Homogeneous states in a gas of *inelastic* and *rough* hard spheres: The undriven and driven cases

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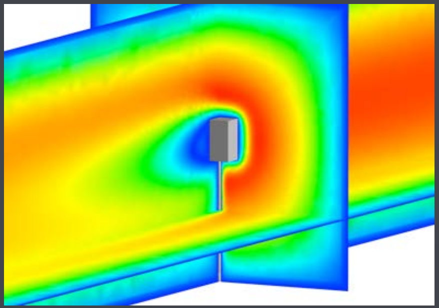
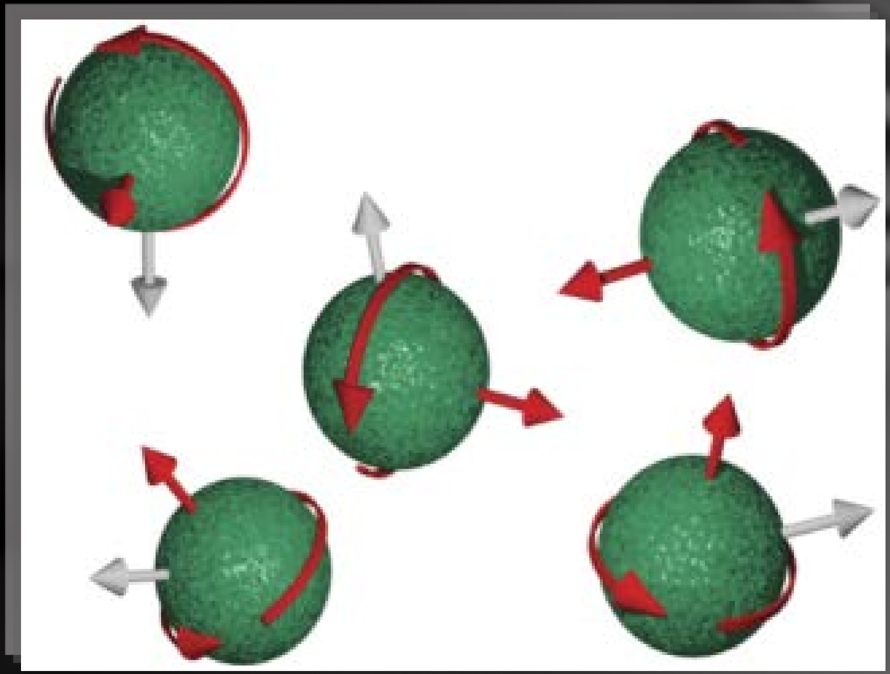


Simple model of a granular gas:
A *collection* of *inelastic* and *rough* hard spheres

Material parameters:

- Mass m
- Diameter σ
- Moment of inertia I $(\kappa=4I/m\sigma^2)$
- Coefficient of normal restitution α
- Coefficient of tangential restitution β
- $\alpha=1$ for perfectly elastic particles
- $\beta=-1$ for perfectly smooth particles
- $\beta=+1$ for perfectly rough particles



This model unveils the inherent breakdown of equilibrium and energy equipartition in granular fluids, even in *homogeneous* and isotropic states

Collision rules

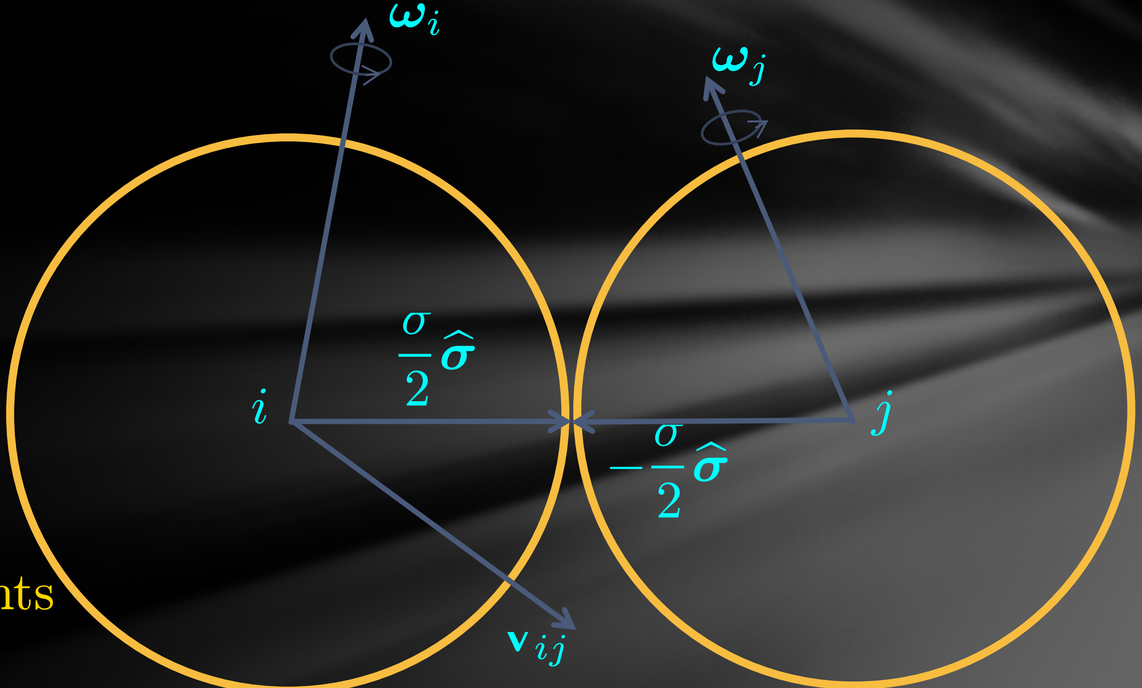
Cons. linear momentum:

$$\mathbf{v}_i' + \mathbf{v}_j' = \mathbf{v}_i + \mathbf{v}_j$$

Cons. angular momentum:

$$I\boldsymbol{\omega}_{i,j}' \mp m\frac{\sigma_i}{2}\widehat{\boldsymbol{\sigma}} \times \mathbf{v}_{i,j}'$$

$$= I\boldsymbol{\omega}_{i,j} \mp m\frac{\sigma_i}{2}\widehat{\boldsymbol{\sigma}} \times \mathbf{v}_{i,j}$$



Relative velocity of the points of the spheres at contact:

$$\overline{\mathbf{v}}_{ij} = \mathbf{v}_{ij} - \frac{\sigma}{2}\widehat{\boldsymbol{\sigma}} \times (\boldsymbol{\omega}_i + \boldsymbol{\omega}_j)$$

$$\boxed{\widehat{\boldsymbol{\sigma}} \cdot \overline{\mathbf{v}}_{ij}' = -\alpha\widehat{\boldsymbol{\sigma}} \cdot \overline{\mathbf{v}}_{ij}, \quad \widehat{\boldsymbol{\sigma}} \times \overline{\mathbf{v}}_{ij}' = -\beta\widehat{\boldsymbol{\sigma}} \times \overline{\mathbf{v}}_{ij}}$$

