$

$$T_1^{\rm tr} < T_2^{\rm tr}$$



Conditions for *mimicry*



The smaller spheres must have a larger solid density than the biggest spheres

$$\frac{m_1}{m_2} \approx \frac{1}{3}\left(1 + 2\frac{\sigma_1}{\sigma_2}\right)$$

Conclusions and outlook

- A very rich interplay between polydispersity, inelasticity, and roughness exists.
- An *intruder* component can “disguise” as the host monocomponent gas (from the point of view of mean kinetic energies): *Mimicry* effect.
- Extensions to steady states (e.g., white-noise thermostat) are straightforward.
- Comparison with computer simulations are planned.
- Next step for monocomponent gases: Derivation of the Navier-Stokes hydrodynamic equations.


THANK
YOU!


What people think about during your conference talk
Hey! That's what I'm working on!
Hee hee! Animated arrows!
Typo on slide 14, line 3, centre-left. Noted.
(RANDOM HAPPY THOUGHTS]
I'm up next I'm up next I'm up next
I'm in the wrong session.
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