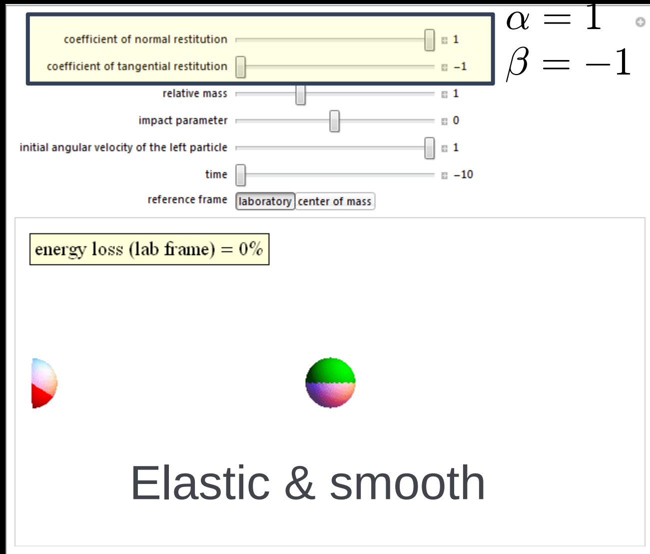
Energy collisional loss

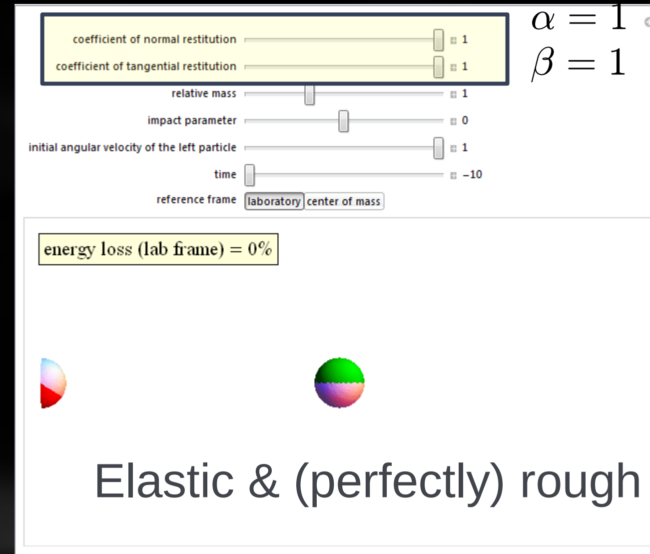
$$E_{ij} = \frac{1}{2}mv_i^2 + \frac{1}{2}mv_j^2 + \frac{1}{2}I\omega_i^2 + \frac{1}{2}I\omega_j^2$$

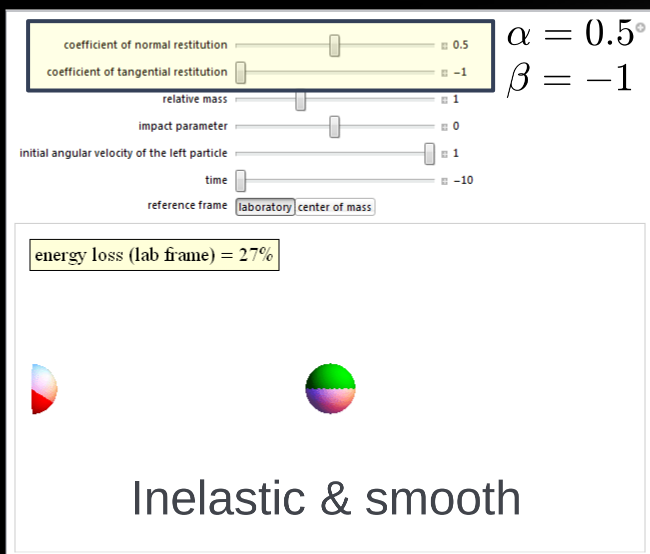
$$\begin{aligned} E'_{ij} - E_{ij} &= -(1-\alpha^2)\times\cdots \\ & \quad -(1-\beta^2)\times\cdots \end{aligned}$$

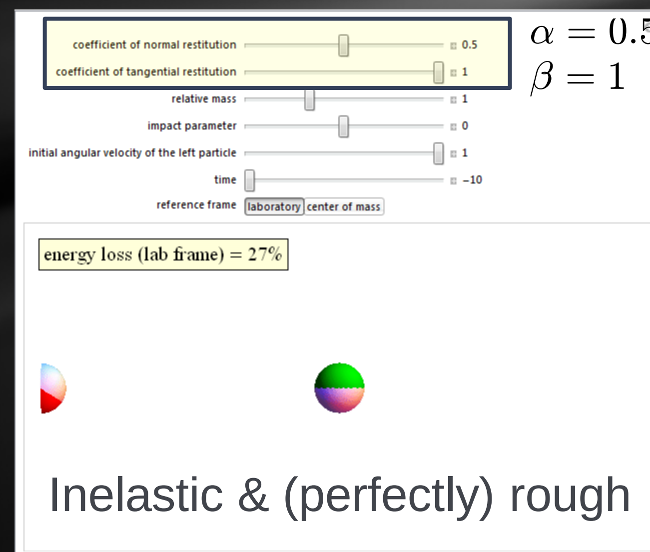
Energy is conserved *only* if the spheres are

- elastic (α=1) and
- either
  - perfectly smooth (β=-1) or
  - perfectly rough (β=+1)







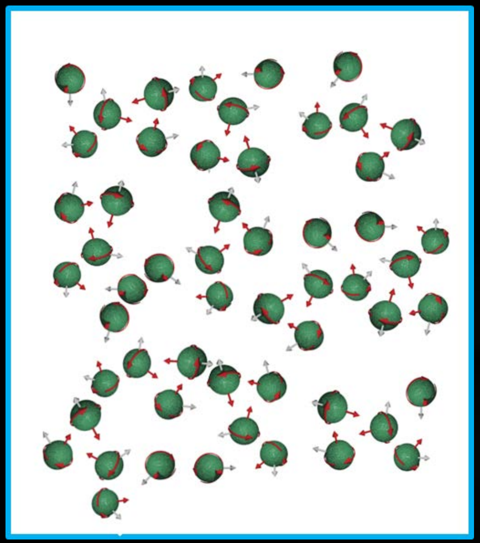


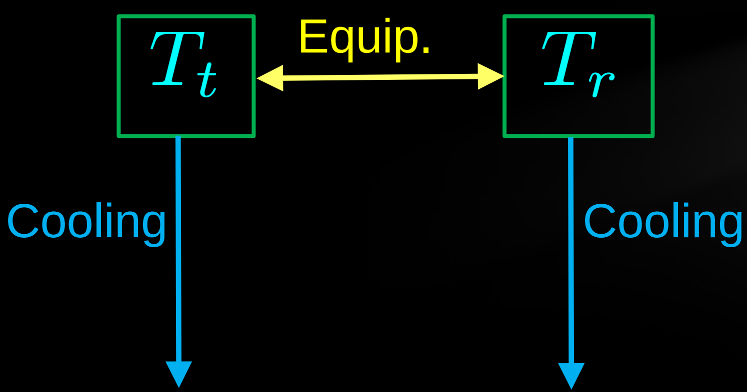
http://demonstrations.wolfram.com/InelasticCollisionsOfTwoRoughSpheres/

Aim of the work

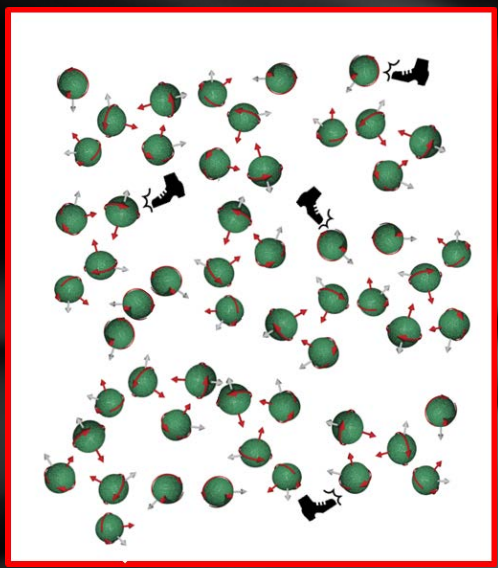
1. Consider a *homogeneous* and *isotropic* granular gas.
2. Measure the basic *nonequilibrium* features: energy nonequipartition and velocity cumulants.
3. Compare the *undriven* (cooling) and *driven* (thermostatted) cases.

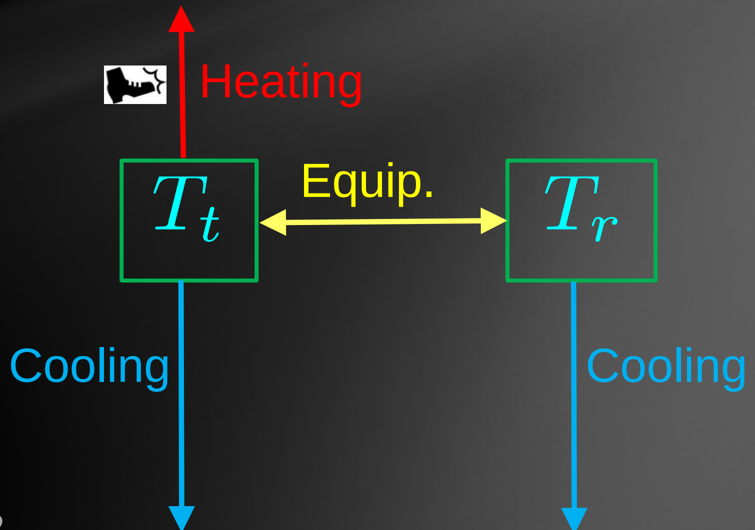
Undriven
(Homogeneous cooling state, HCS)



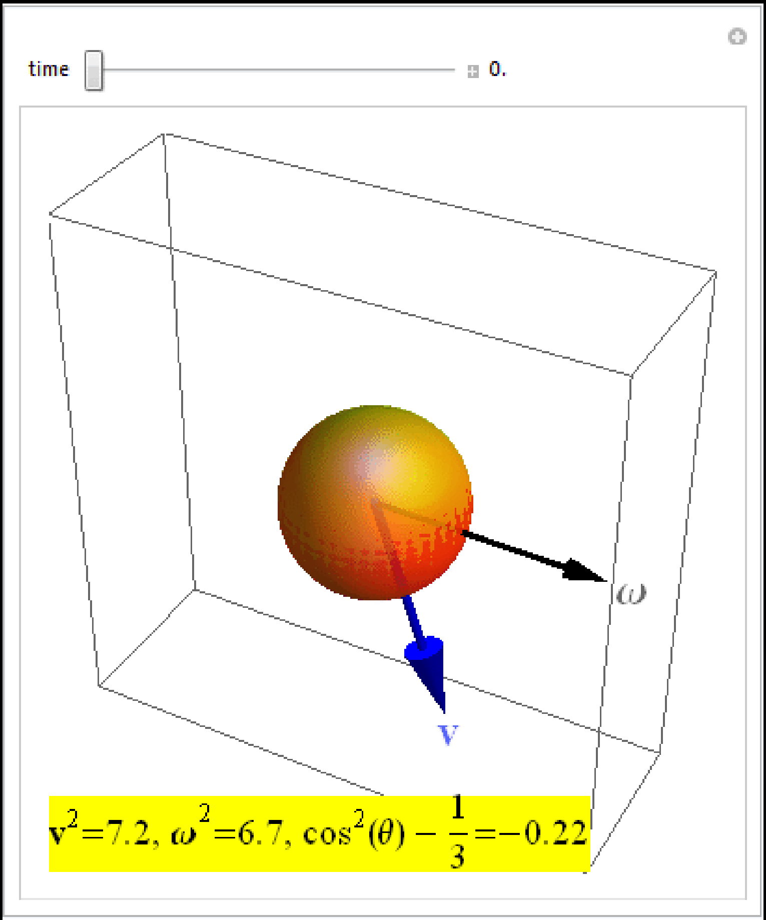


Driven
(White-noise thermostat, WNT)





Granular temperatures and velocity cumulants



translational temperature: $\langle v^2\rangle = \dfrac{3T_t}{m}$

rotational temperature: $\langle \omega^2\rangle = \dfrac{3T_r}{I}$

temperature ratio: $\theta = T_r/T_t$

translational kurtosis: $\langle v^4\rangle = \dfrac{5}{3}\langle v^2\rangle^2\left(1+a_{20}^{(0)}\right)$

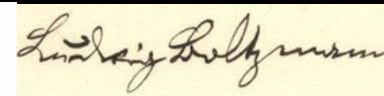
rotational kurtosis: $\langle \omega^4\rangle = \dfrac{5}{3}\langle \omega^2\rangle^2\left(1+a_{02}^{(0)}\right)$

scalar correlations: $\langle v^2\omega^2\rangle = \langle v^2\rangle\langle \omega^2\rangle\left(1+a_{11}^{(0)}\right)$

angular correlations: $\langle (\mathbf{v}\cdot\boldsymbol{\omega})^2\rangle - \dfrac{1}{3}\langle v^2\omega^2\rangle \propto a_{00}^{(1)}$

Boltzmann equation:

$$\partial_t f(\mathbf{v},\boldsymbol{\omega},t) - \underbrace{\frac{\chi_0^2}{2}\left(\frac{\partial}{\partial\mathbf{v}}\right)^2 f(\mathbf{v},\boldsymbol{\omega},t)}_{\text{External driving}} = \underbrace{J[\mathbf{v},\boldsymbol{\omega},t|f]}_{\text{Inelastic+Rough collisions}}$$



(1844-1906)

(Cartoon by Bernhard Reischl, University of Vienna)

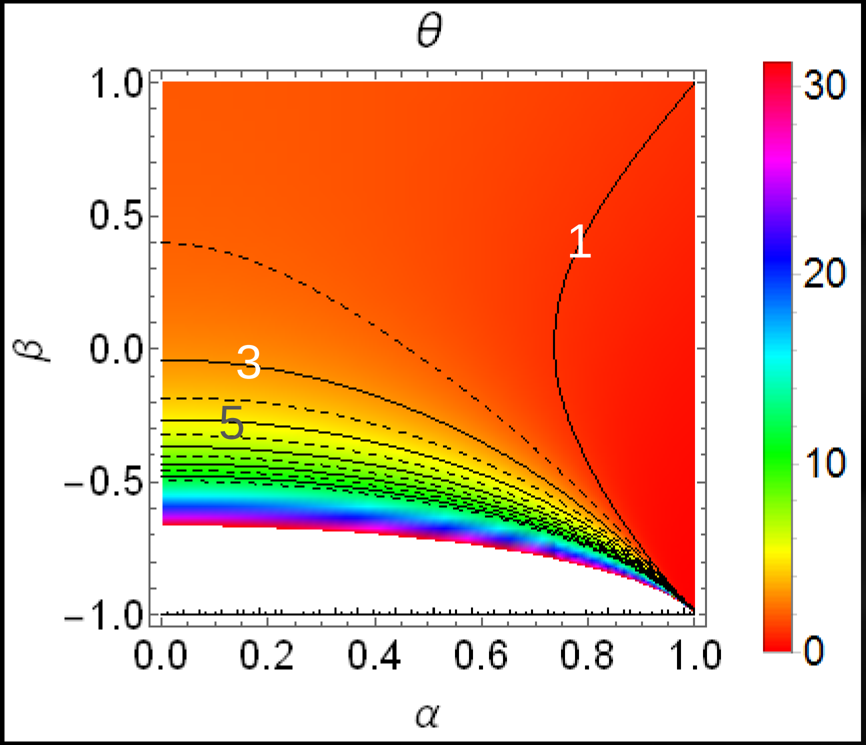
- $\chi_0^2 = 0 \Rightarrow$ Homogeneous cooling state [Phys. Rev. E **89**, 020202(R) (2014)]
- $\chi_0^2 \neq 0 \Rightarrow$ White-noise thermostat [Phys. Fluids **27**, 113301 (2015)]

Tools:

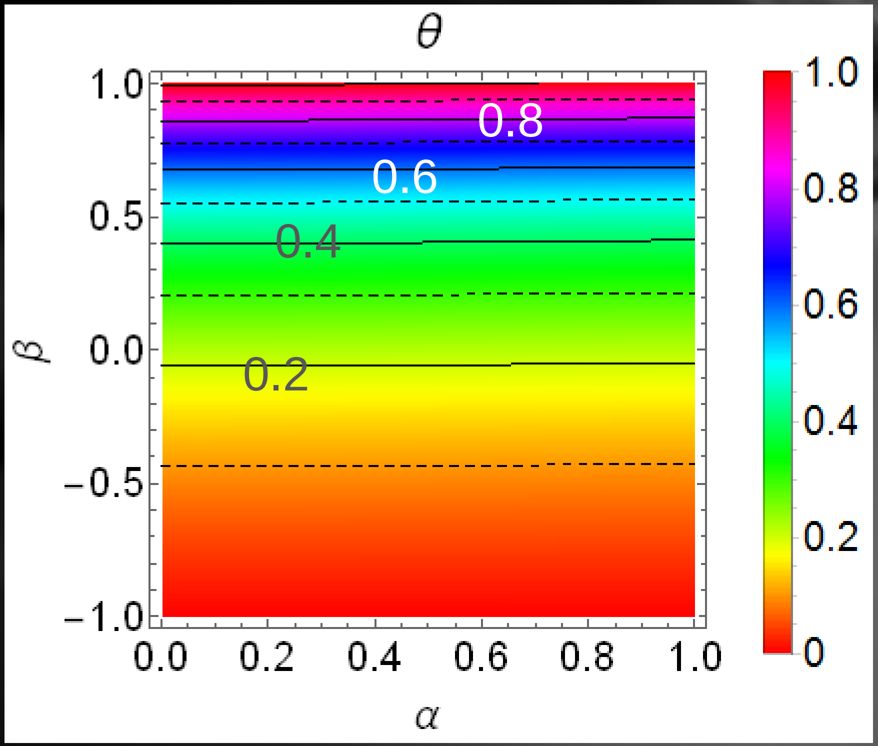
- Theory: Truncated (Sonine) polynomial expansion
- Simulation: DSMC

temperature ratio: $\theta = T_r/T_t$

Undriven



Driven

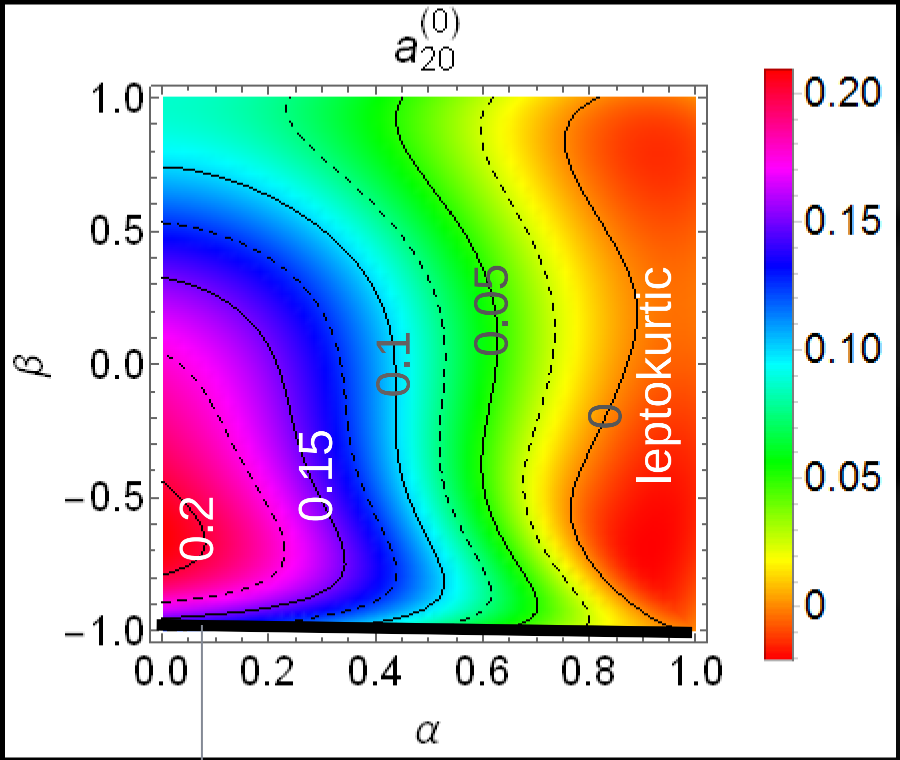


Typically, $T_r > T_t$
$\lim_{\beta\to-1} T_r/T_t \to \infty$

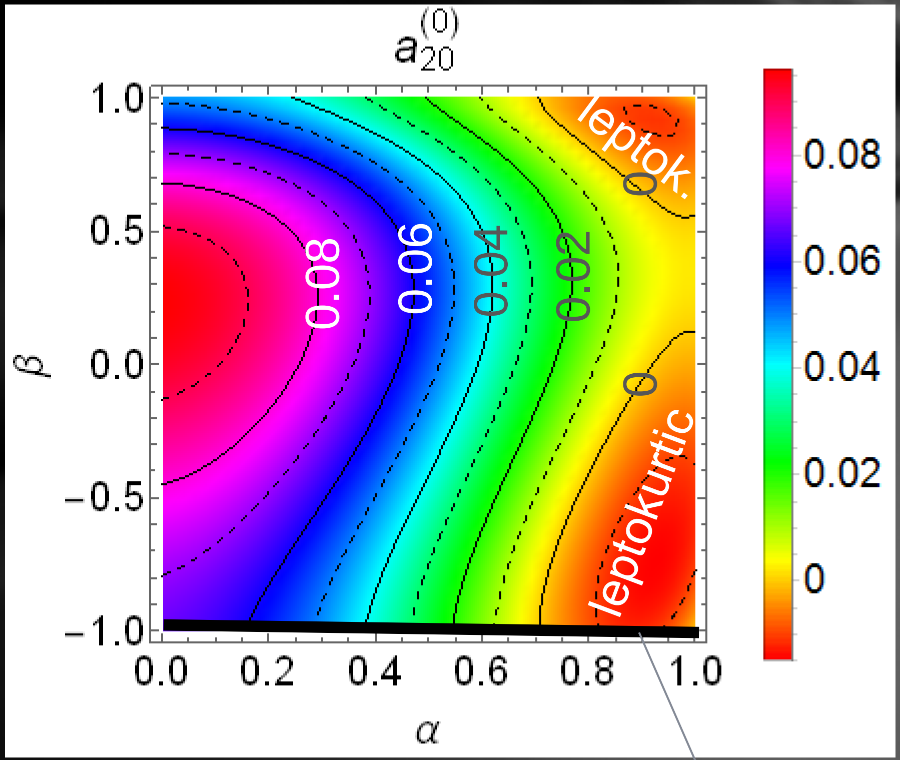
Typically, $T_r < T_t$
$\lim_{\beta\to-1} T_r/T_t \to 0$

translational kurtosis: $\langle v^4 \rangle = \frac{5}{3}\langle v^2 \rangle^2 \left(1 + a_{20}^{(0)}\right)$

Undriven



Driven

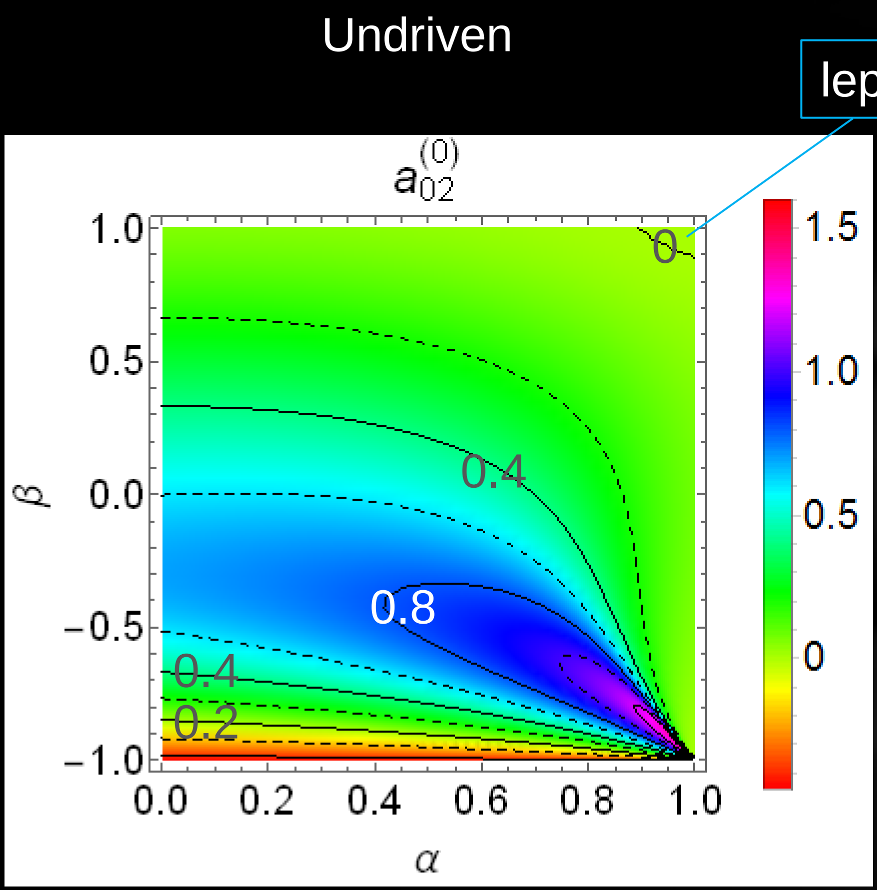


Different from pure smooth spheres

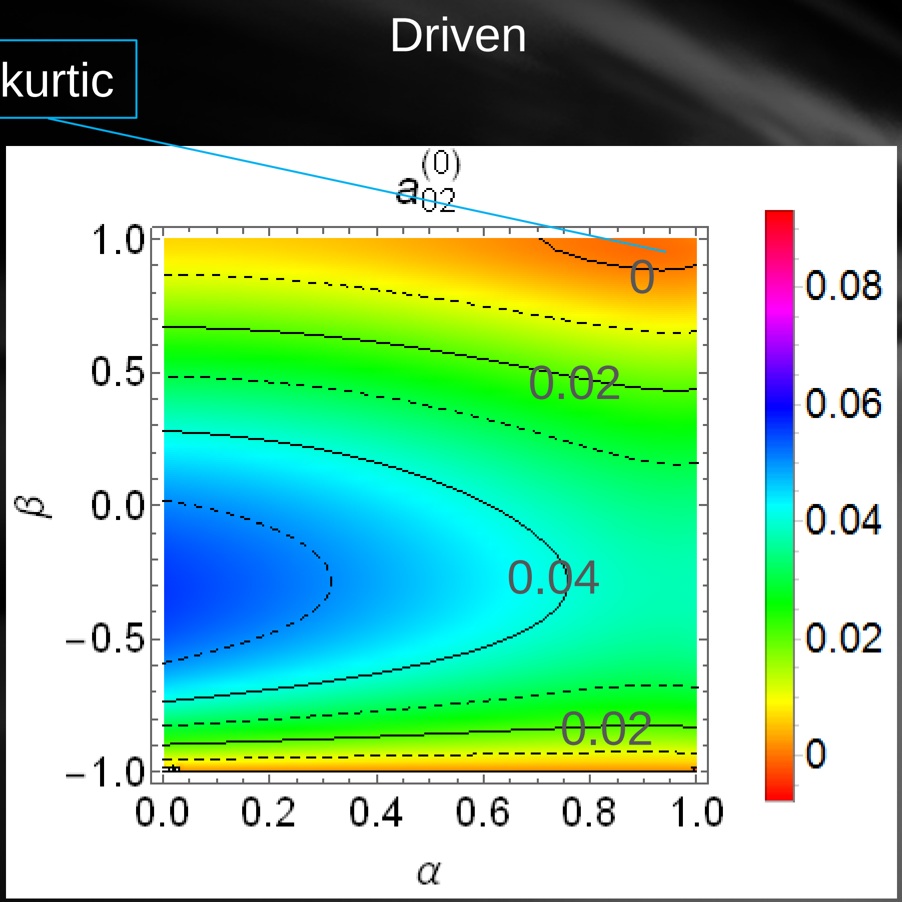
$$a_{20}^{(0)}\Big|_{\mathrm{HCS}} = 2\text{–}3 \text{ times } a_{20}^{(0)}\Big|_{\mathrm{WNT}}$$

Same as pure smooth spheres

rotational kurtosis: $\langle\omega^4\rangle = \frac{5}{3}\langle\omega^2\rangle^2\left(1 + a_{02}^{(0)}\right)$



leptokurtic



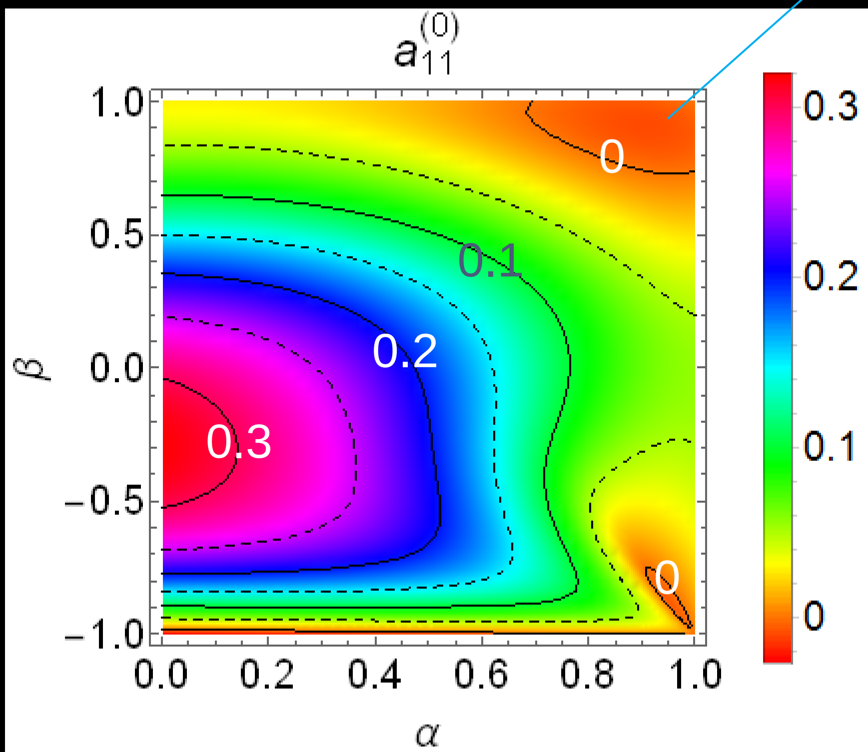
$$a_{02}^{(0)}\Big|_{\text{HCS}} = \text{10–20 times } \; a_{02}^{(0)}\Big|_{\text{WNT}}$$

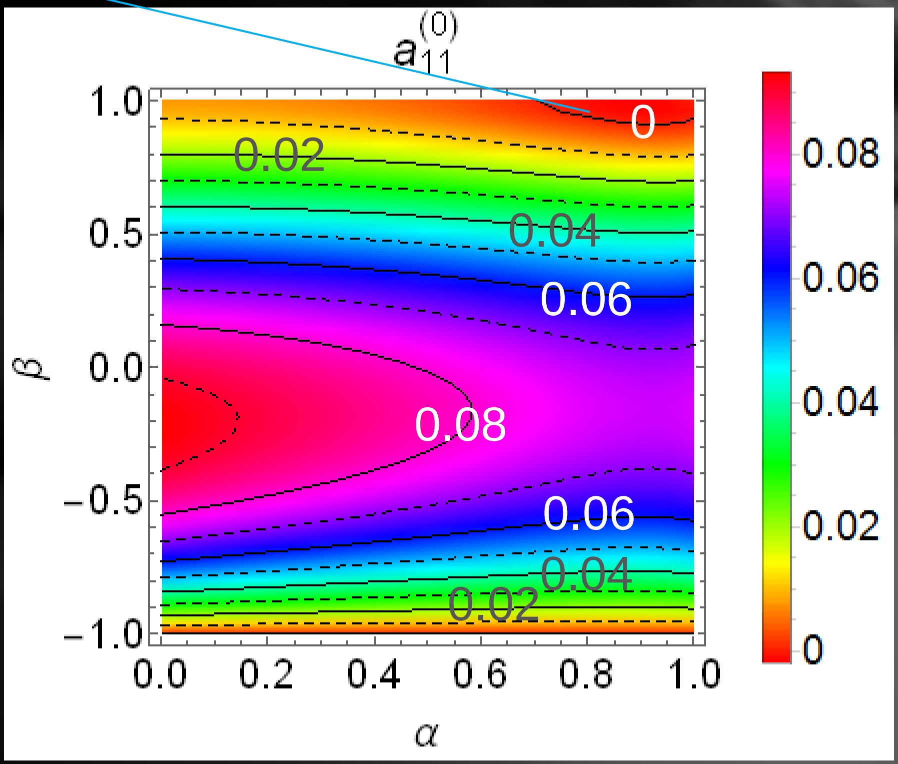
scalar correlations: $\langle v^2 \omega^2 \rangle = \langle v^2 \rangle \langle \omega^2 \rangle \left(1 + a_{11}^{(0)}\right)$

Undriven

$v \uparrow \Rightarrow \omega \downarrow$

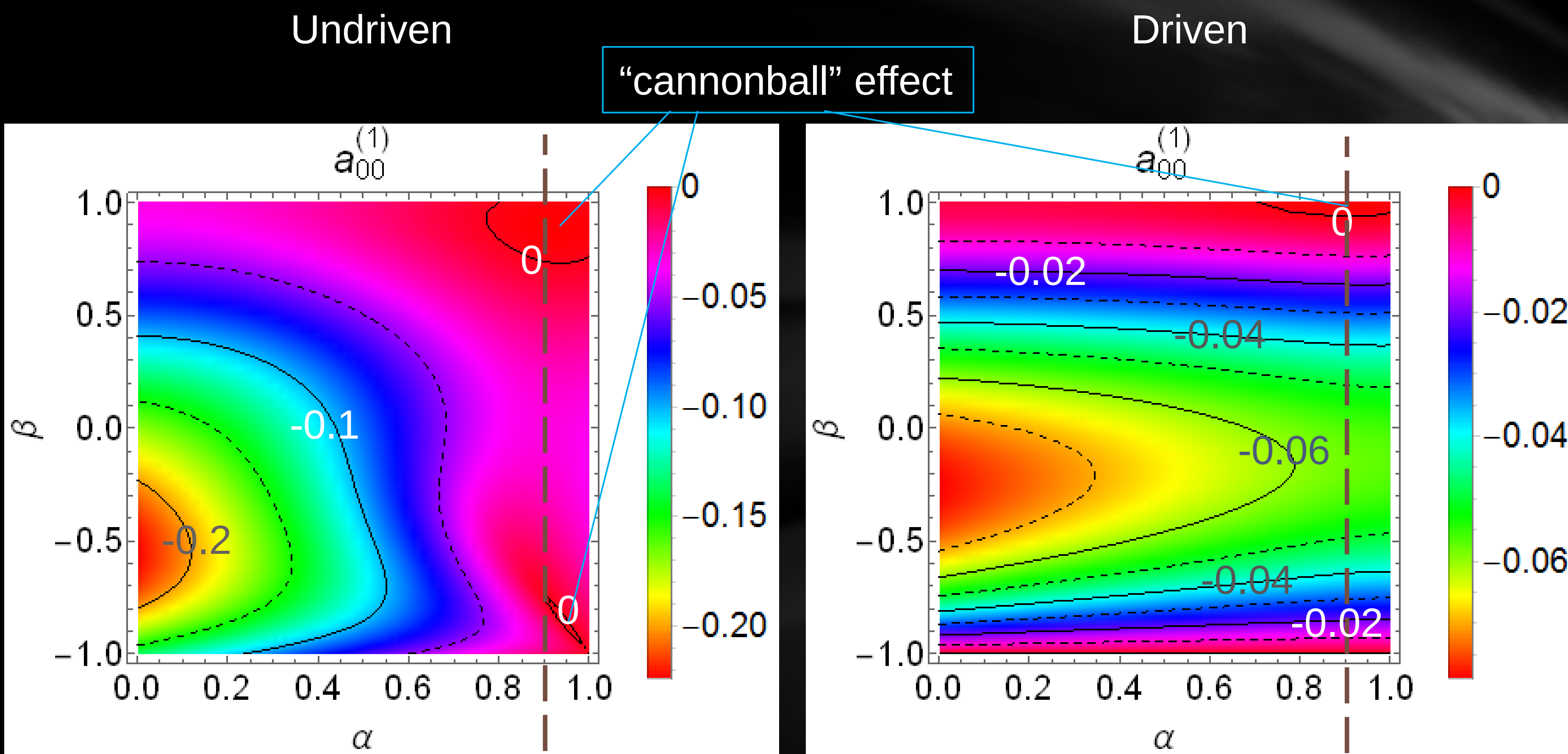
Driven





$$a_{11}^{(0)}\Big|_{\mathrm{HCS}} = 3\text{–}4 \text{ times } a_{11}^{(0)}\Big|_{\mathrm{WNT}}$$

angular correlations: $\langle(\mathbf{v}\cdot\boldsymbol{\omega})^2\rangle - \frac{1}{3}\langle v^2\omega^2\rangle \propto a_{00}^{(1)}$

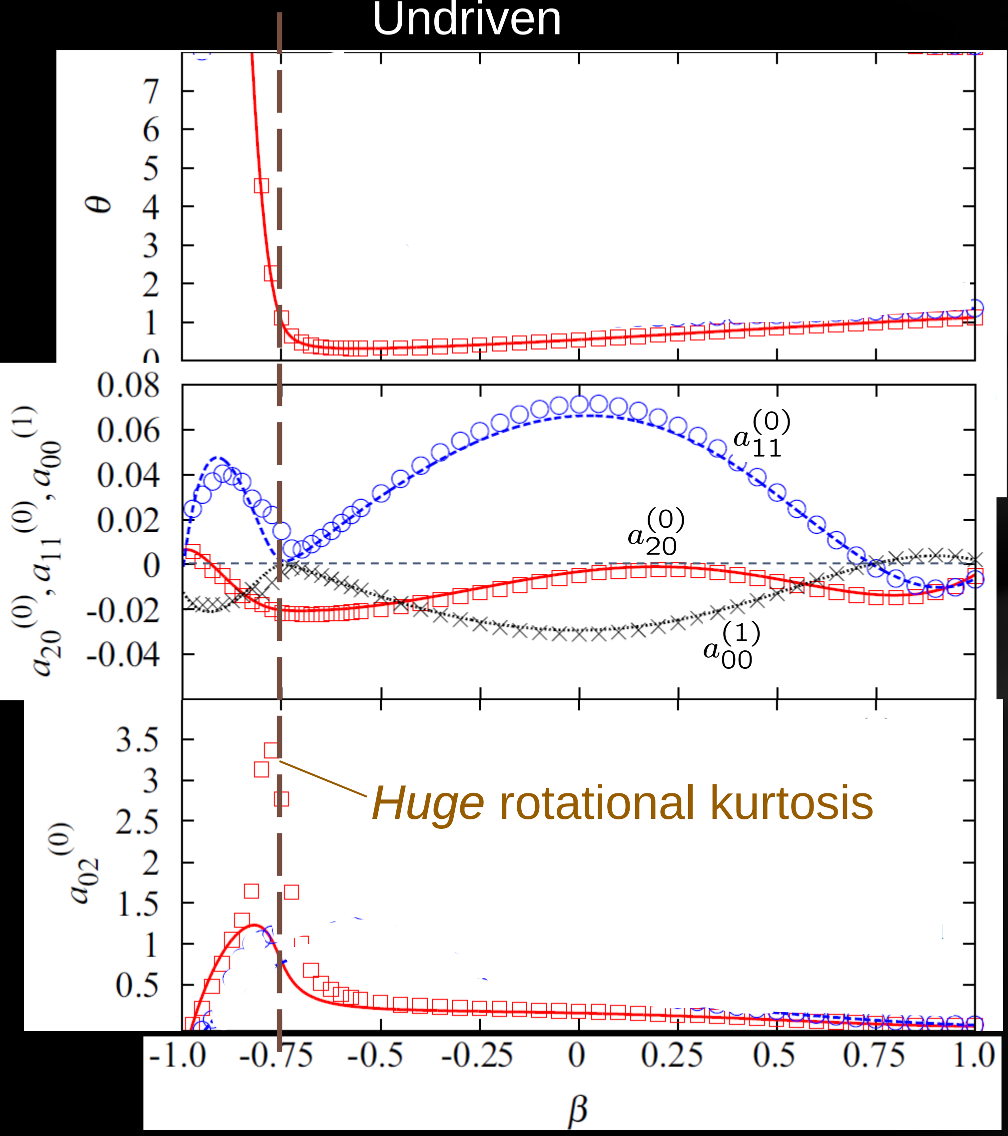


$$a_{00}^{(1)}\Big|_{\mathrm{HCS}} = 2\text{–}3 \text{ times } a_{00}^{(1)}\Big|_{\mathrm{WNT}}$$

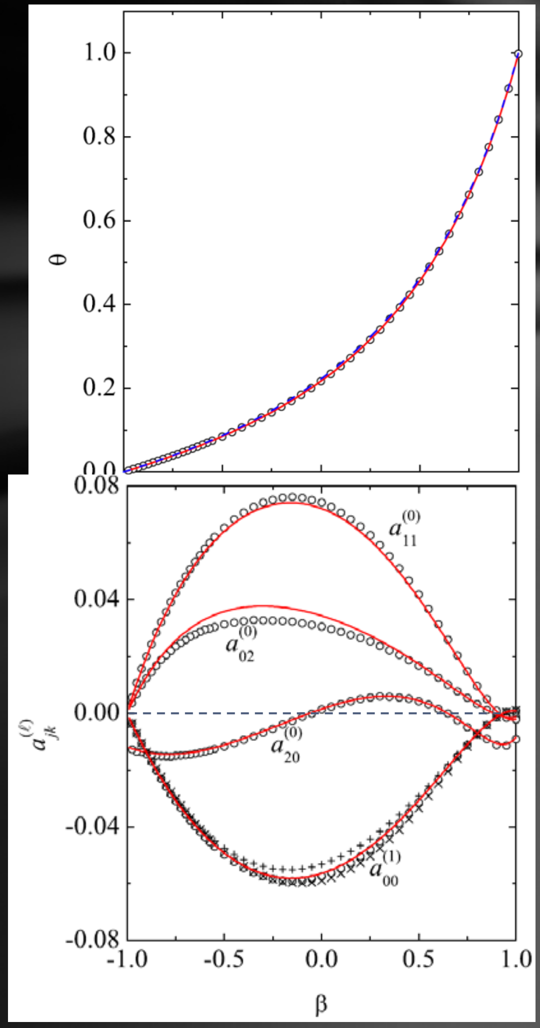
Comparison with simulations

α=0.9

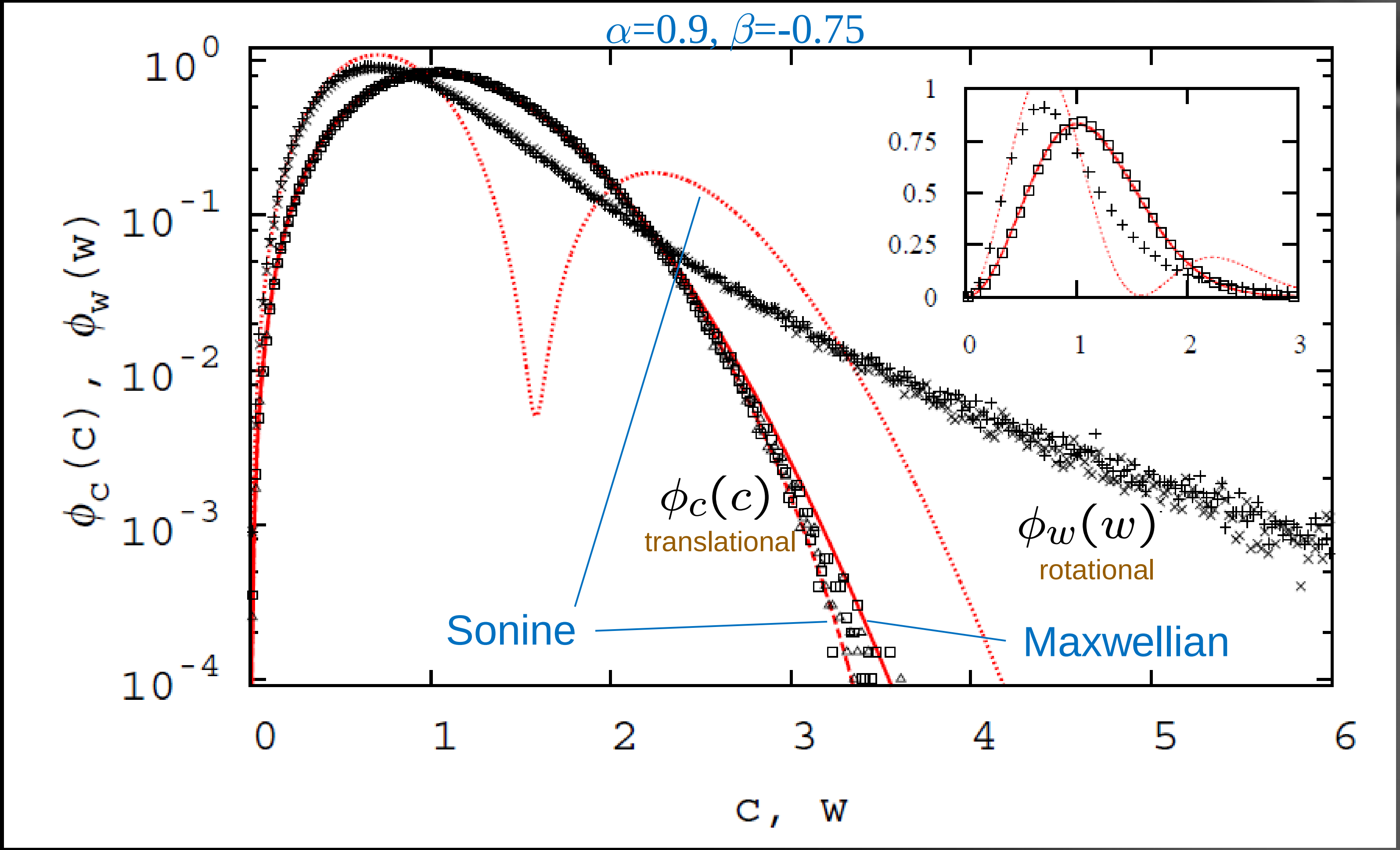
Undriven



Driven



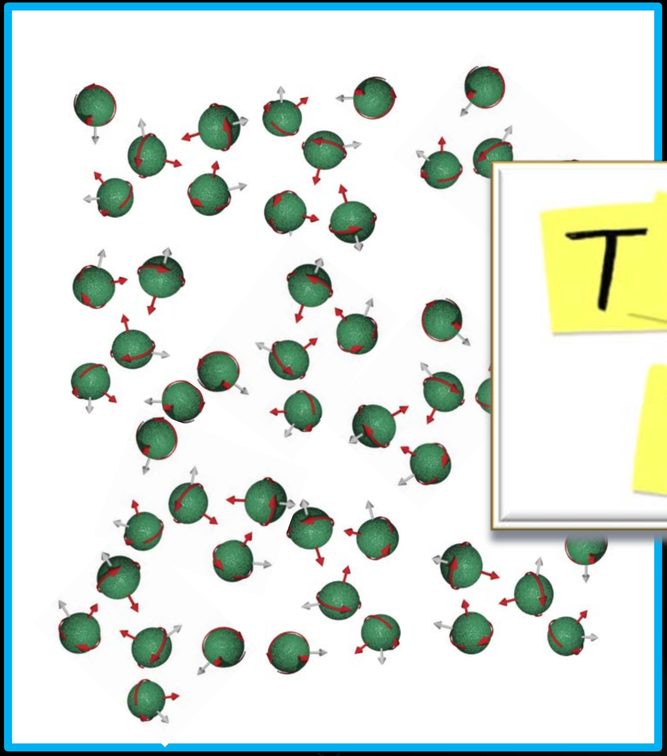
HCS: (Marginal) velocity distributions



THE TAKE-HOME MESSAGE

- Driving has a strong influence on the velocity distribution function of a granular gas of *rough* particles.
- The undriven system exhibits much higher deviations from equilibrium than the driven one.

Undriven
(Homogeneous cooling state, HCS)



Driven
(White-noise thermostat, WNT)